PRACTICAL SHOP MATHEMATICS

VOLUME I—ELEMENTARY

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PRACTICAL SHOP MATHEMATICS

VOLUME I—ELEMENTARY

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PREFACE TO THE THIRD EDITION

One of the two main changes in the third edition of this volume is the inclusion of more work on the theoretical side of trigonometry. Material on the fundamental relations between the trigonometric functions, expressing of one function in terms of each of the other functions, the variation of the functions with varying angles, and other important topics have been added. This will have two advantages. First, the more complete theoretical background will enable the student to apply more easily his knowledge of trigonometry to the solution of shop problems. Second, the student will be better prepared to continue with other work in mathematics.

The second principal change is the insertion of a chapter on the use of the slide rule. A brief discussion of the theory on which the slide rule is based is followed by a detailed explanation of how to use a slide rule for the simple mathematical processes. This material is finally summarized in several formulas that will enable the student quickly to carry out multiplication, division, squares, square roots, proportion, and various combinations of these that include those involving the use of trigonometric functions.

The authors have given this information on the slide rule with the object of saving the student hours of laborious computations. Actual shop problems usually require an accuracy to five significant figures, whereas the slide rule can be depended on for only three significant figures. However, in order to acquire more experience on the geometrical phases of the practice problems, instead of concentrating on the numerical results, the use of the slide rule is very valuable and a great time saver. Furthermore, a slide rule solution may be used as a quick check on any problem.

Other features of this text that have not been disturbed are

the geometrically wide range of shop problems, especially those involving the use of trigonometric functions. These problems have not been confined to the usual limited number of geometrical theorems. Quite the contrary, they fall into many classifications. The authors have carefully selected these shop problems, many of which are types that frequently recur, so that a student becoming thoroughly familiar with them will have little difficulty in forming a solution for any type of geometrical problem.

JOHN H. WOLFE
EVERETT R. PHELPS

PREFACE TO THE FIRST EDITION

For several years a course in shop mathematics has been taught under the guidance of John H. Wolfe at the Ford Apprentice School of the Ford Motor Company. The substance of this course was presented on loose-leaf printed sheets which were frequently revised in an effort to treat the material in the simplest and most understandable manner. The material in this book and in its continuation (Volume II) is the result of this careful revision and includes the work already developed in loose-leaf form with a great deal of new and important material which has never before been presented.

In writing this text, the authors have kept in mind its use not only in factory schools, trade schools, vocational high schools, etc., but also in all high schools to replace the usual geometry course for those students not intending to go to college. The geometry necessary for the solution of practical shop problems, together with the necessary work to give continuity, has been concisely presented by the authors in fifty propositions. These fifty propositions are proved in a formal manner in order that the training value of rigorous proofs may not be lost. The authors feel that the geometry as presented and the numerous practical problems which require a combined application of geometry and plane trigonometry are of much greater value to the high school student who is not going on to college than is the usual geometry course consisting of about one *hundred fifty theorems and the usual* more or less artificial and stereotyped exercises.

The value of this text in teaching the shop mathematics necessary to solve actual shop problems will be highly appreciated by anyone who has worked in this field. The exposition of the principles involved, the solution of many practical problems, and the presentation of hundreds of problems for the student to solve (many of which are accompanied by hints for the solution) teach the student the general methods of deriving solutions which can be applied to all shop problems. Mr. Wolfe's fourteen years of machine shop experience preceding his seventeen years of teaching shop mathematics

have given him the proper background to present the many practical problems which originated in the factory tool rooms, die rooms, and drafting rooms.

One of the features of this book is the use of what the authors call the "variable system." Instead of all dimensions of a problem being given, one has been omitted and its value represented by a letter called the variable. Adjacent to the problem, or immediately after each exercise, six or seven values for the variable are given, any of which may be used for the omitted dimension. Thus the instructor can, by using the six given values for the variable, assign separate problems of the same type to six students, each of whom will obtain a different answer. This helps greatly in preventing students from comparing work and answers. The student's own abilities are consequently developed to a much fuller extent. Of course, if the instructor does not care to make use of this method, he may assign the same value of the variable to all members of the class. Whenever seven values for the variable are given, the answer for the seventh one accompanies the figure.

The explanations in this text are presented very completely so that a student or mechanic can profitably use the book for home study or as a reference source. The solutions accompanying many of the problems are also a great help to such students.

The second volume of this text continues with the application of trigonometry and geometry to shop mathematics. It contains such subjects as solid trigonometry (commonly known as compound angles), the common types of gearing, screw threads, gear ratio and lead screw problems, continued fractions as applied to the cutting of leads and cams, and many special types of problems which occur in machine shop practice.

The authors wish to thank Mr. John W. Busman and Mr. William F. Mueller of the Ford Apprentice School Faculty for their assistance in proofreading.

John H. Wolfe
Everett R. Phelps

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USE OF THE VARIABLE SYSTEM

In all problems, with a very few exceptions, one number or dimension is represented by a letter which is called the variable. This letter or variable has six or seven different numerical values which may be substituted for it to complete the problem as stated. This makes six or seven similar problems, each of which has a different answer. These six or seven values of the variable are given in tabular form at the end of a group of problems. For problems which are stated diagrammatically, the six values of the variable are usually placed to the right of the diagram. A seventh value of the variable and the corresponding answer are usually placed directly under the diagram.

To illustrate the use of the variable system when the variables are given in tabular form, consider problem 1 of page 4 which reads: "Reduce to a mixed number $\frac{191}{A}$." On page 5 immediately following this group of problems is a table of variables which gives for A the values: 16, 18, 20, 22, 24, 26. Thus the $\frac{191}{A}$ of problem 1 becomes $\frac{191}{16}, \frac{191}{18}, \frac{191}{20}, \frac{191}{22}, \frac{191}{24}, \frac{191}{26}$. Each student is to work with only one of these values, that is, one student may work with $\frac{191}{16}$, another with $\frac{191}{18}$, etc.

To illustrate the use of the variable system when variables are given to the right of the diagram, consider problem 1 at the bottom of page 19. x is the dimension to be computed and A represents another dimension, six values of which are given at the right of the figure. Any one of these values of A may be used to complete the statement of the problem. Thus for one student the dimension A is 2.1, for another 2.5, etc.

Throughout the text, x, y, and z are used to represent unknown distances, and any other letter of the English alpha-

bet appearing in the problem is the variable. If the variable dimension is an angle, the Greek letter θ is generally used as the variable. Other Greek letters are used to represent the angular quantities to be computed.

A suggested plan for the use of the variable system in classrooms is presented in the chart below:

Name	No of variable for 1st set of problems	No. of variable for 2d set of problems	No. of variable for 3d set of problems
Brown, John	5	3	1
Collins, Ray.	3	1	5
Grant, Peter	1	4	2
Hale, George	2	6	4
Miller, Henry	6	5	3
Smith, Williams	4	2	6

The foregoing plan may be repeated for each group of six students.

The authors upon request will give further information regarding the use of the variable system.

PRACTICAL SHOP MATHEMATICS

CHAPTER I

COMMON FRACTIONS

DEFINITIONS

A **fraction** is a number expressing one or more of the equal parts of any whole quantity, as: $\frac{2}{7}$ bu., $\frac{5}{8}$ ft., $\frac{1}{2}$ mile.

The **terms** of a fraction are the denominator and numerator, which constitute a common fraction.

The **denominator** is the number below the line and shows the number of parts into which the whole is divided.

The **numerator** is the number above the line and shows how many parts are taken.

Example: $\frac{3}{4}$ of a foot shows that a foot has been divided into four equal parts, and three of the parts have been taken.

Common fractions are divided into the following classes: proper and improper fractions; mixed numbers; compound and complex fractions.

A **proper fraction** is one whose numerator is less than its denominator or whose value is less than unity, as $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{8}$.

An **improper fraction** is one whose numerator equals or exceeds its denominator and whose value is equal to or greater than unity, as $\frac{5}{5}$, $\frac{7}{3}$, $\frac{9}{4}$.

A **mixed number** is a number expressed by an integer and a fraction, as $2\frac{2}{3}$, $4\frac{3}{4}$, $5\frac{5}{8}$.

A **compound fraction** consists of the indicated products of two or more proper or improper fractions, as $\frac{5}{8} \times \frac{3}{7}$, $\frac{1}{4} \times \frac{2}{3} \times \frac{7}{5}$.

A **complex fraction** is one in which one or both of its terms is a fraction or mixed number, as:

$$\frac{\frac{3}{5}}{8}, \quad \frac{\frac{9}{2}}{7}, \quad \frac{5\frac{1}{2}}{2\frac{7}{11}}.$$

REDUCTION OF FRACTIONS

To reduce an improper fraction to a whole or mixed number, divide the numerator by the denominator. The quotient will be the whole number. If there is a remainder, it will be the numerator of the fractional part, while the denominator will be the same as the denominator of the improper fraction.

Example: Reduce $\frac{21}{4}$ to a mixed number.

Solution: 21 contains 4 five times with one remaining. Thus: $\frac{21}{4} = 5\frac{1}{4}$.

To reduce a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, add the numerator to this product, and place the denominator under the result.

Example: Reduce $8\frac{3}{4}$ to an improper fraction.

Solution: $8 \times 4 = 32$, $32 + 3 = 35$; this result written over the denominator is $\frac{35}{4}$.

To reduce a fraction to higher or lower terms, multiply or divide the numerator and denominator by the same number. This does not change the value of the fraction.

LEAST COMMON DENOMINATOR

A common denominator of a group of fractions is a number which contains each of the denominators a whole number of times.

The least common denominator (L.C.D.) of a group of fractions is the *least* number which contains each of the denominators a whole number of times.

To Find the L.C.D. of a Group of Fractions.—Rewrite the denominators in a column, neglecting those denominators which are contained by others a whole number of times. Separate the remaining denominators into their prime factors.[1]

[1] A prime number is a number which is divisible only by itself and one, as 2, 3, 5, 7, 11, 13, 17, etc. A prime factor is one of two or more prime numbers which when multiplied together produce a given product.

The L.C.D. is the product of the different prime factors each taken the greatest number of times that it occurs in any one of the expressions.

Example: Find the L.C.D. of $\frac{1}{9}$, $\frac{1}{8}$, $\frac{1}{24}$, $\frac{1}{18}$, $\frac{1}{10}$. Since 24 and 18 contain 8 and 9, respectively, 8 and 9 are neglected. The remaining denominators separated into prime factors:

$$\begin{cases} 24 = 2 \times 2 \times 2 \times 3. \\ 18 = 2 \times 3 \times 3. \\ 10 = 2 \times 5. \end{cases}$$

The greatest number of times that 2 occurs is three; the greatest number of times that 3 occurs is two; the greatest number of times that 5 occurs is one. Therefore, the L.C.D. is the product of 2 used as a factor three times, 3 twice, and 5 once, or $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

To reduce fractions to equivalent fractions having a L.C.D., divide the L.C.D. by the denominator of the fraction and multiply this quotient by the numerator of the fraction, then write this product as the numerator of the reduced fraction.

Example: Find the L.C.D. of $\frac{2}{3}$, $\frac{7}{9}$, $\frac{1}{2}$, $\frac{3}{4}$. By the foregoing method, the L.C.D. is equal to 36. $36 \div 3 = 12$, $12 \times 2 = 24$; therefore, $\frac{2}{3} = \frac{24}{36}$. Similarly $\frac{7}{9} = \frac{28}{36}$; $\frac{1}{2} = \frac{18}{36}$; $\frac{3}{4} = \frac{27}{36}$.

ADDITION OF FRACTIONS

Rule for Addition of Fractions.—*Reduce the fractions to equivalent fractions having a least common denominator, add their numerators, and write their sum over the common denominator.*

When fractions, mixed numbers, and whole numbers occur in addition of fractions, add the whole numbers and fractional parts separately and unite their sums. If the fractional part of the result is an improper fraction, it should be changed to a mixed number, the whole number part of which should be added to the rest of the whole numbers.

Example a: Add $\frac{2}{5}$, $\frac{4}{15}$, $\frac{4}{9}$, $\frac{3}{4}$. Reducing these to a L.C.D., $\frac{72}{180} + \frac{48}{180} + \frac{80}{180} + \frac{135}{180}$. Adding the numerators: $72 + 48 + 80 + 135 = 335$. Therefore the sum is $\frac{335}{180}$. Reducing to a mixed number in lowest terms, the sum is $1\frac{31}{36}$.

The sum is the result of addition of two or more quantities.

The **difference** is the result of subtraction of two quantities.

Example b: Add $4\frac{3}{8}$, $2\frac{5}{6}$, $5\frac{1}{2}$, $1\frac{1}{4}$.

Reducing the fractional parts to a common denominator, which is 24:

$$\frac{3}{8} = \frac{9}{24}, \qquad \frac{5}{6} = \frac{20}{24}, \qquad \frac{1}{2} = \frac{12}{24}, \qquad \frac{1}{4} = \frac{6}{24}.$$

Adding the numerators: $9 + 20 + 12 + 6 = 47$.

Then the sum of the fractional parts is $\frac{47}{24}$ or $1\frac{23}{24}$.

Adding all of the whole numbers: $1 + 4 + 2 + 5 + 1 = 13$.

Uniting the whole number and fractional part results in a total of $13\frac{23}{24}$.

PROBLEMS

Reduce to a mixed number, expressing the fractional part in its lowest terms:

1. $\frac{191}{A}$. **2.** $\frac{B}{42}$. **3.** $\frac{835}{C}$. **4.** $\frac{D}{29}$.

Reduce to an improper fraction:

5. $5\frac{7}{E}$. **6.** $F\frac{6}{7}$. **7.** $13\frac{13}{G}$. **8.** $15\frac{H}{35}$.

9. Determine the least common denominator of: $\frac{2}{J}$, $\frac{5}{34}$, $\frac{7}{8}$, $\frac{3}{24}$, and $\frac{8}{12}$.

10. Determine the least common denominator of: $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{K}$, and $\frac{1}{3}$.

11. Reduce the following fractions to 72nds: $\frac{L}{36}$, $\frac{L}{12}$, $\frac{L}{8}$, $\frac{L}{9}$, and $\frac{L}{4}$.

12. Determine the sum of the following fractions: $\frac{5}{7}$, $\frac{M}{21}$, $\frac{20}{63}$, and $\frac{10}{21}$.

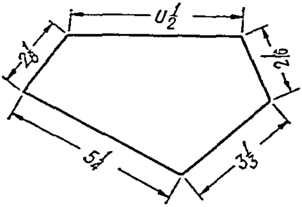
13. Reduce to the lowest terms: $\frac{36}{N}$.

14. How many thirds in P?

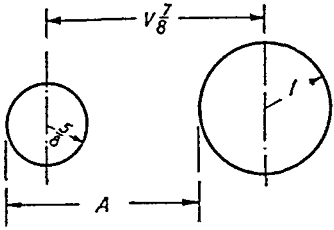
15. How many sixths are in the sum of: $5\frac{5}{6} + 6\frac{1}{6} + R\frac{2}{6} + 8\frac{7}{6}$?

16. Determine the sum of: $S\frac{4}{5} + 3\frac{9}{10} + 1\frac{2}{3} + \frac{19}{20}$.

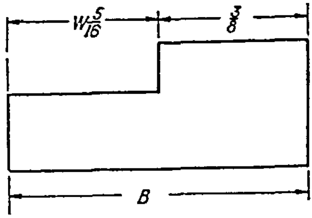
17. Determine the sum of: $3\frac{9}{70} + 2\frac{13}{35} + T\frac{1}{7} + 10\frac{1}{10}$.



18. Determine the distance around the polygon.



19. Determine the distance A.



20. Determine the distance B.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	A	16	18	20	22	24	26
2	B	1511	1531	1551	1571	1591	1611
3	C	10	12	14	16	18	20
4	D	6608	6638	6668	6698	6728	6758
5	E	8	10	12	14	16	18
6	F	49	52	55	58	61	64
7	G	91	88	85	82	79	76
8	H	8	9	10	11	12	13
9	J	16	15	14	13	12	11
10	K	10	11	12	13	14	15
11	L	2	3	5	6	7	10
12	M	14	13	12	11	10	9
13	N	42	44	46	48	50	52
14	P	16	15	14	13	12	11
15	R	9	11	13	15	17	19
16	S	44	42	40	38	36	34
17	T	12	14	16	18	20	22
18	U	1	2	3	4	5	6
19	V	14	13	12	11	10	9
20	W	1	2	3	4	5	6

SUBTRACTION OF FRACTIONS

Rule for Subtraction of Fractions.—*Reduce the fractions to equivalent fractions having a least common denominator. Subtract the numerator preceded by the negative sign from the numerator preceded by the positive sign and place this difference over the common denominator.*

The first term in any expression is understood to be a plus quantity, unless otherwise specified.

Example: $\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8} = \frac{1}{8}$.

To subtract a mixed number from a mixed number, subtract the whole numbers and fractional parts separately and unite these results.

Example: $3\frac{1}{2} - 1\frac{1}{4}$.

Solution: $\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$; $3 - 1 = 2$.

Answer $= 2 + \frac{1}{4} = 2\frac{1}{4}$.

Sometimes in subtracting a mixed number from a mixed number, the fractional part in the subtrahend (the part to be subtracted) is greater than the fractional part in the minuend (the part that is subtracted from), and in this case it becomes evident that one unit must be borrowed from the whole number in the minuend and added to its fractional part.

Example: $4\frac{2}{7} - 2\frac{5}{8}$.

Solution: $\frac{2}{7} - \frac{5}{8} = \frac{16}{56} - \frac{35}{56}$; 35 cannot be subtracted from 16. Therefore one unit ($\frac{56}{56}$) is borrowed from the whole number of the minuend. This added to $\frac{16}{56}$ will be $\frac{72}{56}$. Then $\frac{72}{56} - \frac{35}{56} = \frac{37}{56}$. Next subtract the whole numbers: $3 - 2 = 1$. This united with the fractional part gives the final result, or $1\frac{37}{56}$.

To subtract a fraction from a whole number, borrow one unit from the whole number and express it as a fraction having the same denominator as the fraction to be subtracted. Subtract the fractional parts and annex this remainder to the remaining whole number.

Example: $7 - \frac{2}{5} = 6\frac{5}{5} - \frac{2}{5} = 6\frac{3}{5}$.

After addition or subtraction has been performed, it is customary to reduce the final fraction to its lowest terms.

PROBLEMS

1. What is the value of $\frac{21}{31}$ less $\frac{2}{A}$?
2. What is the value of $\frac{4}{C}$ less $\frac{2}{13}$?
3. What is the difference between $5\frac{1}{3}$ and $3\frac{23}{E}$?
4. What is the difference between $7\frac{2}{F}$ and $5\frac{1}{2}$?
5. Subtract $\frac{2}{3}$ from $3\frac{3}{G}$.

6. $2\frac{5}{8} - \frac{1}{K} - \frac{2}{3} - \frac{1}{8}$ is equal to what fraction in its reduced form?

7. $5\frac{1}{2} - 1\frac{2}{L} - 2\frac{1}{8} - \frac{2}{7}$ is equal to what fraction in its reduced form?

8. $7 - \frac{7}{8} - \frac{1}{10} - \frac{2}{M} - \frac{3}{11}$ is equal to what fraction in its reduced form?

9. $3\frac{1}{8} - \frac{6}{7} - \frac{5}{8} - \frac{3}{N} - \frac{1}{4}$ is equal to what fraction in its reduced form?

10. If $\frac{3}{R}$ is subtracted from a whole quantity, what part of the quantity remains?

11. If $\frac{3}{8}$ is subtracted from a whole quantity, and later $\frac{2}{S}$ is subtracted, what part of the quantity remains?

12. If $\frac{2}{T}$ is subtracted from a quantity, and later $\frac{3}{8}$ is subtracted, what part of the whole quantity remains?

13. Which fraction is the greater in value: $\frac{21}{U}$ or $\frac{15}{67}$?

14. Subtract $\frac{5}{V}$ from $2\frac{2}{3}$ and from the difference take away $\frac{2}{7}$.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	A	13	14	15	16	17	18
2	C	10	9	8	7	6	5
3	E	25	26	27	28	29	30
4	F	8	7	6	5	4	3
5	G	10	11	12	13	14	15
6	K	3	4	5	6	7	8
7	L	15	14	13	12	11	10
8	M	8	9	10	11	12	13
9	N	21	18	15	12	9	6
10	R	14	15	16	17	18	19
11	S	13	12	11	10	9	8
12	T	15	16	17	18	19	20
13	U	36	35	34	33	32	31
14	V	12	13	14	15	16	17

ADDITION AND SUBTRACTION OF FRACTIONS

In any expression, where the plus and minus signs both occur, it is customary to add all of the plus quantities first,

next to add all of the minus quantities, and then to subtract the sum of the minus quantities from the sum of the plus quantities.

Example: $\frac{2}{5} - \frac{7}{8} + \frac{3}{4} - \frac{2}{3} + \frac{1}{2} = ?$

Solution: Reducing the fraction to a common denominator:

$$\frac{48}{120} - \frac{105}{120} + \frac{90}{120} - \frac{80}{120} + \frac{60}{120}.$$

Adding all the plus quantities: $\frac{48}{120} + \frac{90}{120} + \frac{60}{120} = \frac{198}{120}$.

Adding all the minus quantities: $\frac{105}{120} + \frac{80}{120} = \frac{185}{120}$.

Subtracting the sum of the minus quantities from the sum of the plus quantities: $\frac{198}{120} - \frac{185}{120} = \frac{13}{120}$.

PROBLEMS

1. From the sum of $\frac{7}{9}$ and $\frac{5}{6}$ subtract the sum of $\frac{1}{3}$ and $\frac{2}{D}$.

2. From the sum of $4\frac{1}{2}$ and $6\frac{3}{5}$ subtract the sum of $2\frac{1}{E}$ and $3\frac{3}{4}$.

3. Add $5\frac{2}{9}$ and $6\frac{7}{F}$ and from the sum take away $4\frac{8}{9}$.

4. A man did $\frac{5}{G}$ of his work one day and $\frac{1}{3}$ of it the next. (*a*) What part of his work did he finish? (*b*) What part of his work was unfinished?

5. A truck drew $5\frac{3}{7}$ and $3\frac{5}{8}$ tons of pig iron on two successive days; another truck drew $6\frac{1}{2}$ and $7\frac{3}{H}$ tons on the same days. How many more tons did the latter draw than the former?

Simplify the following expressions by performing the operations indicated:

6. $5\frac{2}{5} - 7\frac{1}{8} + 4\frac{4}{7} + 7\frac{3}{4} - 4\frac{5}{J} = ?$

7. $\frac{7}{8} + \frac{6}{7} - \frac{7}{9} + \frac{8}{K} + \frac{3}{4} = ?$

8. $\frac{3}{4} - \frac{7}{8} - \frac{8}{L} + \frac{9}{10} + \frac{1}{3} = ?$

9. $6\frac{1}{2} - 7\frac{1}{3} + 8\frac{1}{3} + 5\frac{1}{2} - 4\frac{1}{M} - 3\frac{1}{2} = ?$

10. $\frac{6}{7} - \frac{4}{5} + \frac{3}{8} - \frac{8}{56} + \frac{5}{7} - \frac{7}{N} = ?$

11. If a man works $15\frac{1}{2}$ hr. on Monday, $10\frac{1}{3}$ hr. on Tuesday, $9\frac{3}{4}$ hr. on Wednesday, $7\frac{5}{6}$ hr. on Thursday, $11\frac{5}{P}$ hr. on Friday, and $8\frac{2}{3}$ hr. on Saturday, how many hours does he work during the week?

12. What is the perimeter (distance around) of a triangular piece, the sides of which measure $28\frac{2}{3}$, $45\frac{2}{R}$, and $67\frac{20}{21}$ in., respectively?

13. What is the perimeter of an irregular polygon, the sides of which measure $7\frac{1}{2}$, $5\frac{2}{3}$, $8\frac{3}{S}$, $2\frac{6}{7}$, and $2\frac{1}{3}$ in., respectively?

14. The perimeter of a four-sided irregular polygon is $24\frac{2}{3}$, three sides of which are $3\frac{2}{T}$, $5\frac{5}{7}$, and $4\frac{7}{8}$ in., respectively. Determine the length of the fourth side.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	D	4	5	6	7	8	9
2	E	13	12	11	10	9	8
3	F	14	15	16	17	18	19
4	G	13	12	11	10	9	8
5	H	8	9	10	11	12	13
6	J	20	19	18	17	16	15
7	K	10	12	14	16	18	20
8	L	34	32	30	28	26	24
9	M	3	4	5	6	7	8
10	N	19	18	17	16	15	14
11	P	10	11	12	13	14	15
12	R	5	6	7	8	9	10
13	S	15	14	13	12	11	10
14	T	21	22	23	24	25	26

MULTIPLICATION OF FRACTIONS

When multiplying fractions, do *not* reduce the fractions to a common denominator. Fractions should be reduced to a common denominator *only* in addition and subtraction.

In the multiplication of fractions, multiply together the numerators for the numerator of the product, and the denominators for the denominator of the product.

Examples: $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$; $\frac{3}{4} \times \frac{5}{8} = \frac{15}{32}$.

CANCELLATION

Cancellation, which is used only in the multiplication of fractions, is the process of dividing a numerator and a denominator of an expression by a common factor. Should a plus or minus symbol occur in the numerator or in the denominator

DIVISION OF FRACTIONS

In division of fractions the **dividend** is that number or fraction which is divided by some other number or fraction.

The **divisor** is that number or fraction which the dividend is divided by, *i.e.*, the **divisor** is the number or fraction which follows the division symbol (÷).

The **quotient** is the result obtained by dividing the dividend by the divisor.

RECIPROCALS

The **reciprocal** of a number is 1 divided by that number. Thus the reciprocal of 8 is $\frac{1}{8}$.

The reciprocal of a fraction is 1 divided by the fraction.

Example: The reciprocal of $\frac{5}{8}$ is $\frac{1}{\frac{5}{8}}$ or $\frac{1 \times 8}{\frac{5}{8} \times 8}$ or $\frac{8}{5}$.

From the foregoing it can be seen that the reciprocal of a fraction is the fraction inverted.

The reciprocal of a mixed number is the mixed number reduced to an improper fraction and then inverted.

Example: The reciprocal of $3\frac{2}{5}$ is $\frac{1}{\frac{17}{5}}$ or $\frac{5}{17}$.

Instead of dividing by a number or a fraction, one can multiply by the reciprocal of the number or fraction and get the same result. Therefore, in division of fractions, invert the divisor and multiply.

	Dividend		Divisor		Quotient
Example a:	$\frac{5}{8}$	$\div$	$\frac{3}{4}$	$=$	$\frac{5}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{3} = \frac{5}{6}$.

Example b: $\frac{5}{7} \div \frac{3}{5} \div \frac{4}{9} \div \frac{5}{14} = \frac{\cancel{5}}{\cancel{7}} \times \frac{5}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{2}{\cancel{14}}}{\cancel{5}} = 7\frac{1}{2}$.

PROBLEMS

1. Divide the product of 6, 9, 10 by the product of D, 3, 5.
2. Divide the product of E, 9, 12 by the product of 6, 8, 21.

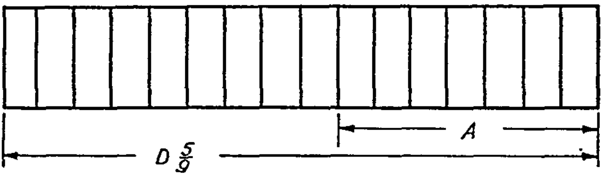
3. $\frac{F}{8} \div \frac{3}{5} = ?$ 4. $\frac{6}{7} \div \frac{J}{5} = ?$ 5. $\frac{5}{8} \div \frac{2}{15} \div \frac{5}{24} \div \frac{45}{M} \div \frac{94}{100} = ?$

6. $\frac{3}{7} \div \frac{2}{3} \div \frac{5}{N} \div \frac{45}{21} \div \frac{5}{6} = ?$ 7. $2\frac{1}{2} \div 3\frac{2}{3} \div 5\frac{6}{P} \div 4\frac{2}{5} \div 2\frac{6}{7} = ?$

8. $\frac{7}{12}$ of the distance from A to B is $R\frac{2}{3}$ in. What is the distance from A to B?

9. If a man chops $1\frac{1}{6}$ cords of wood a day, in what time can he chop $S\frac{4}{5}$ cords?

10. If $\frac{3}{4}$ of a ton of coal costs \$5, how much will $T\frac{2}{3}$ tons cost?



11. Determine the distance A.

12. $\frac{1}{7}$ is $\frac{5}{E}$ of what number? 13. $\frac{3}{7}$ is $\frac{1}{G}$ of what number?

14. $\frac{5}{8}$ is how many times greater than $\frac{1}{J}$?

15. $\frac{6}{7}$ is how many times greater than $\frac{1}{K}$?

16. $\frac{2}{L}$ is what part of $\frac{5}{9}$? 17. $\frac{5}{8} \div \frac{4}{7} \div \frac{2}{5} \div N \div 2 \div \frac{3}{4} \div \frac{1}{8} = ?$

18. $\frac{4}{7} \div 2\frac{3}{5} \div \frac{7}{9} \div 3\frac{2}{3} \div \frac{3}{P} \div \frac{9}{14} = ?$

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	D	11	12	13	14	15	16
2	E	8	7	6	5	4	3
3	F	3	4	5	6	7	8
4	J	7	8	9	10	11	12
5	M	46	48	50	52	54	56
6	N	8	12	16	20	24	28
7	P	18	17	16	15	14	13
8	R	62	60	58	56	54	52
9	S	5	6	7	8	9	10
10	T	19	18	17	16	15	14
11	D	20	22	24	26	28	30
12	E	6	7	8	9	10	11
13	G	2	3	4	5	6	7
14	J	3	4	5	6	7	8
15	K	14	13	12	11	10	9
16	L	3	4	5	6	7	8
17	N	34	32	30	28	26	24
18	P	21	22	23	24	25	26

CHAPTER II

CHECKING MULTIPLICATION AND DIVISION BY THE EXCESS OF NINES

In all branches of mathematics, accuracy is of great importance and therefore one must have some convenient method of checking multiplication and division. A simple method of checking multiplication and division is by a process involving the excess of nines. The excess of nines is the remainder of a number which has been divided by nine.

Example: 38 divided by 9 has 2 as a remainder which is the excess of nines in 38. The excess of nines can also be found by dividing the *sum* of the digits by nine; then the remainder becomes the excess of nines. The excess of nines can be found with greater ease by eliminating the nines as soon as the sum of the digits is equal to, or immediately after it exceeds, nine. The excess of nines of any two digits whose sum is greater than nine is always one more than the last digit of the sum.

Examples: The excess of nines of 10 is $0 + 1 = 1$; the excess of nines of 14 is $4 + 1 = 5$, etc.

Example: Find the excess of nines of 74,685.

Solution: This carried out in detail form is as follows: Begin eliminating nines from left to right: $7 + 4 = 11$ where 2 is the excess of nines. Add this excess of nines to the next digit on the right, $2 + 6 = 8$. In cases like this, keep on adding the successive digits to the right until the sum equals or exceeds nine. Then $8 + 8 = 16$ where 7 is the excess of nines. Add this excess of nines to the next digit on the right and continue this process until all of the digits have been considered. Thus, $7 + 5 = 12$ where the excess of nines is 3. This final excess of nines is called the excess of nines of 74,685.

In order to determine the excess of nines with the greatest ease, the grouping method is recommended. The eye should

be trained to recognize groups of two or three figures whose sums are nine, such as (1 and 8), (2 and 7), (3 and 6), (4 and 5), (2, 3, and 4), (5, 6, and 7). In order that the eye may be able to recognize the foregoing groups quickly, they should be memorized. These groups of digits need not be considered when finding the excess of nines by the foregoing process.

Example: Find the excess of nines of 5,762,382 by the grouping method.

Solution: At a glance it can be seen that 7 and 2, and 6 and 3, are each equal to nine. These groups should be neglected immediately. The remaining figures to be considered are 5, 8, and 2. Since the value of 8 is so close to that of nine it is best to take one from the 5 which completes another group of nine, leaving only 4 and 2 to be considered. 4 and 2 are 6, which is the excess of nines of the number 5,762,382. The following numbers are a few examples of grouping. The different groups will be indicated by the connecting lines: (6,768,654), (75,634), (68,423), (467,523).

To check multiplication by the excess of nines, multiply the excess of nines in the multiplicand by the excess of nines in the multiplier. The excess of nines in the product must equal the product of the excess of nines of the two original numbers. The check of multiplication is more accurate when applied at the completion of each step. To do this, multiply the excess of nines in the multiplicand by the partial multiplier; the product must equal the excess of nines in the product represented by these two quantities. For example, in multiplying 54,876 by 2 the excess of nines are 3 and 2, respectively, and the product 109,752 gives an excess of 6 (which equals 3×2). Checking in this manner, step by step, will make the work much more accurate and will enable the student to find an error immediately.

Illustrative Problem:

To check division by the excess of nines, proceed as follows: Multiply the excess of nines in the quotient by the excess of nines in the divisor. Add to this product the excess of nines in the remainder. The result must be equal to the excess of

```
                 54876................. 3
                 87542................. 8
6.........109752..2 × 3 = 6            24 or 6
3........219504 ..4 × 3 = 12 or 3
6.......274380  ..5 × 3 = 15 or 6
3......384132   ..7 × 3 = 21 or 3
6.....439008    ..8 × 3 = 24 or 6
     4803954792.................       6
```

nines in the dividend. The check of division can also be made more accurate by applying the check to the multiplication at the completion of each step as shown in the illustrative problem.

Illustrative Problem:

```
                    7.5638.........2
4.  ...78646/594870...................  6
1.  ...      ....550522......7 × 4 = 28 or 1
                 443480
2.     .  .. ..393230.....5 × 4 = 20 or 2
                  502500
6.  ...          .471876....6 × 4 = 24 or 6
                   306240
3.              ...235938...3 × 4 = 12 or 3
                    703020
5. .             ...629168..8 × 4 = 32 or 5
7.    .  .       . ..73852.. remainder
```

The final check is $2 \times 4 = 8$, $8 + 7 = 15$, whose excess of nines is 6. This 6 is equal to the 6 which represents the excess of nines in the dividend, and, since these two quantities are equal, the check is complete and indicates that the quotient is correct.

This check by the excess of nines when applied at the completion of each step was found by experience to be 99.9% accurate.

PROBLEMS

What is the excess of nines of:

1. *D*. 2. *E*. 3. *F*. 4. *G*. 5. *H*.

6. Multiply 16,425 by *J* and check each step by the excess of nines. What is the excess of nines of the sum of the excess of nines of each of the results obtained by multiplying the multiplicand by the first four digits (*starting from left to right*) of the multiplier?

7. Multiply *K* by 4295 and check each step by the excess of nines. What is the excess of nines of the sum of the excess of nines of each of the results obtained by multiplying the multiplicand by the four digits of the multiplier?

8. Divide 438,569 by *L* and check the final result and each step of multiplication by the excess of nines. What is the excess of nines in the remainder? Quotient to consist of five significant figures.

9. Divide *M* by 5783 and check the final result and each step of multiplication by the excess of nines. What is the excess of nines in the remainder? Quotient to consist of five significant figures.

Variables

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	*D*	587634	238674	396872	457635	843926	537462
2	*E*	237674	568943	487632	896543	487542	865437
3	*F*	395827	456843	785326	956472	234589	456787
4	*G*	475652	589542	678762	324578	567823	235678
5	*H*	894263	754326	821254	756234	724235	678756
6	*J*	24345	45678	56278	45673	54678	45638
7	*K*	5892	6785	7368	9375	8537	6278
8	*L*	34567	75623	65892	54237	68752	87542
9	*M*	6782	7368	9375	8537	7564	6785

CHAPTER III

DECIMALS

A **decimal fraction** is a fraction whose denominator is 10 or some multiple of 10. The denominator of a simple decimal fraction is always omitted but is expressed by a dot called the decimal point, placed in different positions of a number corresponding to the magnitude of the denominator. One figure to the right of the decimal point indicates that the denominator is 10; two figures to the right of the decimal point indicates that the denominator is 100; three figures to the right of the decimal point indicates that the denominator is 1000; etc.

A decimal number, or **decimal**, is a number involving a decimal fraction. Thus: .237 or 6.346.

The nomenclature of the decimal system is as follows:

thousands	hundreds	tens	units		tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
3	5	7	9	.	5	8	3	4	7	2

The decimal quantity is always read from left to right, annexing the name corresponding to the last decimal figure.

Example: The figure 35.6437 is read thirty-five and six thousand, four hundred thirty-seven ten-thousandths.

To change a decimal to a common fraction: The numerator will be the same as the original figure omitting the decimal point; the denominator will always be one followed by as many

ciphers as there are figures to the right of the decimal point.

Examples: $.73 = \frac{73}{100}$; $5.496 = \frac{5496}{1000}$.

PROBLEMS

Change the following decimal numbers to common fractions:

1. *A*. 2. *B*. 3. *C*.

Change the following common fractions to decimal numbers:

4. *D*. 5. *E*. 6. *F*.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	*A*	.0327	.0433	.0569	.0671	.0723	.0837
2	*B*	2.427	3.567	4.287	5.367	6.487	8.367
3	*C*	.0037	.0033	.0039	.0041	.0043	.0047
4	*D*	$\frac{235}{1000}$	$\frac{356}{1000}$	$\frac{467}{1000}$	$\frac{697}{1000}$	$\frac{752}{1000}$	$\frac{689}{1000}$
5	*E*	$\frac{41}{10000}$	$\frac{51}{10000}$	$\frac{61}{10000}$	$\frac{71}{10000}$	$\frac{81}{10000}$	$\frac{91}{10000}$
6	*F*	$\frac{23}{100}$	$\frac{48}{100}$	$\frac{56}{100}$	$\frac{67}{100}$	$\frac{73}{100}$	$\frac{84}{100}$

ADDITION AND SUBTRACTION OF DECIMALS

Since it is necessary to have a common denominator when adding or subtracting common fractions, and since decimal fractions are only a modified form of common fractions, it becomes evident that to add or subtract decimals the decimal points must be placed in a column directly under each other.

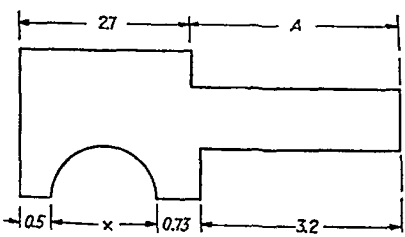
Examples: Add

$$\begin{array}{r} 2.6875 \\ .0789 \\ 35.3000 \\ 6.4789 \\ \hline \text{Sum} = 44.5453 \end{array}$$

Subtract

$$\begin{array}{r} 7.6300 \\ 2.1682 \\ \hline \text{Remainder} = 5.4618 \end{array}$$

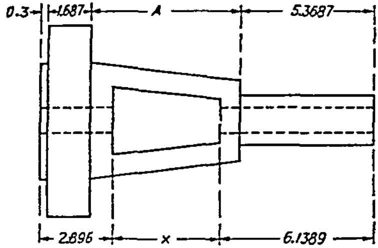
PROBLEMS



VARIABLE

No.	Sym.	Value
1	*A*	2.1
2	*A*	2.5
3	*A*	2.8
4	*A*	2.9
5	*A*	2.3
6	*A*	2.7

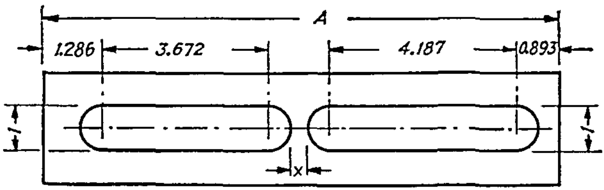
1. Determine the distance x.



VARIABLE

No.	Sym.	Value
1	A	6.8954
2	A	6.9736
3	A	7.0157
4	A	7.1893
5	A	7.3458
6	A	7.4691

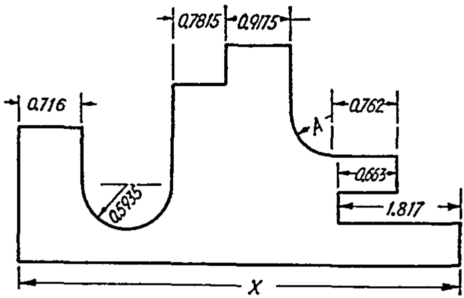
2. Determine the distance x.



VARIABLE

1. A = 11 503	2. A = 11 548	3. A = 11.586
4. A = 11 609	5. A = 11.637	6. A = 11.652

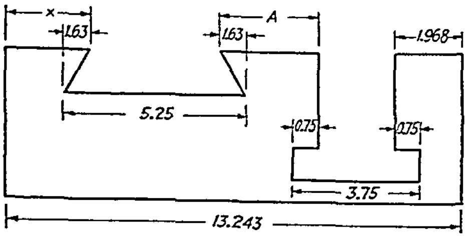
3. Determine the distance x.



VARIABLE

No.	Sym.	Value
1	A	.3685
2	A	.3864
3	A	.3986
4	A	.4017
5	A	.4283
6	A	.4579

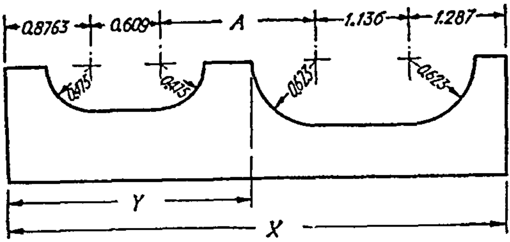
4. Determine the distance x.



VARIABLE

No.	Sym.	Value
1	A	4 0376
2	A	4.1589
3	A	4.2632
4	A	4.3453
5	A	4 4897
6	A	4.5638

5. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	1.7863
2	A	1.8394
3	A	1.9625
4	A	2.0631
5	A	2.2567
6	A	2.3842

6. Determine the distance x.

7. Determine the distance y.

MULTIPLICATION OF DECIMALS

Multiplication is the process of adding a number as many times as there are units in the quantity by which it is multiplied.

The **multiplicand** is the number which is to be multiplied.

The **multiplier** is the number by which the multiplicand is to be multiplied.

The **product** is the result of the multiplication.

The multiplicand and multiplier are both **factors** of the product.

Example: $13 \times 11 = 143$.

In this example 13 is the multiplicand, 11 is the multiplier, and 143 is the product. 13 and 11 are both factors of 143.

In the multiplication of decimal quantities, proceed as in the multiplication of whole numbers. Since the product of two common fractions whose denominators are 10 will produce a fraction whose denominator is 100, the product of two decimal fractions stated in tenths will produce a decimal fraction stated in hundredths. From this it is evident that the number of decimal places in the product is equal to the sum of the decimal places in the multiplicand and multiplier.

Example: Multiply 7.8546 by 487.69.

```
      7.8546... 4 decimal places
      487.69... 2 decimal places
     706914     6 decimal places
    471276      Therefore place the decimal point
   549822        6 decimal places from right to
  628368         left in the product.
 314184
 3830.609874
```

PROBLEMS

1. Multiply 8.6542 by A.
2. Multiply 10.856 by B.
3. Multiply 24.678 by C.
4. Multiply 8.4967 by D.
5. Multiply 4.8976 by E.
6. Multiply 5.9654 by F.
7. Multiply 6.9876 by G and then subtract .87654.
8. Multiply 6.8763 by H and then add 8.6957.
9. Multiply the sum of 3.8756 and J, by the difference of 4.8643 and 2.7632.
10. Multiply the difference of 8.5438 and K by the sum of 5.9875 and 2.8737.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	A	.85375	5.8365	5.8495	.69857	7.3865	.48756
2	B	5.7532	.75985	8.763	.35674	.73745	7.3876
3	C	4.8565	5.9758	8.3957	4.9867	8.3865	.68756
4	D	.87495	.68996	.86075	.94765	.58764	6.8597
5	E	.09843	6.9847	.97846	.48967	5.8398	.97865
6	F	6.4623	3.8576	8.9476	.39874	8.0753	.58746
7	G	8 4965	.57849	3.9875	2.9578	2.7497	2.9175
8	H	4 9687	7.9687	.98576	6.9587	.64865	7.5987
9	J	7.8539	.68932	.75894	.68932	.85964	.57684
10	K	2.2	2.7	2.5	2.6	2.3	2.8

DIVISION OF DECIMALS

Division of decimals is a special application of long division. When a decimal is divided by another decimal, it is essential that the decimal point in the quotient be placed in the proper location. Misplacing the decimal point changes the value greatly. For each place that it is moved to the right, the value of the decimal is multiplied by 10; for each place it is moved to the left, the value is divided by 10. Multiplying both dividend and divisor by the same number does not alter the value of the quotient. To move the decimal point to the right in both dividend and divisor the same number of places is the same as to multiply both by 10, or some power of 10; hence the value of the quotient remains the same.

In division of decimals the divisor may be reduced to a whole number. In order to do this, move the decimal point

to the right as many places as there are figures to the right of the decimal point. Next, move the decimal point in the dividend the same number of places to the right, counting from the original position. Eliminate the original decimal points in the dividend and divisor by drawing a cross through them. If there are fewer figures to the right of the decimal point in the dividend than there are to the right of the decimal point in the divisor, annex enough ciphers to the right of the dividend to take care of the new decimal point. Next, divide as in whole numbers. Write the first figure of the quotient directly above the last figure of the dividend used in the first step. Thereafter each figure annexed to the quotient should be written directly above the successive figures in the dividend. Place the decimal point in the quotient directly above the decimal point in the dividend and proceed as in the following examples.

Example a: Divide 58.759787 by .73867, *i.e.*,

$$.73867\overline{)58.759787}.$$

Solution: Since there are five decimal places in the divisor, the decimal point should be moved five places to the right in both dividend and divisor, thus: $73867.\overline{)5875978.7}$. Find by inspection the number of times the divisor is contained into the first group of figures. Place this partial quotient directly above the last figure used in the dividend as shown in the illustrative problem. Place the next partial quotient directly above the next figure used in the dividend, and so on until five figures have been obtained.

```
            79.548
73867./5875978.7
       517069 ↖Last figure used
        705288  in first step.
        664803
         404857
         369335
          355220
          295468
          597520
          590936
            6584
```

Example b: Divide .03959 by 8.9752, *i.e.*,

$$8.9752\overline{).03959}.$$

Solution:

```
              .00441
89752./0395.900    ← Hundredths figure.
       359 008     ← Tenths figure.
        36 8920
        35 9008
           99120
           89752
            9368
```

Proceed as in Example *a*, but if the divisor is not contained into the dividend by using the tenths figure, a cipher is placed in the tenths place in the quotient. If the divisor is not contained in the dividend by using the hundredths figure, a second cipher is placed in the quotient in the hundredths place. Keep on adding ciphers until the divisor is contained into the dividend.

From the previous examples the student will notice by placing the partial quotient directly above the last figure used in the dividend, that if the last figure used in the dividend is in tens, the partial quotient is in tens; if the last figure used is in units, the partial quotient is in units; if the last figure used is in tenths, the partial quotient is in tenths; etc.

PROBLEMS

1. Divide the product of 7.9854 and 6 5437 by *B*.
2. Divide 6 8647 by *C*. **3.** Divide 7 9754 by *D*.
4. Divide 6 984 by 23.765 and then multiply by *E*.
5. Multiply 5 8746 by .26376 and then divide by *F*.
6. Divide .87654 by *G* and then multiply by 7.9867.
7. Divide 5 9876 by *H*. **8.** Divide *J* by .076543.
9. Divide .008765 by *K* and then add .76534.

VARIABLES

Prob	Sym.	No 1	No 2	No. 3	No 4	No 5	No 6
1	*B*	5 7487	3 8765	7 9754	8 6432	5 9732	2 9648
2	*C*	76542	.58352	95275	48275	92746	92648
3	*D*	.06542	06327	07625	09642	08426	.08532
4	*E*	4 9876	5 9264	6 9375	4 9863	7 9543	6 3965
5	*F*	.98534	95432	7 9532	97532	.96427	.96425
6	*G*	6 9476	8 6543	7 9642	3 9742	8 9542	8.7533
7	*H*	46538	96536	67486	98743	96438	.94672
8	*J*	8 4852	7 5837	5 4375	8 6548	4 8769	6 4653
9	*K*	08764	07378	08754	08765	.05328	.0987

10. Multiply .09867 by 3.7652 and then divide by L.

11. Divide the product of M and 2.3649 by the quotient of 4.9867 divided by .76548.

12. Multiply 7.5876 by N and then divide by 5.9837.

13. Divide 4.8956 by P. **14.** Divide .008765 by R.

15. Divide 5.8646 by S.

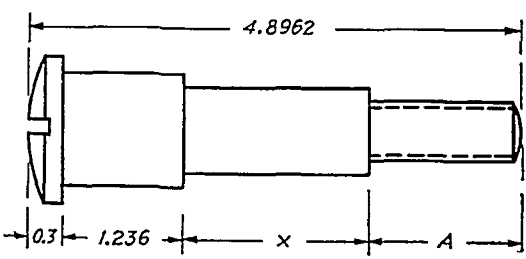
16. Divide .48769 by T and then multiply by .056849.

Change the following fractions to decimal numbers:

17. $\frac{17}{U}$. **18.** $\frac{V}{189}$. **19.** $\frac{262}{W}$.

VARIABLES

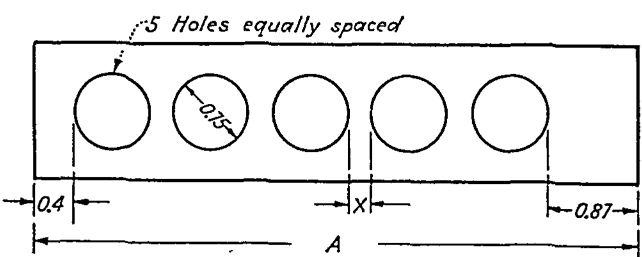
Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
10	L	8.8654	6.0984	8.4653	5.9836	5.3869	4.9724
11	M	4.3856	3.9875	3.6582	3.2857	2.9876	2.6895
12	N	.68954	.62893	.56894	.49873	.47239	.42189
13	P	5.8796	5.9873	6.3897	6.7893	7.1389	7.5693
14	R	.03985	.04763	.05784	.06895	.07289	.08396
15	S	4.6879	4.1389	3.8976	3.6895	3.1896	2.9876
16	T	.06895	.07329	.08395	.08962	.09137	.09654
17	U	21	23	25	27	29	31
18	V	2123	2133	2143	2153	2163	2173
19	W	567	589	603	625	647	669



VARIABLE

No.	Sym.	Value
1	A	.8792
2	A	.8967
3	A	.9063
4	A	.9278
5	A	.9487
6	A	.9672

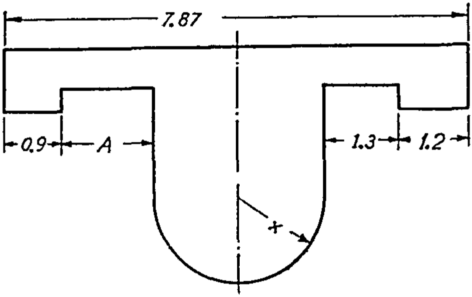
20. Determine the distance x.



VARIABLE

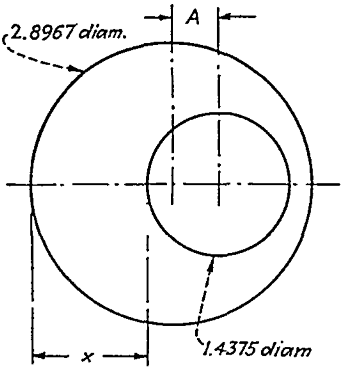
1. A = 10.783 **2.** A = 11.642 **3.** A = 11.875
4. A = 11.964 **5.** A = 12.137 **6.** A = 12.379

21. Determine the distance x.



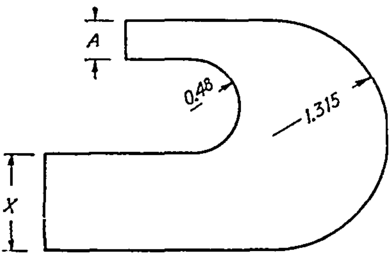
VARIABLE		
No.	Sym.	Value
1	A	1.5
2	A	1.6
3	A	1.7
4	A	1.8
5	A	1.9
6	A	2.1

22. Determine the radius x.



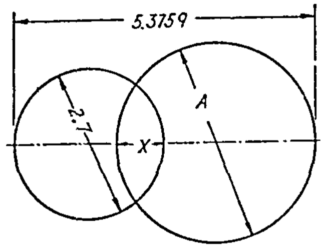
VARIABLE		
No.	Sym.	Value
1	A	.50953
2	A	.51681
3	A	.52834
4	A	.56955
5	A	.57326
6	A	.58437

23. Determine the distance x.



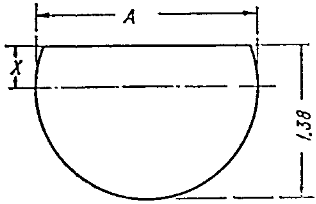
VARIABLE		
No.	Sym.	Value
1	A	.4376
2	A	.4985
3	A	.5367
4	A	.5832
5	A	.6281
6	A	.6898

24. Determine the distance x.



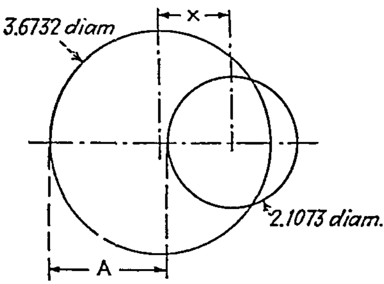
VARIABLE		
No.	Sym.	Value
1	A	2.9
2	A	3.2
3	A	3.7
4	A	3.3
5	A	3.5
6	A	3.1

25. Determine the distance x.



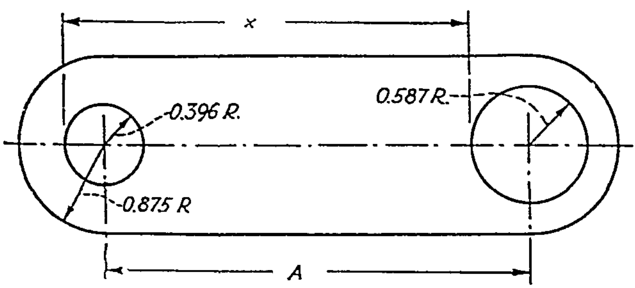
VARIABLE		
No.	Sym.	Value
1	A	1.7
2	A	1.9
3	A	1.8
4	A	1.6
5	A	2.3
6	A	1.5

26. Determine the distance x.



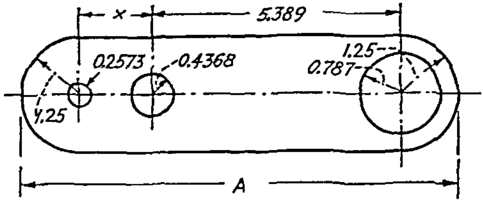
VARIABLE		
No.	Sym.	Value
1	A	2.0365
2	A	2.0674
3	A	2.0983
4	A	2.1762
5	A	2.2041
6	A	2.2467

27. Determine the distance x.



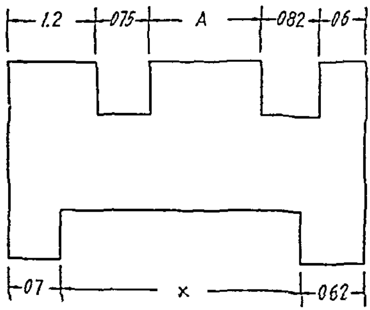
VARIABLE		
No.	Sym.	Value
1	A	5.987
2	A	6.078
3	A	6.193
4	A	6.345
5	A	6.589
6	A	6.793

28. Determine the distance x.



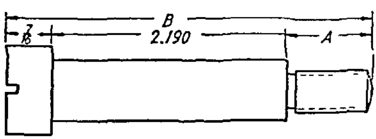
Variable		
No.	Sym.	Value
1	A	8.23
2	A	8.59
3	A	8.78
4	A	8.96
5	A	9.13
6	A	9.37

29. Determine the distance x.



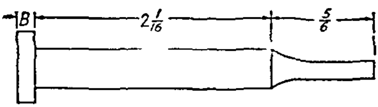
Variable		
No.	Sym.	Value
1	A	1.25
2	A	1.33
3	A	1.41
4	A	1.49
5	A	1.57
6	A	1.65

30. Determine the distance x.



Variable		
No.	Sym.	Value
1	B	3 489
2	B	3 530
3	B	3 571
4	B	3 612
5	B	3.653
6	B	3 694

31. Determine the distance A.



Variable		
No.	Sym.	Value
1	B	.143
2	B	.133
3	B	.123
4	B	.113
5	B	.103
6	B	.093

32. Allowing $\frac{1}{8}$ in. for cutting off each pin, (a) how many pins can be made from a 36 in. bar and (b) how much material will be left?

CHAPTER IV

MICROMETERS, VERNIERS, AND BEVEL PROTRACTORS

MICROMETERS

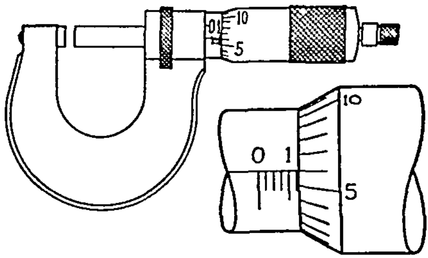


Fig. 1.

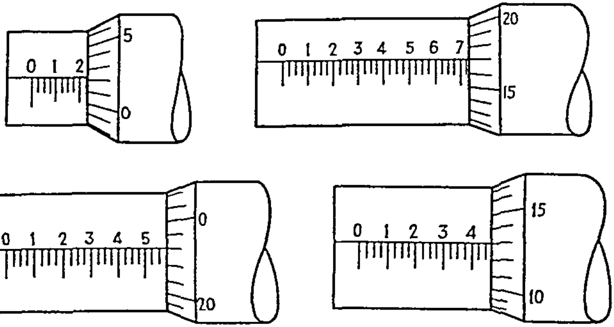
The scale on the barrel of the micrometer is divided into tenths of an inch and each tenth is subdivided into four parts, each part representing twenty-five thousandths of an inch. The screw of the micrometer has 40 threads per inch, so each revolution of the thimble will open the micrometer .025 in. The tapered end of the thimble is graduated into 25 equal divisions. Hence, rotating the thimble one division opens the micrometer .001 in.

To read a micrometer, observe the number of tenths and subdivisions up to the last line exposed to view by the thimble and express this value in thousandths of an inch. Add to this the thousandths given by the reading of the thimble which is just opposite the longitudinal line of reference which runs through the main scale.

In the foregoing figure, the exposed portion of the barrel shows $.1 + .025 = .125$. The reading on the thimble is .006 (to the nearest thousandth). Hence, the reading of the instrument is $.125 + .006 = .131$.

PROBLEMS

Determine the readings for the following settings of a micrometer:



VERNIERS

The vernier caliper has two principal parts, the vernier proper and the main scale. The vernier proper is usually rather short with a definite number of divisions, while the scale may be of great length with a larger number of divisions. In the English-system vernier, the major divisions of the scale are inches; these are subdivided into tenths of an inch; each tenth is subdivided into four or five parts, each part being twenty-five or twenty thousandths, respectively.

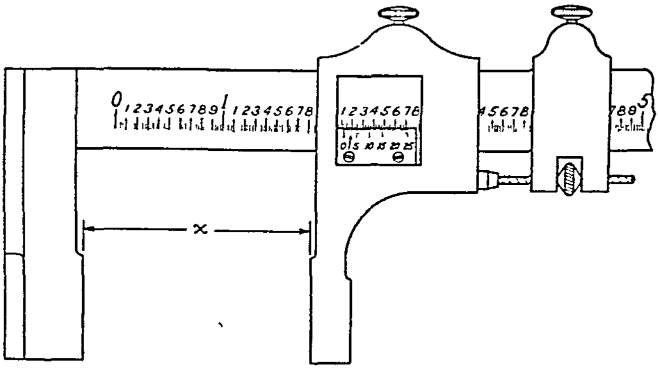


FIG. 2.

The vernier proper of a Starrett vernier caliper is .6 in. in length and is divided into 25 divisions. The scale has 24 divisions within the same length; therefore the two end lines of the vernier proper can be made to coincide with two lines on the scale. Each division of the vernier proper is equal to $\frac{.600}{25}$ or .024 in. and each division of the scale is $\frac{.600}{24}$ or .025 in.

If the two end lines of the vernier coincide with two lines of the scale, the distance from line 1 of the vernier proper to line 1 on the scale is the difference of .025 and .024 in., or .001 in. This value (.001 in.), which is the smallest value that can be read with the instrument, is called the least count. Then if the vernier is moved so that line one of the vernier proper coincides with line one of the scale, the vernier will have moved .001 in.; in like manner if line 5 of the vernier proper coincides with line 5 on the scale, the vernier will have moved .005 in.; if line 15 of the vernier proper coincides with line 15 of the scale, the vernier will have moved .015 in., etc.

When the jaws of the vernier are closed, the zero line of the vernier should coincide with (make a continuous line with) the zero line of the main scale.

The distance between the jaws of the vernier caliper is the same as the distance between the zero line of the vernier and the zero line of the main scale. To determine this, first read the inches and fractional part of an inch up to the line on the main scale, which is to the immediate left of the zero line of the vernier proper, and to this add the number of thousandths as given by the number of the vernier line which coincides with a line of the main scale.

In the foregoing figure, the zero line of the vernier is to the right of the first small division past the 2.1-in. mark, which means that the reading is more than $2.1 + .025 = 2.125$. The arrow indicates that the third line of the vernier proper coincides with a line of the main scale. Hence, the true reading of the instrument is $2.125 + .003 = 2.128$.

For some machines there are special verniers. The following problem will show how any vernier may be read.

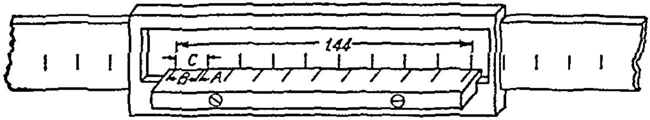


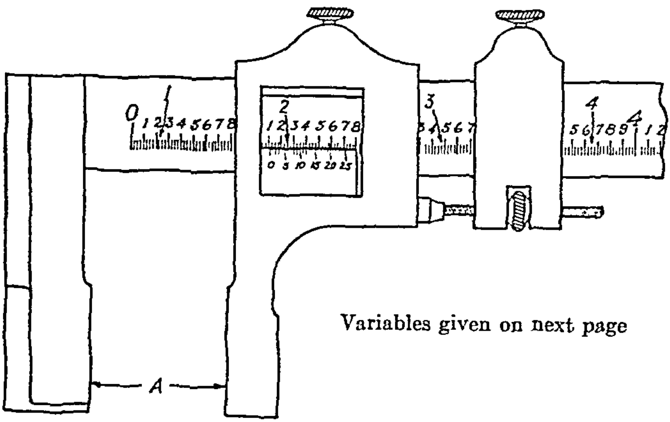
FIG. 3.

Illustrative Problem: The smallest main scale division is $C = \frac{1.44}{9} = .160$. The smallest vernier proper division is $B = \frac{1.44}{10} = .144$. Thus A, which is the distance between the first line on the vernier and the first line on the main scale, $= .160 - .144 = .016$. Hence, if the first line of the vernier coincides with the first line of the main scale, the vernier must have moved .016. If the fifth line of the vernier coincides with the fifth line of the main scale, the vernier proper has moved $5 \times .016 = .08$, etc.

PROBLEMS

If the zero line on the vernier proper is just beyond the line indicated by the arrow on the scale, and the Nth line of the vernier proper makes a continuous line with a line on the scale, what is the reading of the vernier?

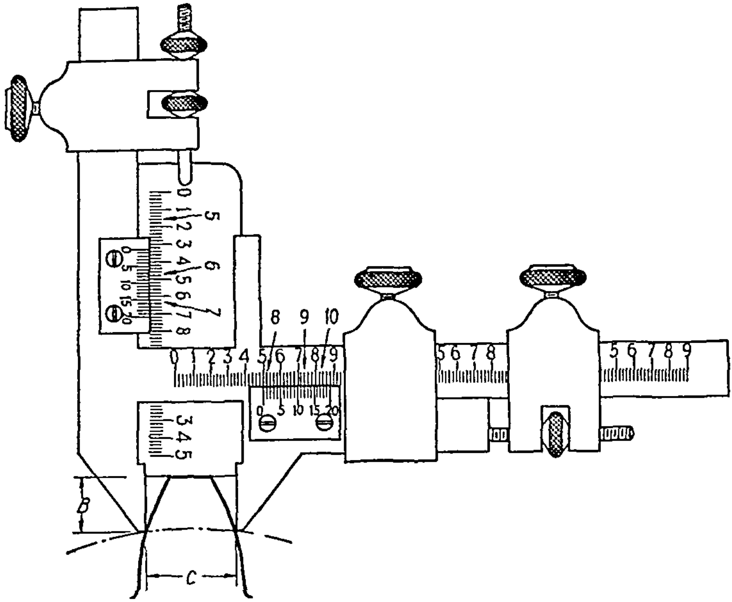
The number attached to each arrow indicates the different settings.



Variables given on next page

1. Determine A in setting 1.
2. Determine A in setting 2.
3. Determine A in setting 3.
4. Determine A in setting 4.

The number attached to each arrow indicates the different settings.



5. Determine B in setting 5.
6. Determine B in setting 6.
7. Determine B in setting 7.
8. Determine C in setting 8.
9. Determine C in setting 9.
10. Determine C in setting 10.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	N	4	6	8	10	12	14
2	N	23	5	4	3	2	9
3	N	14	16	18	20	22	24
4	N	3	5	7	9	11	13
5	N	17	20	18	16	14	2
6	N	11	9	7	5	3	20
7	N	19	15	2	4	6	8
8	N	7	9	11	13	15	17
9	N	10	12	14	16	18	20
10	N	19	20	18	3	5	7

BEVEL PROTRACTORS

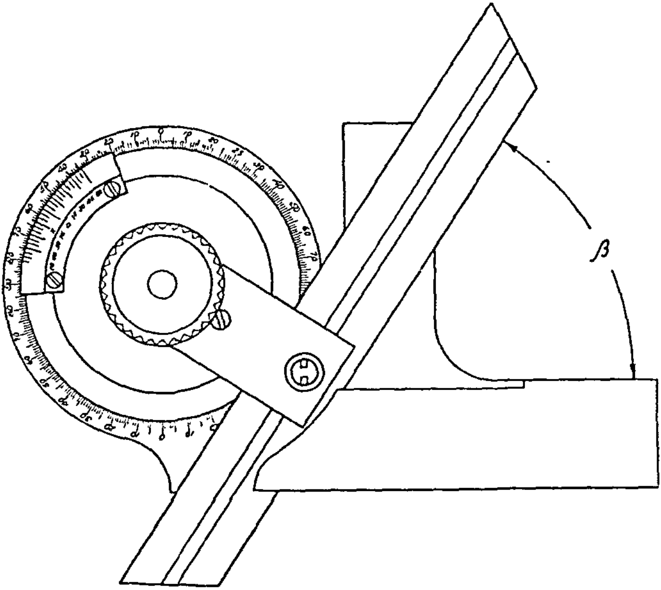


Fig. 4.

The bevel protractor used in shops is usually of the vernie type. The main scale is a stationary circular scale which divided into degrees (see Fig. 4). The vernier proper attached to a circular disk which rotates inside the mai circular scale. The Brown and Sharpe instrument has double vernier proper, each half of which is divided int 12 divisions.

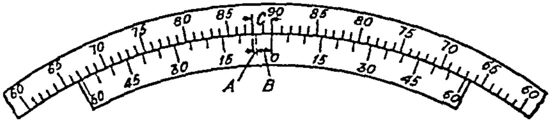


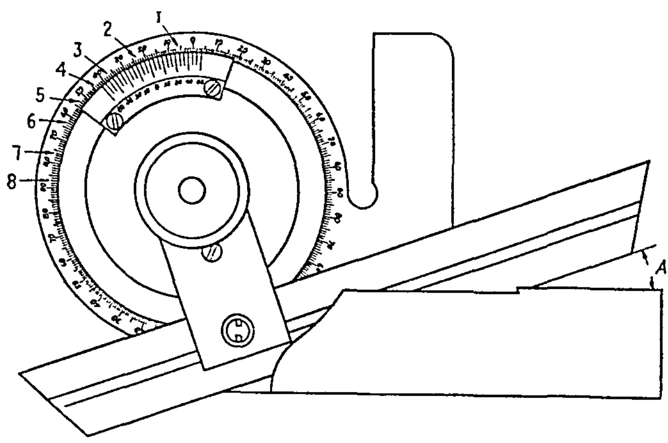
Fig. 5.

The principle of reading a vernier attached to a protract is the same as that of a vernier attached to a scale. The onl difference is that instead of the vernier being within a certai

number of inches it is within a certain number of degrees. Reduce these degrees to minutes and proceed as before. There are 60 minutes (60′) in one degree. First of all observe that the vernier is within 23 degrees (23°). Reduce 23° to minutes, which gives 23×60 or 1380′. Next note that the vernier is divided into 12 equal parts, each part being equal to one-twelfth of 1380′ or 115′ indicated by B (Fig. 5). Since there are 60′ in 1°, then in 2° there are 120′ indicated by C (Fig. 5). $C - B = A$ or $120' - 115' = 5'$ which is the curved distance the vernier moves from one line to the next consecutive line on the protractor scale.

Hence, if the fourth line of the vernier proper coincides with a line of the main scale, four times 5′ or 20′ must be added to the reading in degrees on the main scale as determined by the position of the zero line of the vernier scale. As shown by Fig. 5, the main scale is graduated in degrees in both directions from zero. If the zero of the vernier proper is to the left of the zero of the main scale, the left half of the double vernier proper is used, and *vice versa*. Thus, in the illustration (Fig. 4) the reading is $57° + 3 \times 5' = 57°\ 15'$.

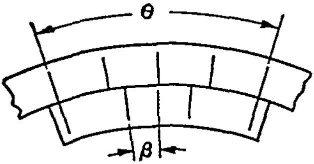
PROBLEMS



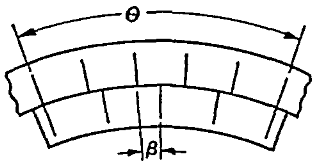
If the zero line on the vernier proper is just beyond the line on the outer dial indicated by the arrow, and the Lth line of the vernier proper makes a continuous line with a graduation on the outer dial, what is the value of angle A?

1. Determine angle A in setting 1.
2. Determine angle A in setting 2.
3. Determine angle A in setting 3.
4. Determine angle A in setting 4.
5. Determine angle A in setting 5.
6. Determine angle A in setting 6.
7. Determine angle A in setting 7.
8. Determine angle A in setting 8.

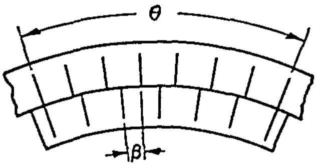
The answers to the following four problems are to be expressed in degrees and minutes.



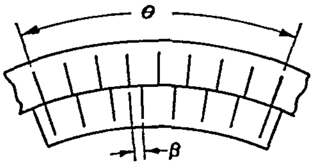
9. Determine the angle θ.



10. Determine the angle β.



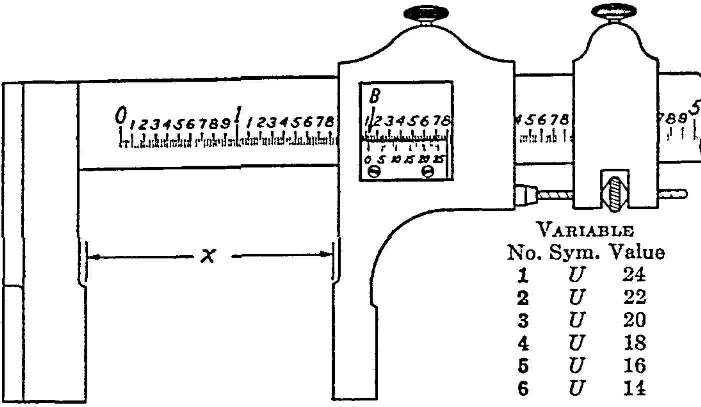
11. Determine the angle β.



12. Determine the angle β.

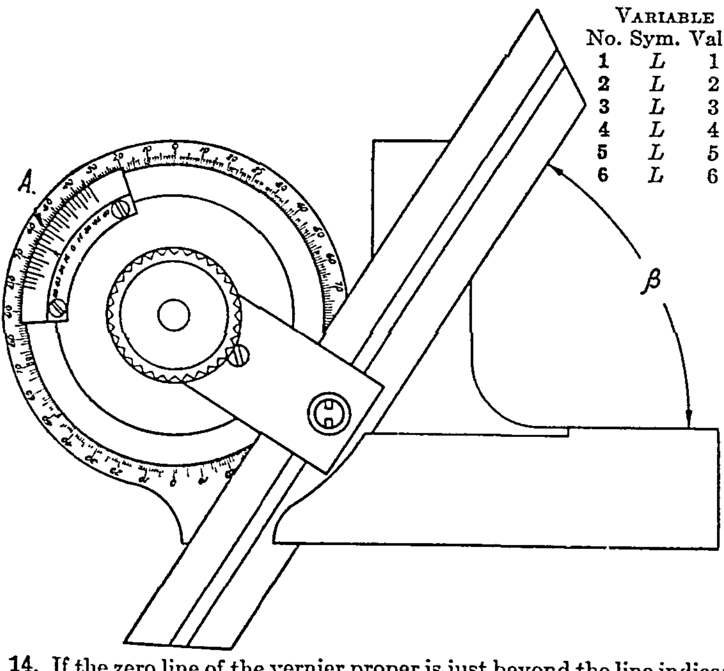
VARIABLES

Prob.	Sym	No. 1	No. 2	No. 3	No. 4	No. 5	No 6
1	L	2	3	4	5	6	7
2	L	10	9	8	7	5	6
3	L	4	6	8	10	9	7
4	L	7	9	11	3	1	10
5	L	8	6	4	2	7	5
6	L	5	4	3	2	1	9
7	L	5	6	7	8	9	10
8	L	1	2	3	4	5	6
9	θ	20°	21°	22°	23°	24°	25°
10	θ	16°	17°	18°	19°	20°	21°
11	θ	19°	20°	21°	22°	23°	24°
12	θ	22°	23°	24°	25°	26°	27°



Variable		
No.	Sym.	Value
1	U	24
2	U	22
3	U	20
4	U	18
5	U	16
6	U	14

13. If the zero line on the vernier proper is just beyond the line indicated by the arrow (B), and the Uth line on the vernier proper makes a continuous line with a line on the scale, determine the distance x.



Variable		
No.	Sym.	Value
1	L	1
2	L	2
3	L	3
4	L	4
5	L	5
6	L	6

14. If the zero line of the vernier proper is just beyond the line indicated by the arrow (A), and the Lth line on the vernier proper makes a continuous line with a line on the protractor scale, determine the angle β.

CHAPTER V

ALGEBRA

Algebra is like arithmetic in that the principal operations used are addition, subtraction, multiplication, and division.

Algebra differs from arithmetic in that the quantities involved are often represented by letters instead of by numbers.

Letters are frequently used to represent unknown quantities in practical problems as a means of leading to a statement that two quantities are equal. Such a statement of equality is called an equation. One of the main functions of algebra is the solving of such equations, thus leading to the determination of the unknown quantity.

In this chapter those processes of algebra particularly used in the solution of shop problems will be discussed.

USE OF POSITIVE AND NEGATIVE NUMBERS

Numbers are commonly used to represent the magnitude of quantities. Thus the temperature on a certain summer day may be 70°F. (meaning 70 above 0°F.), a man may possess \$5, or a certain place may have an altitude of +500 ft. (meaning 500 ft. above sea level).

Suppose it is now desired to represent a temperature of 6° below zero, the fact that a man owes \$5, or the altitude of a place which is 200 ft. below sea level. These quantities are best represented by −6°F., −\$5, and −200 ft., respectively. Thus negative numbers are used to represent magnitudes in the opposite sense to those which are arbitrarily chosen as positive. Usually the sign of operation is omitted in the case of positive numbers. Thus: 11 means +11.

The numerical value of a number regardless of its sign is called its absolute magnitude. The absolute magnitudes of −4, +8, and −3 are 4, 8, and 3, respectively.

ADDITION AND SUBTRACTION OF POSITIVE AND NEGATIVE NUMBERS

To add two positive numbers, add their absolute magnitudes and prefix the plus sign.

Example: $+7 + 22 = +29$.

To add two negative numbers, add their absolute magnitudes and prefix the minus sign.

Example: $-8 - 34 = -42$.

To add a positive number and a negative number, obtain the difference of their absolute magnitudes and prefix the sign of the number having the greater magnitude.

Examples: $-27 + 19 = -8, 44 - 18 = 26, 37 - 52 = -15$.

MULTIPLICATION OF POSITIVE AND NEGATIVE NUMBERS

The product of two numbers having like signs is positive and the product of two numbers having unlike signs is negative.

Examples: $5 \times 8 = 40$, $-20(-3) = 60$, $-12 \times 8 = -96$, $4(-13) = -52$. Note that the parentheses often take the place of a multiplication sign (see page 42).

DIVISION OF POSITIVE AND NEGATIVE NUMBERS

In division one number, called the divisor, is contained in another number, called the dividend, a certain number of times. This latter number is called the quotient. Thus:

$$\frac{48 \text{ (dividend)}}{8 \text{ (divisor)}} = 6 \text{ (quotient)}.$$

From this it follows that the dividend = divisor $\times$ quotient.

Since the law regarding signs in multiplication must apply to this, it follows that the law of signs for division is:

If the signs of the dividend and divisor are alike, the quotient is positive; if they are unlike, the quotient is negative.

$$\textit{Examples:}\ \frac{28}{4} = 7, \frac{56}{-4} = -14, \frac{-72}{9} = -8, \frac{-96}{-6} = 16.$$

When the plus (+) and minus (−) signs occur with multiplication ($\times$) and division ($\div$) signs in an expression, the multiplication and division operations must be performed first,

and then the addition and subtraction operations may be performed in the order in which they are written.

Examples:

$$12 \times 8 - 6 + 4 \times 12 = 96 - 6 + 48 = 138.$$
$$102 \div 6 - 6 \times 2 + 3 = 17 - 12 + 3 = 8.$$

It is a good policy to add all the plus quantities first, then add all of the minus quantities, and finally subtract the quantity having the lesser magnitude from the quantity having the greater magnitude and prefix the sign of the quantity having the greater magnitude to the result.

Example a: $12 - 6 + 4 - 2 + 9 - 18 + 5 = ?$

Solution: The sum of the plus quantities is $12 + 4 + 9 + 5 = 30$. The sum of the minus quantities is $-6 - 2 - 18 = -26$. The subtraction of the lesser magnitude from the greater is $30 - 26 = 4$.

Example b: $8 - 4 - 16 + 3 - 7 + 2 = ?$

Solution: The sum of the plus quantities is $8 + 3 + 2 = 13$. The sum of the minus quantities is $-4 - 16 - 7 = -27$. The subtraction of the lesser magnitude from the greater is $13 - 27 = -14$.

When several numbers are multiplied together, the product will be the same regardless of the order in which the multiplications are performed. Thus: $6 \times 35 \times 48$; $48 \times 6 \times 35$; $6 \times 48 \times 35$; $35 \times 48 \times 6$, etc., all have the same product.

PROBLEMS

1. Represent the following by the use of positive and negative numbers: a bank balance of \$95, a debt of \$20, a temperature of 10° below zero, a temperature of 72° above zero, the altitude of a place which is 800 ft. above sea level, the altitude of a place which is 100 ft. below sea level.

2. On a certain winter day, 20 tons of coal were burned in a factory and A tons were delivered. What is the net gain in the coal pile for the day?

3. At 6 A.M. a thermometer reads 60°F. The temperature rises $B°$ during the next 3 hr.; rises 10° more during the next 3 hr.; falls 2° during the next 3 hr.; falls 12° during the next 3 hr. Compute the temperature at (*a*) 9 A.M., (*b*) noon; (*c*) 3 P.M., and (*d*) 6 P.M.

4. $4 - 8 + C - 17 + 4 = ?$

5. Multiply: (*a*) $+4$ and $+9$, (*b*) -5 and $+6$, (*c*) 8 and -7, (*d*) -6 and -13, (*e*) 21 and -8.

6. Divide: (*a*) 72 by 8, (*b*) 96 by -12, (*c*) -144 by 16, (*d*) -182 by -13, (*e*) 63 by -7.

Perform the indicated operations:

7. $8(-7) + D - 10 + 60.$

8. $E - 8(-3) = ?$

9. $-20 \times 6 + F - 28 + \dfrac{-40}{-4} = ?$

10. $G + \dfrac{35}{-7} = ?$

11. $\dfrac{30}{-5} + 6 - H + 7 \times 6 = ?$

12. $18 - J + 6 \times 8 = ?$

13. $8 - K + 12 \div 3 + 4 \times 9 = ?$

14. $27 - L + 3 = ?$

15. $16 \div 2 + 4 - 7 - M = ?$

16. $-9 + N - 24 \div 6 = ?$

17. $4 \times 8 - 9 - P = ?$

18. $12 - Q + 2 - 15 = ?$

19. $15 - R + 7 - 12 = ?$

20. $58 - 38 \div 2 - S = ?$

21. $23 + 7 - 48 \div 6 - 3 \times 7 + T - 26 + 8 = ?$

22. $55 \div 5 + 6 - 56 \div 8 + 2 \times 7 - U + 9 \times 3 = ?$

23. $12 + 7 - 20 - 36 - 52 - V + 2 \times 9 = ?$

24. $75 - 32 - 46 - W + 7 + 8 + 9 = ?$

25. $84 \div 2 - 4 \times 6 - 12 \div 2 - A + 3 - 8 = ?$

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
2	*A*	40	42	44	46	48	50
3	*B*	16	18	20	22	24	26
4	*C*	10	12	14	16	18	20
7	*D*	3	4	5	6	7	8
8	*E*	5	7	9	11	13	15
9	*F*	70	74	78	82	86	90
10	*G*	7	9	11	13	15	17
11	*H*	14	16	18	20	22	24
12	*J*	18	19	20	21	22	23
13	*K*	2	4	6	8	10	12
14	*L*	35	37	39	41	43	44
15	*M*	5	7	9	11	13	15
16	*N*	18	20	22	24	26	28
17	*P*	41	43	45	47	49	51
18	*Q*	5	6	7	8	9	10
19	*R*	10	11	12	13	14	15
20	*S*	5	7	9	11	13	15
21	*T*	2	3	4	5	6	7
22	*U*	50	52	54	56	58	60
23	*V*	25	27	29	31	33	35
24	*W*	30	32	34	36	38	40
25	*A*	5	8	11	14	17	20

PARENTHESES AND GROUPING SYMBOLS

In a series of operations it frequently becomes necessary to use grouping symbols, such as parentheses (), brackets [], braces { }, or a vinculum ———. These symbols indicate that certain addition and subtraction operations should precede multiplication and division. They also indicate that the operations within should be carried out completely before the remaining operations are made. After these have been completed, the grouping symbols may be removed. If there are more than one pair of grouping symbols in an expression, the innermost pair should be removed first.

Examples: $7 + (6 - 2) = 7 + 4 = 11;$
$6 \times (8 - 5) = 6 \times 3 = 18.$

In an expression where grouping symbols are immediately preceded or followed by a number or quantity with the signs of operation omitted, multiplication is understood.

Examples: $8 + 6(4 - 1) = 8 + 18 = 26;$
$(6 + 2)(9 - 5) = 8 \times 4 = 32.$

Parentheses or other grouping symbols are often used in connection with subtraction and multiplication of negative quantities. Thus: plus 4 less negative 7 is written: $4 - (-7)$; plus 4 times negative 7 is written $4(-7)$.

To remove parentheses (or other grouping symbols) which are preceded by negative signs, the signs of all terms inside the grouping symbols must be changed (from plus to minus and minus to plus).

Examples: $4 - (-7) = 4 + 7 = 11;$
$8 - (7 - 4) = 8 - 3 = 5.$

Parentheses (or other grouping symbols) which are preceded by a plus sign may be removed without changing the signs of the terms within the grouping symbols.

Examples: $3 + (-8) = 3 - 8 = -5;$
$7 + (4 - 19) = 7 + (-15) = 7 - 15 = -8.$

When one set of grouping symbols is included within another set, remove the innermost set first.

Examples: $3[40 + (7 + 5)(8 - 2)] = 3[40 + 12 \times 6] = 3[40 + 72] = 3 \times 112 = 336;$

$$7 - 3\left[\frac{16 + 4}{5(8 - 6)} + 4\right] = 7 - 3\left[\frac{20}{5 \times 2} + 4\right] = 7 - 3[2 + 4] = 7 - 3 \times 6 = 7 - 18 = -11.$$

When several terms connected by + or − signs contain a common quantity, this common quantity may be placed in front of a parenthesis (or other grouping symbol) which encloses the results of dividing each of the several terms by the common quantity (called the common factor).

Example: In the expression $8x + 12$ the quantity 4 may be "factored out" giving $4(2x + 3)$. This is easily seen to be the reverse procedure of removing parentheses.

PROBLEMS

1. $M - (8 - 3) + 7 = ?$
2. $(7 - 2)(8 + N) = ?$
3. $L + 16 - 2[30 - (8 - 6)(2 + 7)] + 12 \div 4 = ?$
4. $K - \left[\frac{17 - 2}{5(6 - 4)}\right]5 + \frac{36 + 4}{8} = ?$
5. $P + 4(8 + 6)(9 - 2) \div 7 + 5 \times 8 = ?$
6. $Q - (8 + 2)(6 + 3) - 5 + 16 + 2 \times 8 + 72 = ?$
7. $5 - (-8) + R + (-14) = ?$
8. $[15 + 2(6 + 3)][3(A - 7) - 8] = ?$
9. $(8 - 3)(9 + B) - 6(8 + 7) - 21 = ?$
10. $6 \times 2 + 7(6 + 7) - 8(3 - 5)(C + 3) - 3 = ?$

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	M	12	23	34	45	56	67
2	N	7.5	8.6	9.7	10.8	11.9	12.3
3	L	41	52	63	74	85	96
4	K	2	4	6	8	10	12
5	P	7	11	16	19	21	23
6	Q	56	67	78	89	95	99
7	R	6	8	10	12	14	16
8	A	2	3	4	5	6	7
9	B	8	7	6	5	4	3
10	C	3	5	7	9	11	13

11. $12 \div 4 + 18 \div 3 - D(8 - 2)(6 + 4) = ?$
12. $5[8 - 4(E - 4)][(6 - 3) + 2(7 + 2)] + 10 = ?$
13. $F + 12 \div 4 - 18 \div 6 - 24 \div 4 = ?$
14. $(12 - 4)(G - 5) + (8 - 12)(5 - 10) + 3 = ?$
15. $3 + 2 - H - 8 \div 4 - 3 \times 5 - 15 \div 5 + 7 = ?$
16. $3\{J + 2[6 - 3(6 - 3) + 3] - 6 \div 2\} + 10 = ?$
17. $2\{8 - 3[(4 + 5) - 3(6 \div 2)] + 5\}[S - 4(6 + 2)] = ?$
18. $5(6 + 5) - (T + 3)(7 - 5) + 3(8 \div 4)(5 \times 3 - 7) = ?$

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
11	D	7	6	5	4	3	2
12	E	8	10	12	14	16	18
13	F	15	17	19	21	23	25
14	G	5	6	7	8	9	10
15	H	7	9	11	13	15	17
16	J	6	8	10	12	14	16
17	S	5	7	9	11	13	15
18	T	12	11	10	9	8	7

ALGEBRAIC SYMBOLS AND SIMPLE EQUATIONS

Frequently when the numerical value of a quantity is unknown, it is represented by a letter called an algebraic symbol. If enough data are given, the numerical value represented by the algebraic symbol can be obtained.

A **factor** of an expression is any one of the numbers or letters or groups which multiplied together give the expression.

Examples: The factors of 12 are 3 and 4. The factors of $5ac$ are 5, a, and c. The factors of $8x + 12$ are 4 and $2x + 3$. (See the example preceding the problems on page 43.)

An **equation** is a statement of equality between numbers *or numbers and algebraic symbols.*

The part of the equation which is to the left of the equality sign is called the left member (or left side) and the part to the right of the equality sign is called the right member (or right side).

Examples of equations:

$$12 = 6 \times 2; \; 13 + 5 = 18;$$
$$2x + 9 = 15; \qquad y - 7 = 4y + 5.$$

Equations involving algebraic symbols to the first power only (the same symbol may occur more than once but only to the first power) is called a **simple equation** or linear equation.

Examples: $2x + 4 = 10$; $4x + 2 = 14x$;
$3x + 4y + 6 = 2y + 4$.

An **exponent** is the small number written at the right and a little above another number (or quantity) called the **base number** (or quantity). The exponent indicates the power of the base number (or quantity), *i.e.*, it expresses the number of times the base number (or quantity) is taken as a factor.

The **power** of a number is the result obtained by using the base number the specified number of times as a factor.

Example: $3^4 = 3 \times 3 \times 3 \times 3 = 81$. In the exponential expression 3^4, 3 is the base number, 4 is the exponent denoting that 3 is taken as a factor 4 times, and the result 81 is the fourth power of 3.

$a^2 = a \times a$. In this case, the base quantity is a, the exponent is 2 denoting that a is taken as a factor twice, and a^2 is the second power or "square" of a.

ADDITION AND SUBTRACTION OF EXPRESSIONS INVOLVING ALGEBRAIC SYMBOLS

In adding or subtracting several quantities, some of which involve algebraic symbols, only those terms which involve the same symbols and power can be combined. Thus:

$$10x + 14 - 7y^2 - 11a + 2x - 4 + 3y^2 - 4a + 8 =$$
$$10x + 2x - 7y^2 + 3y^2 - 11a - 4a + 14 - 4 + 8 =$$
$$12x - 4y^2 - 15a + 18.$$

PROBLEMS

Perform the indicated operations and combine similar terms.

1. $10x + 14 + 3y + 8 + 6x = ?$ *Ans.* $16x + 3y + 22$
2. $13 - (3x + 2) + (x - 8) = ?$
3. $4(2x + 8) - 9y - 3(4x + 7) = ?$
4. If $x = 2$, is $4x + 2 = 10$ a true equation?

Transposition.—The effect of adding the same number to both members or subtracting the same number from both members is seen in Examples *a* and *b* to be equivalent to removing the number from one side of the equality sign and placing it on the opposite side with its sign changed. This is called transposition.

Example a: $2x + 3 = 11$.

Solution: Transposing the 3, $2x = 11 - 3 = 8$.

Dividing by 2, $x = 4$.

Example b: $6x - 7 = 11$

Solution: Transposing the -7, $6x = 11 + 7 = 18$.

Dividing by 6, $x = 3$.

SOLUTION OF PROBLEMS

Consider the following problem: Divide 35 into two parts, so that four times the lesser equals three times the greater.

Let x = lesser number.

Then $35 - x$ = greater number.

By the condition of the problem,

$$4x = 3(35 - x).$$

Removing the parentheses, $4x = 105 - 3x$.

Transposing the $-3x$, $4x + 3x = 105$.

Adding the like terms, $7x = 105$.

Solving for x (by dividing both sides by 7), $x = 15$ = lesser number. $35 - x = 35 - 15 = 20$ = greater number.

These two numbers are seen to satisfy the conditions of the problem, since $4 \times 15 = 60$ and $3 \times 20 = 60$.

From the solution of the above problem, the general procedure for the solution of problems involving simple equations is seen to be as follows:

1. Represent the unknown number (or one of the unknown numbers) by some letter (such as x).

2. If there is a second unknown number, represent it in terms of the same letter according to the conditions given in the problem.

3. Use the relations given by the statement to form an equation.

4. Solve the equation.

5. Check the results by showing that they fulfill the conditions of the problems.

Success in solving problems of the above type will depend upon obtaining from a careful reading of the problem the necessary relation for Steps 2 and 3.

PROBLEMS

Solve the following equations and check the result in each case.

1. $11x + 4 = 37.$ *Ans.* $x = 3$
2. $14 + 3y = -5y - 18.$
3. $-3n + 6 = 14 - 5n.$
4. $3(2x - 4) = -4x + 28.$
5. $4m + 8(3m - 9) = 6 - \frac{72}{-9}.$ *Ans.* $m = 3.0714$
6. $12 - 6x + 4 - 8x - 2(x - 8) = 0$
7. $7 - 3(4 + 5)(3R - 2) - 6R - 7(2R + 3) = 0$
8. $(3x - 6)(7 - 2) - 2x - 7 + 3x(8 - 3) = 0$
9. $19 - 2y - (6 - 4)(7y - 2) + 3y + 15 = 0$
10. $12m - 6 + 2m(8 - 5) - (3m - 6)(7 - 2) = 0$ *Ans.* $m = -8$
11. $7 + 3x(2 - 5) - 5x + 4(2 - 3x) = 0$
12. $10 + 3R - 7 - 6(9 - 3)(3 - 5R) + 9R \div 3 = 0$
13. $(15 - 6)2y + 3(4 - 7) - 2 + 3y - 3(2 - 4y) = 0$
14. $17 - 3x(7 - 2x)(8 + 3) + 5x - 3(2x - 5) + 5x = 0$
15. The sum of two numbers is 76 and their difference is 4. Find the numbers. *Ans.* 36 and 40
16. If 8 is added to three times a number, the result exceeds twice the number by 17. Find the number.
17. The age of A is three times that of B, but in 5 years A will be only twice as old as B. Find their present ages.
18. A boy is one-half as old as his father and one-fourth as old as his grandfather. The sum of the three ages is 105. How old is each?
19. The sides of a square have been increased and decreased 8 and 6 in., respectively, without changing its area. Determine the length of the side of the square.
20. A cistern can be filled by three pipes operating separately in 10, 15, and 25 min. respectively. In what time can it be filled if all the pipes operate simultaneously? *Ans.* 4.8387 min.

RATIO AND PROPORTION

The **ratio** of one quantity to another is the first divided by the second. The ratio of a to b is $a \div b$, or $\frac{a}{b}$. The ratio of 7 to 3 is $7 \div 3$ or $\frac{7}{3}$.

An **inverse ratio** is the reciprocal of a given ratio and is, hence, equal to the ratio inverted. The inverse ratio of $\frac{a}{b}$ is $\frac{b}{a}$.

A **proportion** is an equality of two ratios. For example, $\frac{a}{b} = \frac{c}{d}$. A proportion is often written $a:b::c:d$, and in either form is read a is to b as c is to d. Another example of a proportion is $\frac{7}{3} = \frac{14}{6}$ or $7:3::14:6$.

In any proportion, the four terms are numbered in the order in which they occur. Thus in the proportion $a:b::c:d$ $\left(\frac{a}{b} = \frac{c}{d}\right)$, a is the first term, b the second, c the third, and d the fourth.

The first and fourth terms of a proportion are called the **extremes,** and the second and third are called the **means.** Thus in the proportion $\frac{a}{b} = \frac{c}{d}(a:b::c:d)$, a and d are the extremes and b and c are the means. In a ratio like $\frac{x}{y} = \frac{y}{z}$ $(x:y::y:z)$, y is called the **mean proportional** between x and z.

FUNDAMENTAL THEOREMS OF PROPORTION

Seven fundamental theorems of proportion, which will be referred to later in some of the geometry proofs, will now be stated and derived and numbered with Roman numerals for future reference.

I. In any proportion, the product of the extremes is equal to the product of the means.

Given: $\frac{a}{b} = \frac{c}{d}$. *To prove:* $ad = bc$.

Multiply both sides by bd (Axiom III), $\frac{a \times bd}{b} = \frac{c \times bd}{d}$.

Canceling the b terms on the left and the d terms on the right (see page 9):

$$ad = bc.$$

Numerical Illustration: If $\frac{4}{2} = \frac{2}{1}$, then $4 \times 1 = 2 \times 2$.

II. The mean proportional between two quantities is equal to the square root of their product.

If $\frac{x}{y} = \frac{y}{z}$, then by I, $xz = y \times y$, or $xz = y^2$.

Extracting the square root of both sides (Axiom V):

$$y = \sqrt{xz}.$$

III. If four quantities are in proportion, they are in proportion by **alternation**; *i.e.*, the first term is to the third as the second term is to the fourth.

Given: $\frac{a}{b} = \frac{c}{d}$. *To prove:* $\frac{a}{c} = \frac{b}{d}$.

By I, $ad = bc.$

Divide both sides by cd (Axiom IV), $\frac{ad}{cd} = \frac{bc}{cd}$.

Canceling the d's on the left and the c's on the right:

$$\frac{a}{c} = \frac{b}{d}.$$

IV. If four quantities are in proportion, they are in proportion by **inversion**; *i.e.*, the second term is to the first as the fourth is to the third.

Given: $\frac{a}{b} = \frac{c}{d}$. *To prove:* $\frac{b}{a} = \frac{d}{c}$.

By I, $bc = ad.$

Dividing both sides by ac and canceling (Axiom IV):

$$\frac{b\cancel{c} = \cancel{a}d}{a\cancel{c} = \cancel{a}c}$$

or

$$\frac{b}{a} = \frac{d}{c}.$$

V. If four quantities are in proportion, they are in proportion by **composition**; *i.e.*, the sum of the first two terms is to the second term as the sum of the last two terms is to the last term.

Given: $\frac{a}{b} = \frac{c}{d}$. *To prove:* $\frac{a+b}{b} = \frac{c+d}{d}$.

Adding one to each side of the given proportion (Axiom I):

$$\frac{a}{b} + 1 = \frac{c}{d} + 1.$$

This may be written

$$\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$$

or

$$\frac{a + b}{b} = \frac{c + d}{d}.$$

It also may be shown that $\frac{a + b}{a} = \frac{c + d}{c}$ (by first inverting $\frac{a}{b} = \frac{c}{d}$ and then applying V).

VI. If four quantities are in proportion, they are in proportion by **division**; *i.e.*, the difference of the first two is to the second as the difference of the last two is to the last.

Given: $\frac{a}{b} = \frac{c}{d}$. *To prove:* $\frac{a - b}{b} = \frac{c - d}{d}$.

Subtracting one from each side of the given proportion (Axiom II):

$$\frac{a}{b} - 1 = \frac{c}{d} - 1,$$

which may be written

$$\frac{a}{b} - \frac{b}{b} = \frac{c}{d} - \frac{d}{d}$$

or

$$\frac{a - b}{b} = \frac{c - d}{d}.$$

VII. In a series of equal ratios, the sum of all the numerators is to the sum of all the denominators as any one numerator is to its denominator.

Given: $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$. *To prove:* $\frac{a + c + e}{b + d + f} = \frac{a}{b}$.

Let the equal ratios all equal r.

Then since $r = \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$,

$$a = br,\ c = dr,\ e = fr \text{ (Axiom III).}$$

$\therefore a + c + e = br + dr + fr$ because equals added to equals give equals.

Factoring out the common term r on the right side (see page 43),

$$a + c + e = r(b + d + f).$$

Dividing both sides by $b + d + f$ (Axiom IV),

$$\frac{a + c + e}{b + d + f} = r.$$

But

$$r = \frac{a}{b}.$$

$$\therefore \frac{a + c + e}{b + d + f} = \frac{a}{b}.$$

DIRECT AND INVERSE PROPORTION

Problems in ratio and proportion are proportional either directly or indirectly (often written inversely). They are directly proportional when an increase in one denomination will produce an increase in the other; thus, if 5 drills cost \$4, then 7 drills must cost more. This greater cost in dollars is represented by x. By comparing quantities of like denominations is meant comparing men to men, hours to hours, bushels to bushels, dollars to dollars, etc.

In any proportion, the greater quantity of one denomination is to the lesser quantity of the same denomination as the greater quantity of a second denomination is to the lesser quantity of this second denomination.

Thus:

$$\begin{array}{c} \text{drills} \qquad\qquad \text{dollars} \\ \text{greater} : \text{lesser} :: \text{greater} : \text{lesser} \end{array}$$

$$\frac{7 \text{ drills}}{5 \text{ drills}} = \frac{x \text{ dollars}}{4 \text{ dollars}}.$$

$$5x = 7 \times 4 \qquad \text{by I}$$

$$\therefore x = \frac{28}{5} = \$5\frac{3}{5} = \$5.60.$$

A ratio is indirectly (or inversely) proportional when an increase in one quantity will produce a decrease in the other. For example, if 5 men can do a piece of work in 8 hr., 10 men can do it in fewer hours, which will be represented by x hr.

Since an increase in the number of men produces a decreas in the number of hours required, the proportion is indirect.

As in the previous problem, the greater quantity of on denomination is to the lesser quantity of the same denomina tion as the greater quantity of a second denomination is to th lesser quantity of this second denomination. Thus in thi problem:

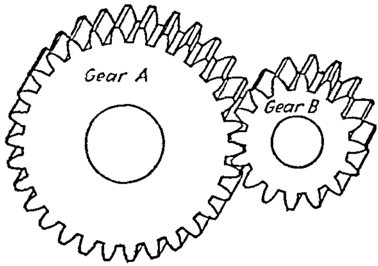
$$\begin{array}{c} \text{men} \qquad\qquad \text{hours} \\ \text{greater}:\text{lesser}::\text{greater}:\text{lesser} \end{array}$$

$$\frac{10 \text{ men}}{5 \text{ men}} = \frac{8 \text{ hr.}}{x \text{ hr.}}$$

$$10x = 5 \times 8 \qquad \text{by I.}$$

$$\therefore x = \frac{40}{10} = 4 \text{ hr.}$$

PROBLEMS



1. The numbers of teeth in gears A and B are 42 and N, respective What is the ratio of the numbers of teeth in the gears A and B?

2. The ratio of the numbers of teeth in the gears A and B is $\frac{5}{3}$. Wh number of teeth must B have if A has M?

3. If the numbers of teeth in the gears A and B are 73 and R, resp tively, and if B makes 35.7 revolutions, how many revolutions will make?

4. If 11 reamers cost T dollars, what would 7 reamers cost?

5. What is the ratio of 7.4859 and S? Answer to be a decin number.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No 6
1	N	26	27	28	29	30	31
2	M	25	30	35	40	45	50
3	R	45	48	52	56	59	62
4	T	12.50	13.25	14.0	14.75	15.5	16.25
5	S	8.1342	8.2641	9.2635	9.7654	10.308	10.86

6. If a rod of steel F in. long weighs 35.738 lb., what would a rod of the same steel weigh if it were 21.5 in. long?

7. What is the ratio of G and 5.9732? Answer to be a decimal number.

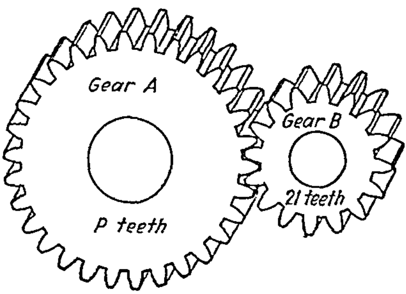
8. What common fraction must K be multiplied by to obtain 35?

9. If from a steel rod 23.7 ft. long L tapered pins can be machined, how many tapered pins can be machined from a steel rod 13.7 ft. long?

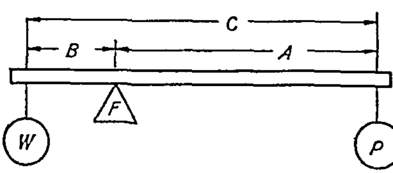
10. What is the ratio of $\frac{3}{7}$ and H? Answer to be a fraction.

11. What fraction must $\frac{4}{5}$ be multiplied by to obtain D?

12. If C pieces of work can be machined in 7 hr., how many hours will it take to machine 9 pieces of work?



13. When gear A makes 7.5 revolutions, how many revolutions will gear B make?



P = applied force = variable. W = weight. F = fulcrum.
A = 17 in. B = 5 in.

14. Determine the weight W.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
6	F	12.5	14.3	15.6	17.4	18.8	19.5
7	G	1.106	1.241	1.385	1.824	2.244	2.361
8	K	42	46	51	57	62	65
9	L	25	27	29	31	33	35
10	H	12	15	18	21	25	33
11	D	$\frac{7}{8}$	$\frac{5}{8}$	$\frac{9}{11}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{3}{8}$
12	C	55	58	68	75	82	93
13	P	41	44	47	51	53	56
14	P	21	32	53	64	75	96

COMPOUND RATIO AND PROPORTION

A **compound ratio** is the product of simple ratios.

If a quantity is multiplied by a ratio greater than unity, the quantity is increased: thus

$$7 \times \frac{17}{14} = 8.5.$$

If a quantity is multiplied by a ratio less than unity, the quantity is decreased; thus

$$7 \times \frac{11}{14} = 5.5.$$

In many problems the quantity to be determined is affected by more than one condition. The effect of each of these conditions may be expressed by a simple ratio, and the effect of all of the conditions acting simultaneously may be obtained by multiplying these simple ratios together.

If $\begin{pmatrix}\text{decreasing}\\ \text{increasing}\end{pmatrix}$ one of the conditions causes an increase in the quantity to be determined, the two are said to be in $\begin{pmatrix}\text{indirect}\\ \text{direct}\end{pmatrix}$ proportion, and the quantity to be determined is obtained by multiplying the original value of that quantity by a ratio greater than unity.

If $\begin{pmatrix}\text{decreasing}\\ \text{increasing}\end{pmatrix}$ one of the conditions causes a decrease in the quantity to be determined, the two are said to be in $\begin{pmatrix}\text{direct}\\ \text{indirect}\end{pmatrix}$ proportion, and the quantity to be determined is obtained by multiplying the original value of that quantity by a ratio less than unity.

Example: If 9 men working 8 hr. per day can unload 24 carloads of castings in 3 days, how many days would be required for 5 men working 10 hr. per day to unload 18 carloads of castings?

Solution: Start with the quantity of the same denomination as the question, which in this case is 3 days. Then compare

the men, 9 in the first case and 5 in the second. In arranging the ratio, the number of days required is the only thing that should be considered. Thus 5 men will require more days than 9 men, therefore, the ratio should be $\frac{9}{5}$. Next, the hours per day should be considered. Since working 10 hr. per day in the second case will require fewer days, the ratio should be $\frac{8}{10}$. Finally, the number of carloads must be considered. 18 carloads will require fewer days than 24 carloads. Therefore, this ratio should be $\frac{18}{24}$. The solution of the problem then becomes

$$\cancel{3} \times \frac{9}{5} \times \frac{\cancel{8}}{\underset{5}{\cancel{10}}} \times \frac{\overset{9}{\cancel{18}}}{\underset{\cancel{3}}{\cancel{24}}} = \frac{81}{25} \text{ or } 3.24 \text{ days.}$$

PROBLEMS

1. If 9 iron bars 7 ft. long, 3 in. broad, and 1.2 in. thick, weigh B lb., what will be the weight of 5 bars of the same material 10 ft. long, 4 in. broad, and 2.3 in. thick?

2. If C men working 9 hr. per day, 7 days per week, can machine 236 castings in 23 weeks, how many weeks will it take 11 men working 6 hr. per day, 5 days per week, to machine 729 castings?

3. A pulley D in. in diameter runs at a speed of 125 r.p.m. and drives another pulley at a speed of 425 r.p.m. What is the diameter of the other pulley?

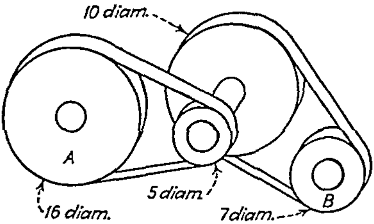
4. If 7 men do E pieces of work in 8 hr., how long will it take 9 men to do 55 pieces of the same work?

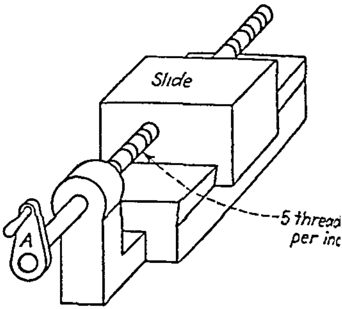
5. The diameter of two pulleys connected by a belt are, respectively, 35 and G in., and the smaller makes 217 r.p.m. Find the number of r.p.m. of the larger pulley.

6. If a piece of steel 16 ft. long, 5 ft. wide, and 2.3 ft. thick weighs R lb. what will be the weight of a block of the same kind of steel 9 ft. long, 2 ft. wide, and 1.2 ft. thick?

7. A grinder spindle is to be driven from a main shaft line making 167 r.p.m. It is found necessary to employ two countershafts. The pulley on the main shaft line is S in. in diameter and drives a pulley 7 in. in diameter. On the same shaft with the 7-in. pulley is a 12-in. pulley which in turn drives a 9-in. pulley, and on the same shaft with the 9-in. pulley is a 16-in. pulley which in turn drives a 5-in. pulley on the grinder spindle. What is the speed of the grinder spindle?

8. If 12 men working 10 hr. per day, 6 days per week, can dig a trench 4 ft. wide, 7 ft. deep, and Q ft. long, in 3 weeks, how many men working 8 hr. per day, 7 days per week, will be required to dig a trench 2.5 ft. wide, 7 ft. deep, and 10,587 ft. long in 4 weeks?





9. When shaft A makes T revolutions, how many revolutions will B make?

10. When A makes H revolutions, how far will the slide move?

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	B	912	953	993	1,034	1,074	1,115
2	C	81	108	135	162	189	217
3	D	2.9411	3.235	3.5294	3.8235	4.1176	4.4117
4	E	13	12	11	10	9	8
5	G	130.20	142.6	155	167.4	179.8	192.2
6	R	241	252.39	259.43	275.16	287.6	295.35
7	S	17.9	19.6	21.4	23.6	25	26.8
8	Q	50,639	48,706	44,775	40,901	39,024	35,845
9	T	164.57	187.42	214.85	242.28	269.71	297.14
10	H	442	464	506	528	550	592

PERCENTAGE

Per cent means hundredths. For example, 5 per cent means five hundredths (which may be written .05 or $\frac{5}{100}$). The symbol for per cent is %, which is written after the number. Thus, 9 per cent is written 9%. A quantity is always $\frac{100}{100}$ or 100% of itself.

Example a. Find 24% of 85.

Since per cent means hundredths, we must find .24 of 85, which is $.24 \times 85 = 20.4$.

Example b. 15 is what per cent of 65? That is, 15 is how many hundredths of 65?

$$\frac{15}{65} = .23076, \text{ which is } 23.076\%.$$

This problem may also be considered as a simple proportion.

Thus: Since 65 is 100%, $\quad \dfrac{15}{65} = \dfrac{x\%}{100\%}$

$$x = 100 \times \frac{15}{65} = 23.076\%.$$

PROBLEMS

1. N is what per cent of 73? **2.** 21 is what per cent of L?

3. If in a certain machine G of the energy supplied to the machine is lost in friction, what is the per cent of efficiency?

4. 13 is $M\%$ of what number? **5.** H is 3% of what number?

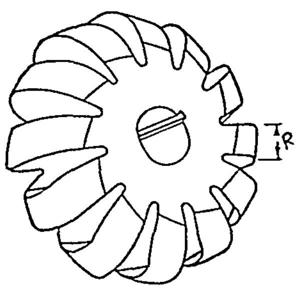
6. The usual allowance for shrinkage when casting iron pipes is $\frac{1}{8}$ in. per foot. What is the per cent of the allowance?

7. The indicated horsepower of an engine is F, the actual effective horsepower is 12.3. The actual horsepower is what per cent of the indicated horsepower?

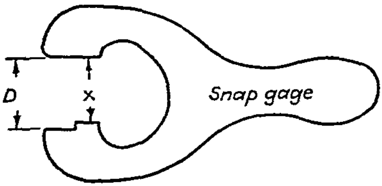
8. What is the net price per barrel of oil, the list price of which is C dollars, subject to a discount of $15\frac{1}{4}\%$ and 5% off for cash?

9. Find the cost of an article that is listed at S dollars, 35% and 7% off for cash.

10. The clearance between a punch and die for a certain metal is 7% of the thickness of the stock. For stock of thickness T, determine the value of the clearance.



11. The thickness R of a tooth on a certain rotary cutter should measure $\frac{7}{8}$ in. The company manufacturing this cutter agreed to take a loss of \$1 for each per cent that the specified dimension is undersize. The dimension R was found to be $\frac{13}{16}$ in., and accordingly the manufacturing company received N dollars. What was the list price?



12. A snap gage is to be made for checking a shaft which has an allowance of .06% undersize. If the correct diameter is D, determine the distance x on the gage.

13. One of the best bearing metals is a Babbitt having the following composition:

Tin	84%
Antimony	10%
Copper	5%
Lead	1%

Find the number of pounds of each of the constituents in a bearing weighing A lb.

14. Certain alloys having low melting points are used in overhead safety sprinkler systems. Of such alloys, Wood's metal has about the lowest melting point (154°F.) and has the following composition:

Bismuth	50%
Lead	25%
Tin	$12\frac{1}{2}$%
Cadmium	$12\frac{1}{2}$%

Calculate the weight of each constituent in a mass of this alloy which weighs B lb.

Variables

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	N	6	8	11	15	17	19
2	L	118	135	154	178	193	209
3	G	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{2}{7}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$
4	M	4	5	6	7	8	9
5	H	15	17	19	21	23	27
6		Complete	Complete	Complete	Complete	Complete	Complete
7	F	13.2	13.7	14.1	14.6	14.9	15.1
8	C	5.5	5.75	6.25	6.75	7.4	7.75
9	S	18.44	22.64	26.39	29.18	39.75	49.55
10	T	.093	.102	.112	.117	.125	.156
11	N	28 9	35.4	38.5	41.25	45.75	47.4
12	D	.431	.562	.623	.684	.775	.916
13	A	9.875	9.75	9.625	9.5	9.375	9.25
14	B	639	619	599	579	559	539

Illustrative Problem: How much lead must be added to 485 lb. of tinsmith's solder (59% tin, 41% lead) to change it to plumber's solder which is 35% tin and 65% lead?

Solution: In 100 lb. of the tinsmith's solder, there are 59 lb. of tin and 41 lb. of lead. In the plumber's solder the percentage is to be 35% tin and 65% lead. Hence

$$\frac{x \text{ lb. of lead}}{59 \text{ lb. of tin}} = \frac{65\% \text{ of lead}}{35\% \text{ of tin}}$$

or

$$\frac{x}{59} = \frac{65}{35}.$$

$$x = 59 \times \frac{65}{35} = 109.57 \text{ lb. of lead.}$$

Since the tinsmith's solder already contains 41 lb. of lead per 100 lb. of solder, the amount of lead to be added per 100 lb. of tinsmith's solder is $109.57 - 41 = 68.57$ lb.

Since 485 lb. is 4.85 times as great as 100 lb., the total amount of lead to be added is $4.85 \times 68.57 = 332.56$ lb.

15. How many pounds of lead must be added to change E lb. of a batch of solder which is 42% tin and 58% lead to a new batch of solder which is 30% tin and 70% lead?

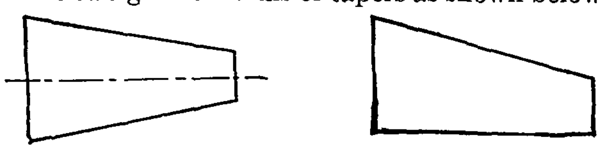
16. In making cloth for upholstering a certain car, colored fibers were mixed in the following proportions: yellow 20%, green 30%, red 15%, black 10%, and blue 25%. It was found that a better color mixture would be obtained if the following proportions were used: yellow 30%, green 25%, red 10%, black 10%, and blue 25%. How many pounds of each color must be added to J lb. of the original mixture to produce the desired mixture?

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
15	E	1476	1567	1689	1742	1821	1935
16	J	485	435	395	362	337	296

TAPER PER FOOT

There are two general forms of tapers as shown below:



Tapers of the form shown in Fig. 6 are used for tapered plugs and gages, tapered spindles and bearings, and taper fits as in the case of a hub on an axle, etc.

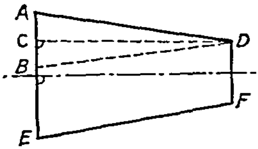


FIG. 8.

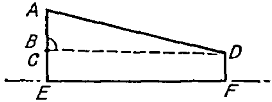


FIG. 9.

Tapers of the form shown in Fig. 7 are used for the gibs of milling and grinding machines, rams of shaper heads, etc.

In Figs. 8 and 9, the taper is AB for the length CD. BD is drawn parallel to EF and CD is drawn parallel to the axis. By **taper per foot** is meant the distance corresponding to AB when CD is 1 ft.

The problem of determining taper per foot is obviously one of simple proportion.

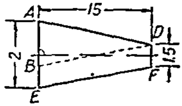


FIG. 10.

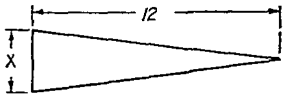


FIG. 11.

Illustrative Example a: Determine the taper per foot for Fig. 10.

Solution: Draw BD parallel to EF, which shows the taper to be .5 in. for a length of 15 in. Draw another figure, which may be called the master figure, having the same shape and a length of 12 in. (see Fig. 11). Then x is the taper per foot.

$$\frac{x}{.5} = \frac{12}{15} \text{ or } x = .5 \times \frac{12}{15} = .4.$$

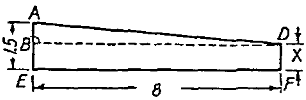


FIG. 12.

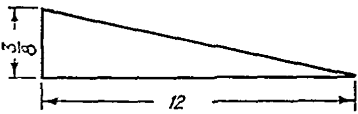


FIG. 13.

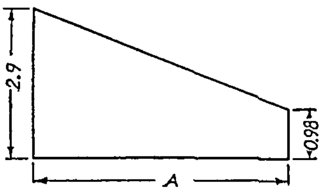
Illustrative Example b: Determine the distance x in Fig. 12. Taper per foot $= \frac{3}{8}$. (*Solution on next page.*)

Solution for preceding problem: Draw BD parallel to EF, and construct a triangle similar to ABD having a length of 12 in. (Fig. 13).

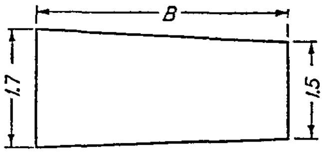
$$\frac{AB}{\frac{3}{8}} = \frac{8}{12} \text{ or } AB = \frac{3}{8} \times \frac{8}{12} = \frac{1}{4}.$$

Hence $$x = 1.5 - .25 = 1.25 \text{ in.}$$

PROBLEMS



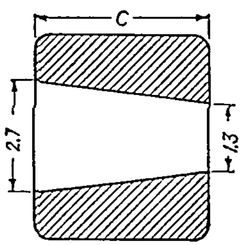
1. Determine the taper per foot.



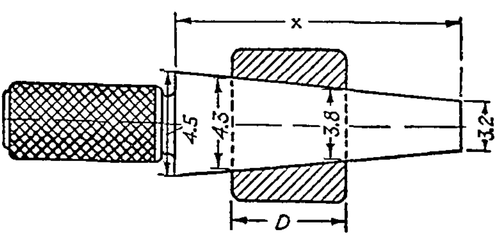
2. Determine the taper per foot.

VARIABLES

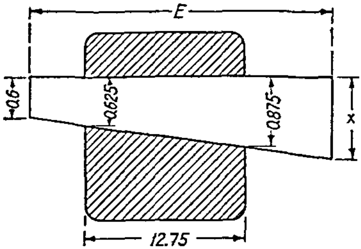
Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	A	5.5	5.7	5.9	6.1	6.3	6.5
2	B	3.25	3.6	3.95	4.45	4.78	5.32



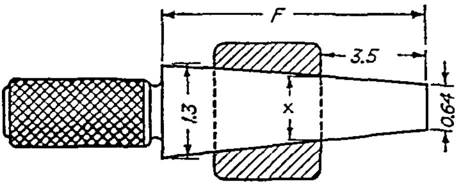
3. Determine the taper per foot.



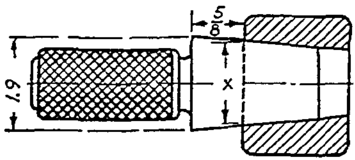
4. Determine the distance x.



5. Determine the distance x.

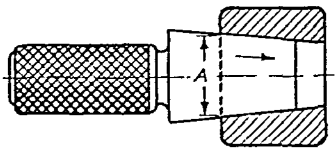


6. Determine the distance x.



G = taper per foot.

7. Determine the diameter x.

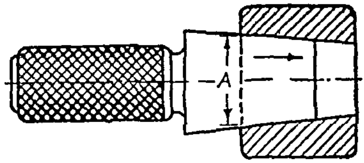


H = taper per foot.

8. If A were made .017 larger, how far would the taper plug advance?

VARIABLES

Prob.	Sym.	No 1	No 2	No 3	No 4	No 5	No. 6
3	C	14 3	16 2	16 6	16 9	17 4	17 8
4	D	4 8	5 2	5 6	5 9	6 2	6.5
5	E	15	15 2	15.4	15 6	15 8	16
6	F	9.2	9.5	9.9	10.3	10.5	10.9
7	G	.125	.1875	.25	.3125	.375	.4375
8	H	.4375	375	.3125	.25	.1875	.125



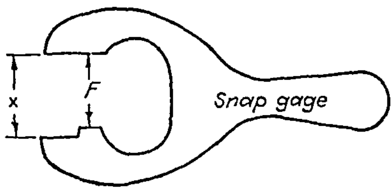
J = taper per foot.

9. How much must the diameter A be increased in order that the taper plug will advance .007?

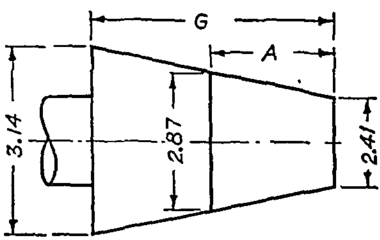
VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
9	J	2.51	2	3	4	3.5	4.56

ADDITIONAL PROBLEMS ON RATIO AND PROPORTION, PERCENTAGE, AND TAPER PER FOOT



1. The above snap gage for checking shafts has an allowance of .06% oversize. Determine the distance x.



2. Determine the distance A.

3. H is what per cent of 91?

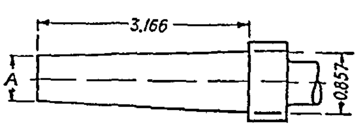
4. J is 7.8% of what number?

5. How much is 4.6% of K?

6. How much is 6.7% of L?

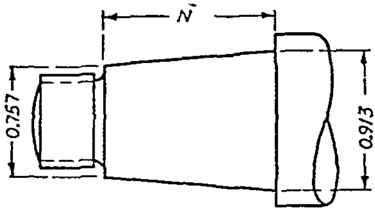
7. What decimal would express the ratio between 19.2 and S?

8. If 10 men can grind T castings in 21 days by working 9 hr. a day, how many days will it require 13 men working 8 hr. a day to grind 8913 castings?

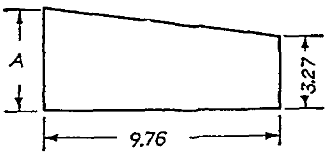


M = taper per foot.

9. Determine the distance A.

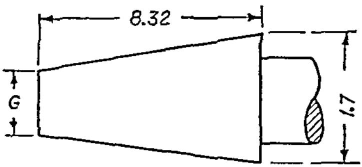


10. Determine the taper per foot.



F = taper per foot.

11. Determine the value of A.



12. Determine the taper per foot in the figure above.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	F	.505	.525	.545	.565	.585	.605
2	G	7.25	6.75	6.25	5.75	5.25	4.75
3	H	13	16	19	22	25	28
4	J	89.2	87 2	85 2	83.2	81.2	79.2
5	K	459.29	479.39	499.49	519.59	539.69	559.79
6	L	196.4	189.6	183 4	179.5	174.3	168.6
7	S	40.8	39.6	38.4	37.2	35.8	34.6
8	T	6625	6675	6725	6775	6825	6875
9	M	.406	.431	.456	.481	.496	.521
10	N	3.331	3.352	3.373	3.384	3.395	3.406
11	F	.211	.221	.241	2.61	.281	.301
12	G	.783	.763	.743	.723	.703	.683

13. If 18 pipes, each delivering 5 gal. per minute, fill a tank in 2 hr. and 10 min., how long will it take 11 pipes each delivering H gal. per minute to fill a tank three times as large as the first?

14. If 7 men can build a wall in J days, how long would it take 13 men to build a wall of the same size?

15. What per cent of his time does a man rest when he sleeps K hr. out of every 24?

16. A man had $1200. He gave 30% of this to his son, and L% of the remainder to his daughter. How much did the daughter receive?

17. If it requires 4500 tiles, 8 in. long by 4 in. wide, to pave a courtyard 40 ft. long by 32 ft. wide, how many tiles, 10 in. square, will be needed to pave a hall M ft. long and 28 ft. wide?

18. If 9 gages cost $27.63, how much would N of the same kind of gages cost?

19. Determine the cost of an article listed at P dollars with 27% and 5% off for cash.

20. If 6 men working 9 hr. a day can build Q rods of fence in 5 days, how many rods of fence can 11 men build by working 7 hr. a day for 13 days?

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
13	H	2.75	3	3.25	3.5	3.75	4
14	J	13	12	11	10	9	8
15	K	8.5	9	9.5	10	10.5	11
16	L	32	30	28	26	24	22
17	M	85	87	89	91	93	95
18	N	22	21	20	19	18	17
19	P	56.75	59.75	62.75	65.75	58.75	71.75
20	Q	849	859	869	879	889	899

SQUARE ROOT

The **square root** of a number or quantity is one of the two equal factors which, when multiplied together, will produce the given number or quantity.

The **radical sign** ($\sqrt{\ }$) preceding a number, or group of numbers, indicates that the square root of the number, or group of numbers, is to be found.

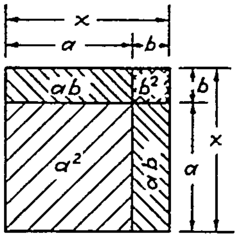
The **square** of a number is the number multiplied by itself.

Example: The square of 4 is 4×4.

The length of the **vinculum** (———) attached to the radical sign indicates the extent of the figures to be considered when finding the square root.

The number from which the square root is to be extracted is called the **radicand.**

The algebraic method for obtaining square root can be readily understood from the accompanying diagram.



If the square root of a quantity (x^2) is desired, that quantity may be represented by the area of the largest square in the figure above.

This largest square (x^2) is seen to be made up of a large square (a^2), a small square (b^2), and two rectangles (each equal to ab). Thus:

$$x^2 = (a + b)^2 = a^2 + b^2 + ab + ab$$

or

$$x^2 = (a + b)^2 = a^2 + 2ab + b^2.$$

Hence the square of $a + b$ is seen to be the square of the first term plus two times the product of the first and second terms plus the square of the second term.

Similarly the square of $a - b$ is $(a - b)^2 = a^2 - 2ab + b^2$.

From the above relation, $x^2 = (a + b)^2 = a^2 + 2ab + b^2$, it follows that $\sqrt{a^2 + 2ab + b^2}$ is $a + b$ or x (by Axiom V).

The details of obtaining $a + b$ as the square root of the quantity $a^2 + 2ab + b^2$ will not be given in this text but a full explanation for obtaining the square root of a number which is based on this algebraic process is given as follows:

Rules for Extracting Square Root.

1. *Separate the radicand into groups consisting of two figures. This must be done by starting at the decimal point and counting to both the left and right. Indicate the groups by a prime symbol. Should the last group to the right of the decimal point consist of a single figure, a cipher should be added to complete the group.*

2. *Find the largest number which when squared will be contained by the first group. Write this number as the first figure of the root directly above the first group.*

3. *Subtract the square of the first figure of the root from the first group of figures and annex to this the second group to form the new partial radicand.*

4. *Form the trial divisor,*[1] *by multiplying the root by 2, and add a small cipher. This small cipher represents the next figure of the root and should be replaced by that figure after it has been obtained. Write this trial divisor to the left of the partial radicand.*

5. *Find how many times the partial radicand contains the trial divisor and write this figure over the second group as a second figure of the root, and also write it immediately above the small cipher to complete the exact divisor.*

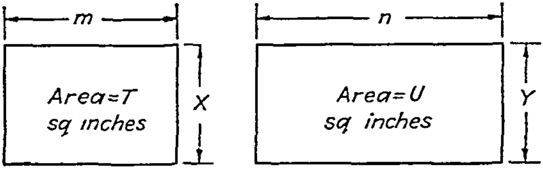
6. *Multiply this exact divisor by the last figure of the root. Write this product under the partial radicand and subtract. (Note: If the product is larger than the partial radicand, a smaller number must be used for the last digit of the exact divisor (see Example a). Annex to this remainder the next group of figures to form the new partial radicand.*

7. *Continue to apply Rules 4, 5, and 6 until sufficient figures are obtained in the root.*

8. *Place the decimal point in the root directly above the decimal point in the radicand.*

Example a: Extract the square root of 762.879

[1] This is called the trial divisor because its last figure is not complete. The last figure will always be the same as the next figure in the root.



$m = 1\ 5x$ $\qquad$ $n = 2.25y$

25. Determine the distance x **26.** Determine the distance y.

VARIABLES

Prob	Sym	No 1	No 2	No 3	No 4	No 5	No 6
1	*A*	48361	50149	82743	39133	67876	91435
2	*B*	196043	276135	342753	876345	991873	146373
3	*C*	26 – 37	50 – 65	37 – 82	65 – 82	26 – 50	50 – 82
4	*D*	78543	12371	23945	76198	67342	81148
5	*E*	38 141	22 345	32 176	85 131	96 127	33 236
6	*F*	00235	00721	00684	00875	00937	00751
7	*G*	9 1436	9 2639	9 1832	9 1935	9 7623	9 9891
8	*H*	784 136	724 733	234 132	625 142	321 123	438 143
9	*J*	00432	00178	00812	00625	00562	00473
10	*K*	43215	67823	14538	78912	49625	86535
11	*L*	04623	07948	091191	08608	07843	04535
12	*M*	45 631	32 1812	25 2253	16 3484	17 3515	18 3546
13	*N*	000076	000067	000057	000045	000032	000027
14	*P*	59121	75261	34375	20857	91546	46172
15	*R*	16 464	25 789	37 078	49 750	64 952	82 023
16	*S*	532 31	1230 9	1856 8	2709 6	3972 6	5196 1
17	*N*	51 387	53 251	55 377	57 451	59 459	59 673
18	*P*	4 5632	5 6387	5 9693	6 1636	6 3637	6 5756
19	*R*	16387	18936	20137	20443	22445	24934
20	*S*	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	*A*	226 35	238 42	247 55	262 91	277 25	289 33
22	*B*	5 962	6 143	6 896	7 385	7 956	8 236
23	*C*	9 894	10 278	10 536	10 897	11 286	11 957
24	*D*	19 289	19 546	19 972	20 132	20 495	20 867
25	*T*	4 364	4 512	4 937	5 139	5 426	5 634
26	*U*	7 193	7 483	7 720	7 916	8 287	8 542

MEANING OF FORMULAS AND METHOD OF SUBSTITUTION IN FORMULAS

A formula is a rule expressed in letters or symbols. The letters or symbols used in a formula simply represent given

figures which are to be substituted in their respective places when the formula is evaluated. The multiplication sign in a formula is generally omitted. When a number, letter, or symbol immediately precedes or follows another letter or symbol without any operation symbol between them, it is understood that multiplication should be performed.

The evaluation of an expression is the process of determining its value by substituting definite numbers for the letters and then performing the operations as indicated.

The formula $A = \pi r^2$, where A stands for the area of a circle, r for the radius of that circle, and π for the constant 3.1416, tells us that the area of any circle may be obtained by squaring the radius of that circle and multiplying that result by π.

Evaluation: Compute the area of a circle having a radius of 10 in.

$$A = \pi r^2 = 3.1416\ (10)^2 = 3.1416 \times 100 = 314.16 \text{ sq. in.}$$

As a second example of evaluating a formula by substituting numbers for letters in a formula, compute the volume of the frustum of a right circular cone by using the formula:

$$\text{Volume} = .2618\ H(D^2 + d^2 + Dd).$$

Where $H = 2.5$, $D = 5.1$, and $d = 3.4$,

$$\begin{aligned} \text{Volume} &= .2618 \times 2.5(5.1^2 + 3.4^2 + 5.1 \times 3.4) \\ &= .6545(26.01 + 11.56 + 17.34) = .6545(54.91) \\ &= 35.94 \text{ cu. in.} \end{aligned}$$

Usually in a formula the one quantity desired is placed on the left side of the equation, and all the other quantities involved are on the right side. If the formula is in the form of a fraction without plus or minus signs between terms in either the numerator or the denominator, the quantity desired is directly proportional to all quantities in the numerator and indirectly (inversely) proportional to all quantities in the denominator.

PROBLEMS

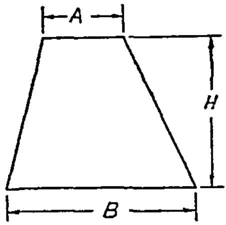
Determine the left member of the following formulas:

1. $A = \frac{h}{2}(b + c)$. $h = 13$. $b = 7$. $c =$ variable.
2. $h = r + \frac{1}{2}\sqrt{4r^2 - c^2}$. $c = 6.5$. $r =$ variable.

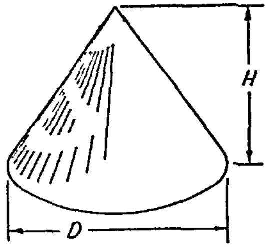
3. $d = \frac{t}{3}\sqrt{rs}$. $t = 7$. $r = 4.2$. s = variable.
4. $A = B + C - P$. $C = 7.3$. $P = 2.6$. B = variable.
5. $P = \frac{N + n}{2C}$. $N = 38$. $C = 5.7$. n = variable.
6. $S = \frac{1.157}{P} - A$. $A = .035$. P = variable.
7. $C = \frac{N - n}{2P}$. $n = 21$. $P = 8$. N = variable.
8. $W = V + r - R$. $V = 20$. $r = 3$. R = variable.
9. $P = \frac{2P}{SP - N}$. $S = 9$. $N = 20$. P = variable.
10. $t = T\frac{C - F}{C}$. $C = 4.5$. $F = 2.8$. T = variable.
11. $A = \frac{BC}{D} + C$. $C = 6.8$. $D = 2.5$. B = variable.
12. $M = AB^2 + D$. $A = 3.9$. $D = 4.3$. B = variable.
13. $D = .3183NP$. $P = .437$. N = variable.
14. $C = \frac{3.1416d}{L}$. $d = 4.875$. L = variable.
15. $A = 2\sqrt{2S(D - 2S)}$. $S = 3.6$. D = variable.
16. $D = \frac{2CN}{N + n}$. $C = 5.5$. $n = 20$. N = variable.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	c	11.2	12.3	13.4	14.5	15.6	16.7
2	r	3.5	4.3	5.4	6.5	7.6	8.7
3	s	3	4	5	6	7	8
4	B	2.3	3.4	4.5	5.6	6.7	7.8
5	n	15	16	17	18	19	20
6	P	8	9	10	11	12	13
7	N	50	52	54	56	58	60
8	R	4	5	6	7	8	9
9	P	3	4	5	6	7	8
10	T	23	25	27	29	31	33
11	B	4.6	5.3	6.8	7.4	8.7	9.2
12	B	3.7	4.6	5.5	6.4	7.3	8.2
13	N	21	22	23	24	25	26
14	L	3.2	4.4	5.3	6.7	7.6	8.9
15	D	12.8	14.7	16.5	18.3	19.6	20.4
16	N	25	28	31	34	37	40



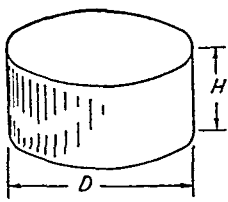
Area $= \frac{1}{2}H(A + B)$
$A = 3.5$
$B = 5.7$



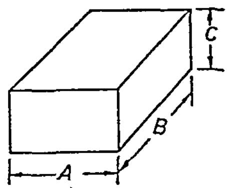
Volume $= .2618D^2H$
$H = 4.1$

17. Determine the area.

18. Determine the volume.



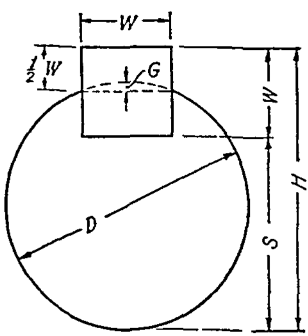
Gallons $= .0034D^2H$
$D = 14.5$



Gallons $= .004329ABC$
$A = 12.6$
$C = 5.8$

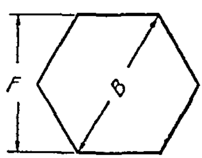
19. Determine number of gallons.

20. Determine number of gallons.

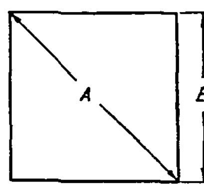


$$G = \frac{D - \sqrt{D^2 - W^2}}{2}. \quad W = .5.$$

$$S = D - \frac{W}{2} - G.$$



$B = 1.1547F.$

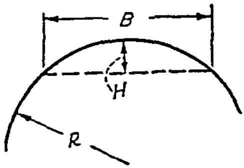


$A = 1.4142E.$

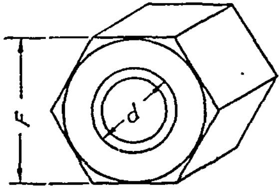
21. Determine S.
22. Determine G.

23. Determine B.
24. Determine B.

25. Determine A.
26. Determine A.



$B = 3.5.\quad R = \dfrac{B^2 + 4H^2}{8H}.$

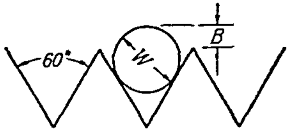


$F = 1.5d + .125.$

27. Determine the radius R.
28. Determine the radius R.

29. Determine the distance F.
30. Determine the distance F.

Sharp V Thread



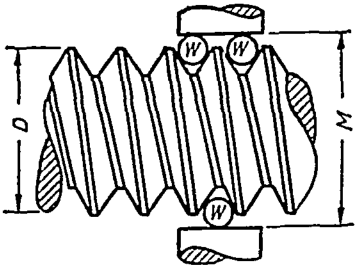
P = pitch = distance between two successive peaks = .625.
$B = 1.5W - .866P.$

31. Determine the distance B.
32. Determine the distance B.

Variables

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
17	H	7 8	7.9	8.1	8.2	8.3	8.4
18	D	4.6	4.7	4.8	4.9	5.1	5.2
19	H	7.3	7.5	7.7	7.9	8.1	8.3
20	B	18.7	18.9	19.1	19.3	19.5	19.7
21	D	1.325	1.437	1.525	1.781	1.785	1.937
22	D	1.325	1.437	1.525	1.781	1.785	1 937
23	F	1.25	1.375	1.625	1.775	1.875	2.225
24	F	2 3	2.5	2.7	2.9	3.2	3.5
25	E	1.375	1.4375	1.775	1.876	1.937	1.875
26	E	1.799	1.909	2.019	2.129	2.239	2.349
27	H	.75	.875	.9375	1.0625	1.125	1.25
28	H	1.057	.997	.937	.877	.817	.757
29	d	.525	.613	.687	.775	.844	.912
30	d	.436	.576	.642	.723	.875	.967
31	W	.375	.403	.437	.468	.505	.544
32	W	.623	.675	.714	.778	.826	.868

American National Thread



P = pitch = distance between two successive peaks = .5625. W = .375.
$M = D - 1.5155P + 3W.$

33. Determine the value of M.
34. Determine the value of M.
35. $n = \dfrac{2CP}{CP + T}$. $C = 3.7$. $P = 8$. T = variable.
36. $D = \dfrac{2\pi R}{L}$. $R = 6.3$. $\pi = 3.1416$. L = variable.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
33	D	3.177	3.288	3.399	3.4	3.511	3.622
34	D	1.654	1.797	1.824	1.963	2.186	2.25
35	T	3	4	5	6	7	8
36	L	5.6	6.7	7.2	8.3	9.5	10.4

SOLUTION OF QUADRATIC EQUATIONS

A quadratic equation is an equation involving an unknown quantity to the second power. The unknown quantity may also occur in the equation to the first power. Thus the following are quadratic equations:

(1) $x^2 - 9x + 20 = 0$. (2) $2y^2 = 50$.
(3) $7x^2 = 2x + 4$. (4) $4.16z^2 + 2.37z = 20.98$.

The standard form of the quadratic equation is

$$ax^2 + bx + c = 0$$

where x is the single unknown and a, b, and c represent numbers. Note that all terms are on the left side of the equation.

In the first equation above, x is the unknown and $a = 1$, $b = -9$, and $c = 20$. To put the second equation in the

standard form, the 50 must be transposed giving $2y^2 - 50 = 0$. Then it is seen that the unknown is y and $a = 2$, $b = 0$, and $c = -50$.

The standard form of the third equation is $7x^2 - 2x - 4 = 0$ (obtained by transposing the $2x$ and the 4). In this equation, the unknown is x, and $a = 7$, $b = -2$, and $c = -4$.

The fourth equation in the standard form is $4.16z^2 + 2.37z - 20.98 = 0$, so in this case z is the unknown and $a = 4.16$, $b = 2.37$, and $c = -20.98$.

The general solution of the standard form of the quadratic equation is obtained by a method involving the completion of a square and the extraction of the square root. The details of this process will not be given in this text, but the resultant formula will be stated, and the method of obtaining the value of the unknown by use of this formula will be explained.

For any quadratic equation of the standard form

$$ax^2 + bx + c = 0,$$

the solution is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Applying this formula to equation (1), $x^2 - 9x + 20 = 0$,

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 1 \times 20}}{2 \times 1}$$

$$= \frac{+9 \pm \sqrt{81 - 80}}{2} = \frac{+9 \pm 1}{2} = \frac{10}{2} \text{ or } \frac{8}{2} = 5 \text{ or } 4.$$

Note that there are two solutions, *i.e.*, two values of x which will satisfy the equation. In general the number of solutions of an unknown in an equation is equal to the highest power to which the unknown occurs in the equation.

The values of the unknown obtained should always be substituted in the equation as a check. Thus in the above problem substituting $x = 5$ in the original equation gives

$$5^2 - 9 \times 5 + 20 = 0 \text{ or } 25 - 45 + 20 = 0 \text{ or } 0 = 0,$$

which proves that $x = 5$ is a correct solution. Similarly checking the value $x = 4$,

$$4^2 - 9 \times 4 + 20 = 0 \text{ or } 16 - 36 + 20 = 0 \text{ or } 0 = 0.$$

The simple method for obtaining y in equation (2) is to divide both members by 2 giving $y^2 = 25$ and to extract the square root giving $y = \pm 5$. To show that the general formula solution will give the same results,

$$2y^2 - 50 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0 \pm \sqrt{0^2 - 4 \times 2 \times (-50)}}{2 \times 2}$$

$$= \frac{\pm\sqrt{400}}{4} = \frac{\pm 20}{4} = \pm 5.$$

Substituting $y = +5$ in the original equation gives $2 \cdot (5)^2 = 50$ or $50 = 50$ and substituting $y = -5$ gives $2 \cdot (-5)^2 = 50$ or $50 = 50$.

Equation (3) in the standard form is $7x^2 - 2x - 4 = 0$.

Applying the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 7 \times (-4)}}{2 \times 7}$$

$$= \frac{+2 \pm \sqrt{4 + 112}}{14} = \frac{2 \pm \sqrt{116}}{14} = \frac{2 \pm 10.7703}{14}$$

$= .9122$ or $-.6265$. Check for $x = .9122$.

$$7(.9122)^2 = 2 \times .9122 + 4 \text{ or } 5.8247 = 5.8244.$$

The slight discrepancy in the check is due to the fact that the last number in .9122 is not exactly 2 (it is nearer 2 than 3 or 1). The student should check the value $x = -.6265$.

Equation (4) in the standard form is $4.16z^2 + 2.37z - 20.98 = 0$.

Applying the formula,

$$z = \frac{-2.37 \pm \sqrt{(2.37)^2 - 4 \times 4.16 \times (-20.98)}}{2 \times 4.16}$$

$$= \frac{-2.37 \pm \sqrt{5.6169 + 349.107}}{8.32} = \frac{-2.37 \pm 18.8341}{8.32}$$

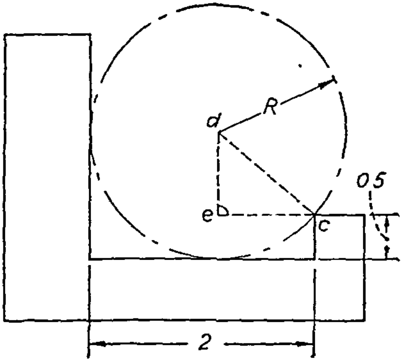
$$= \frac{-2.37 + 18.8341}{8.32} = 1.9788$$

or

$$z = \frac{-2.37 - 18.8341}{8.36} = -2.5363$$

The student should check both values of z.

As an example of how a quadratic equation may originate, consider the following geometrical problem.



Compute the radius of a cylindrical plug which will touch the gage at the point C and be tangent to the other two surfaces.

$R^2 = \overline{cd}^2 = \overline{ce}^2 + \overline{ed}^2$ (see geometric proposition 31 on page 138).

$$ce = 2 - R \text{ and } ed = R - .5.$$

Hence $R^2 = (2 - R)^2 + (R - .5)^2$,

$$R^2 = 4 - 4R + R^2 + R^2 - R + .25 \qquad \text{(see page 68).}$$

Collecting and putting in the standard form

$$R^2 - 5R + 4.25 = 0$$

Hence the above quadratic equation has been formed from the conditions given in the problem. The actual value of the radius may be obtained by solving this equation by means of the general quadratic formula as follows:

$$R = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 4.25}}{2 \times 1}$$

$$= \frac{5 \pm \sqrt{25 - 17}}{2} = \frac{5 \pm \sqrt{8}}{2} = \frac{5 \pm 2.8283}{2} = \frac{7.8283}{2} \text{ and}$$

$$\frac{2.1717}{2} = 3.9142 \text{ and } 1.0859, \text{ respectively.}$$

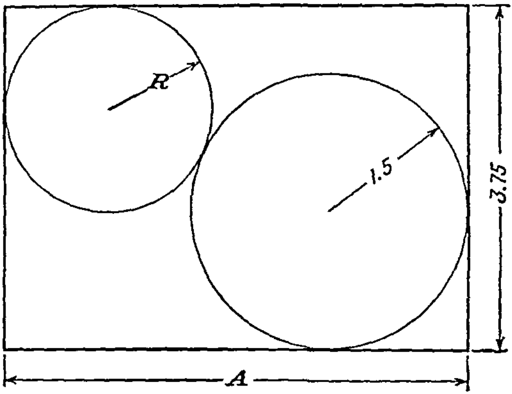
Of the two possible values given by the solution on page 80, only the value less than 2 (*i.e.*, 1.0859) is seen to fit the condition of the problem.

PROBLEMS

Determine the value of the unknown symbol:

1. $x^2 - 6x + 8 = 0$. *Ans.* $x = -2, -4$.
2. $2y^2 - 6 = 9y - 7$.
3. $5R^2 + 5R - 12 = 0$.
4. $x^2 + 7x - 12 = 0$.
5. $2y^2 - y = 4 + 3y$. *Ans.* $y = 1.2268, -12.2268$.
6. $3R^2 - 12R = 7R^2 + 6$.
7. $y^2 + 11y - 15 = 0$.
8. $2x^2 - 8x - 5 = 0$.
9. $7y^2 + 2 = y^2 - 3y + 7$.
10. $5x^2 = 9x^2 + 2 + 7x$. *Ans.* $x = -.35961, -1.3904$.
11. $6R^2 - 7R + 3 = 0$.
12. $x^2 - 12x + 27 = 0$.
13. $3x^2 + 12x - 36 = 0$.
14. $34x - x^2 - 225 = 0$.
15. $16x^2 - 16x + 3 = 0$. *Ans.* $x = .25, .75$.
16. $3R^2 - 10R + 3 = 0$.
17. $2y^2 - 12y + 10 = 0$.
18. $5x^2 - 3x - 2 = 0$.
19. $9R^2 - 24R + 16 = 0$.
20. $y^2 - 4 = 4y - 7$. *Ans.* $y = 1, 3$.
21. $R^2 - 2R + 3 = 0$.
22. $6x^2 - 5x - 1 = 0$.
23. $y^2 - 14y - 51 = 0$.
24. $R^2 - 6R + 8 = 0$.
25. $5x^2 - 4x - 1 = 0$. *Ans.* $x = 1, -.2$.
26. $2y - y^2 = 4y - 3$.
27. $R^2 + R - 20 = 0$.
28. $x^2 - x - 12 = 0$.
29. $5y^2 - 2 = 7y + 6$.
30. $3R^2 + 5R = 7$. *Ans.* $R = .90672, -2.5734$.
31. $2x^2 - 3x - 4 = 0$.
32. $3x - 7 = 7x^2 + 4$.
33. $9R^2 + 8R - 6 = 0$.
34. $y^2 - 2y - 1 = 0$.
35. $6x^2 - 4x - 5 = 0$. *Ans.* $x = 1.3051, -.63849$.
36. $4R^2 + 9R - 6 = 0$.
37. $y^2 - 3y - 7 = 0$.
38. $8x = 3x^2 - 5$.
39. $2R^2 + 7R + 4 = 0$.
40. $x^2 - 9x + 7 = 0$. *Ans.* $x = .85995, 8.1400$.

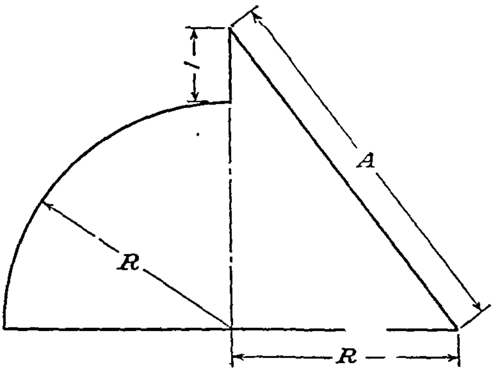
The following eight geometrical algebraic problems are to be solved by the aid of the general quadratic formula. Each problem has a variable A, and for convenience the value of A should be inserted in the problem before forming a solution.



Variable		
No.	Sym.	Value
1	A	4.25
2	A	4.37
3	A	4.50
4	A	4.62
5	A	4.87
6	A	4.93

$A = 5.0$
Ans. $R = 1.1265$

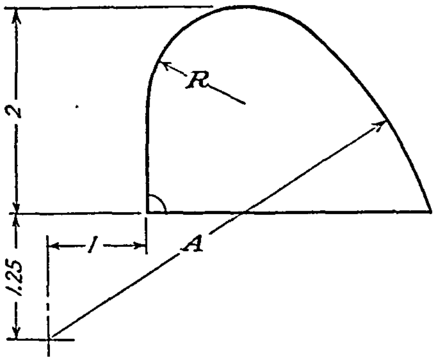
1. Determine the radius R.



Variable		
No.	Sym.	Value
1	A	4.00
2	A	4.25
3	A	4.50
4	A	4.75
5	A	5.00
6	A	5.25

$A = 5.50$
Ans. $R = 3.3468$

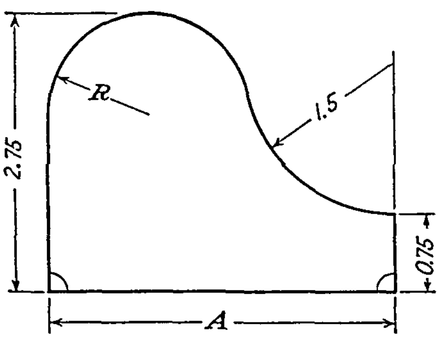
2. Determine the radius R.



VARIABLE		
No.	Sym.	Value
1	A	3.76
2	A	3.80
3	A	3.84
4	A	3.88
5	A	3.92
6	A	3.96

$A = 4.00$
Ans. $R = .98865$

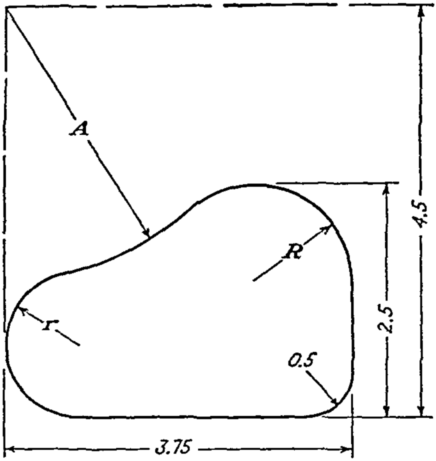
3. Determine the radius R.



VARIABLE		
No.	Sym.	Value
1	A	3.20
2	A	3.25
3	A	3.30
4	A	3.35
5	A	3.40
6	A	3.45

$A = 3.50$
Ans. $R = 1.0279$

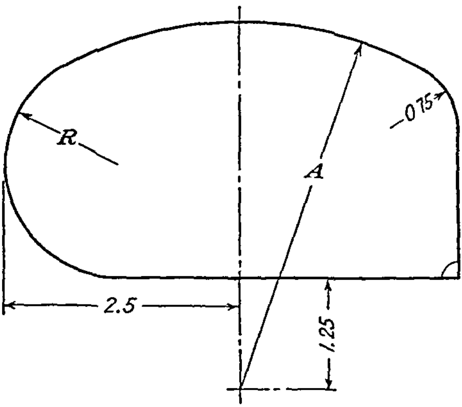
4. Determine the radius R.



Variable		
No.	Sym.	Value
1	A	3 75
2	A	3 31
3	A	3 25
4	A	3 18
5	A	3.12
6	A	3.06

$A = 3.00$
$R = 1\ 0758$
Ans. $r = \ .79185$

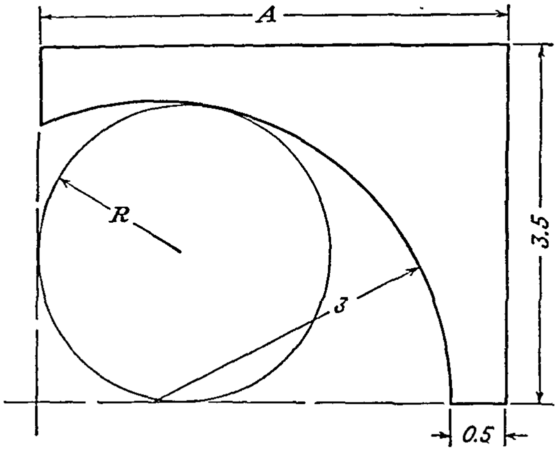
5. Determine the radius R.
6. Determine the radius r.



Variable		
No.	Sym.	Value
1	A	3 81
2	A	3 84
3	A	3 87
4	A	3 90
5	A	3 93
6	A	3 96

$A = 4\ 00$
Ans. $R = 1\ 2186$

7. Determine the radius R.



Variable		
No.	Sym.	Value
1	A	4.37
2	A	4.43
3	A	4.50
4	A	4.56
5	A	4.62
6	A	4.68

$A = 4.75$
Ans. $R = 1.4903$

8. Determine the radius R.

CHAPTER VI

THE SLIDE RULE

The slide rule is an instrument that greatly simplifies the common mathematical processes of multiplication, division, proportion, squares, square roots, etc.

Practical shop problems occurring in toolrooms, die rooms, and machine repair departments, which must be solved by the mechanic, the draftsman, and the engineer, require accuracy to five significant figures. Slide-rule computations are reliable to only three significant figures, so for most actual machine-shop problems, the slide rule cannot be used. However, in carrying out practice problems, where the main object is to obtain practice on the geometrical phases of the problem and the numerical result is of secondary importance, the use of the slide rule in getting approximate answers will save the student hours of time. Furthermore, a slide-rule solution may be used as a rapid check on the ordinary method of multiplication, division, etc.

BRIEF THEORY OF THE SLIDE RULE

In Chap. V on algebra, the idea was exemplified that when quantities expressed with exponents are multiplied, the exponents are added. Thus: $(a^3)\,(a^2) = a^5$. When quantities are divided, the exponents are subtracted. Thus:

$$\frac{a^5}{a^3} = a^{5-3} = a^2.$$

In common logarithms the base number is 10, and the exponent is the degree of the power to which 10 must be raised to give the number. Thus: $10 = 10^1$, $100 = 10^2$, $1000 = 10^3$. A number between 10 and 100 will have an exponent between 1 and 2. Thus: $83 = 10^{1.91908}$, a number between 100 and 1000 will have an exponent between 2 and 3. Thus: $624 = 10^{2.79518}$. The integer part (1 in the case of 83

and 2 in the case of 624) is called the **characteristic** and is determined by inspection. The fractional part of the exponent is called the **mantissa** and has been carefully worked out for all numbers and is given in tables of common logarithms.

To multiply 2 by 3, the characteristics are seen to be 0 (any number between 1 and 10 has a characteristic 0). The mantissas are sought in a five-place "log" table and found to be .30103 and .47712, respectively. Thus $2 \times 3 = (10^{0.30103})(10^{0.47712}) = 10^{0.30103+0.47712} = 10^{0.77815}$, by adding exponents. Reversing the procedure for finding the mantissa of a number, the number having the mantissa of .77815 is found from the table to be 6. Thus $2 \times 3 = 6$. This seems a lot of work to obtain the result, but the amount of work and time is no greater in multiplying 347 by 728.

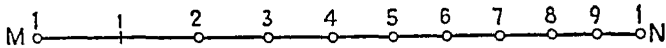


FIG. 14.

In the ordinary slide rules the scales C and D are laid off to represent the mantissas of numbers from 1 to 10.

To show how this is done consider that the line MN in Fig. 14, which is 3 in. long, is to be made into the C scale of a slide rule. The point M is labeled 1, since the log 1 is zero. Point N is also labeled 1, although we may better think of it as 10 for this discussion.

The mantissa of the log 2 is .30103. Hence the number to represent 2 must be .30103 of the distance from M to N (1 to 10). Thus .30103 × 3 in. = .90309 in., which is the distance of 2 from M. Likewise, the mantissa of the log of 3 being .47712, the number 3 is located .47712 × 3 in. = 1.43136 in. to the right of M. Similarly, the digits 4, 5, 6, 7, 8, and 9 are located.

Subdivisions on the scale are determined in the same manner. Thus the location of the small 1 between the left digit 1 and the digit 2 (which represents 1.1) is obtained by multiplying the mantissa of the log of 1.1 (which is .04139) by 3 in. (which gives .12417 in.).

The D scale is made identical to the C scale. Hence, when the left index (the 1) of scale C is placed above the large

number 2 on the scale D and the indicator is moved to the large 3 on the C scale, the distance representing the mantissa of 2 is being added to the distance representing the mantissa of 3, thus giving on the D scale under the indicator the mantissa of 6.

To divide 6 by 3, we have

$$\frac{6}{3} = \frac{10^{0.77815}}{10^{0.47712}} = 10^{0.77815-0.47712} = 10^{0.30103}.$$

The table of logarithms shows that the number having the exponent (mantissa) .30103 is 2.

To divide 6 by 3 using the slide rule, set the large 3 of the C scale directly over the large 6 of the D scale using the indicator (the hairline on slide) to line them up carefully. The answer 2 is found on the D scale directly under the left index of the C scale. The indicator should be moved to this index of the C scale in order to read the answer on the D scale as accurately as possible. The student should realize that in this process the exponent to which 10 must be raised to give 3 (the length on the C scale from left index to the 3) has been subtracted from the exponent to which 10 must be raised to give 6 (the length on the D scale from the left index to the 6), thus giving the exponent to which 10 must be raised to give 2 (the length on the D scale from the left index to the 2).

USE OF THE SLIDE RULE

Multiplication

Rule.—*To multiply two numbers, set the index (the figure* 1) *of the C scale directly above one of the numbers on the D scale and read the answer on the D scale under the other number on the C scale.*

Note: If setting the *left* index of the C scale over one number brings the other number beyond the range of the D scale, the *right* index of the C scale must be used.

The above procedure is summarized in formula (1), page 97.

Example a: Multiply 2×3.

Set the left index of the C scale directly over the large 2 of D

scale (see Fig. 15) and read, under the large 3 of the C scale, the answer 6 on the D scale.

The student should note that the distance between successive integers on the C and D scales diminishes as the numbers increase. For that reason the space between 1 and 2 is first divided into 10 parts (divisions numbered) and each of these

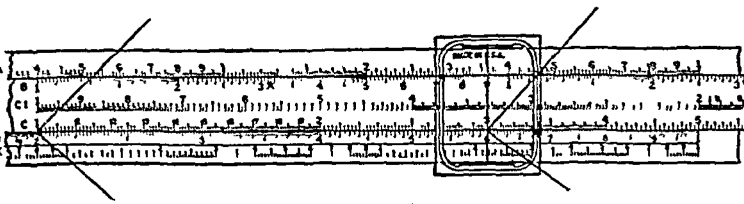


FIG. 15.[1]

parts is subdivided in 10 parts (divisions not numbered). Thus 1.5, which must be midway between 1 and 2, is at the graduation line labeled with the small 5 located between 1 and 2. Since there are 10 graduated divisions between 1.5 and 1.6, each large division represents 1 unit; hence 1 small division beyond this small 5 is 1.51, etc.

Between the large 2 and the large 3 there are also 10 main divisions (not numbered) each of which is divided into 5 subdivisions. The longest line (midway between the large 2 and the large 3) is 2.5. Since there are only 5 graduations between 2 5 and 2.6, each graduation represents 2 units. Hence the first small line beyond 2.5 is 2.52. Halfway along this small division, since each graduation represents 2 units, is one greater than the previous number, or the reading is 2.53, etc.

The space between 3 and 4 is divided in the same manner as the space between 2 and 3 except that the divisions are smaller.

The space between 4 and 5 (and 5 and 6, etc) is divided into 10 main divisions, each of which is divided into 2 subdivisions. Thus the longest line (midway between 4 and 5) is 4 5. Since there are only 2 graduations between 4.5 and 4.6, each graduation represents 5 units, and the next (small) line beyond 4.5 represents 4.55. The student should learn to estimate readings on this part of the scale (from 4 to the right index, which

[1] The slide rule cuts are by courtesy of the Keuffel & Esser Co.

is 10) to three figures. Thus if the hairline of the indicator seems to be about two-fifths (slightly less than half) of the division beyond that corresponding to 4.55, the reading is estimated to be 4.57. If the hairline is at four-fifths of that division (nearly to the 4.60 line), the reading is estimated to be 4.59, etc.

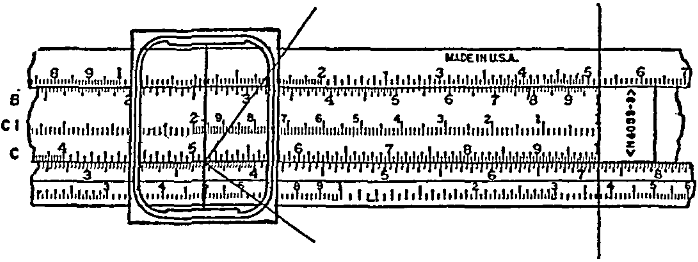


Fig. 16.

The student must also understand that the large 3 may represent 3, in which case 4 represents 4, 5 is 5, etc., or the large 3 may represent 30, in which case the 4 represents 40, the 5 is 50, etc. Similarly, the large 3 may represent 300, 3000, .3, .003, etc. The 1.51 already mentioned can also represent 15.1, 151, 1510, .151, .0151, etc.

Example b: Multiply 72 × 51.

Set the *right* index of the C scale directly over 72 (same place as 7.2) of the D scale as in Fig. 16 and under 51 (same as 5.1) of the C scale read the answer 3672 on the D scale. Actually the slide-rule reading would give only the first three figures 367, which would give an answer of 3670, but in this case it is noted that the product of the last two figures (1 × 2) is 2.

Determination of the Position of the Decimal Point.—To determine the location of the decimal point the student should mentally carry out the process using simple numbers which approximate the actual numbers. Thus in the previous problem, 72 × 51 carried out mentally is 70 × 50, which is equal to 3500. This shows that the answer is 3670 rather than 367 or 36.7.

Example c: Multiply 2.47 × 34.2.

Set the *left* index of C directly over 2.47 on the D scale

(2.47 is halfway between the lines representing 2.46 and 2.48) as in Fig. 17 and under 34.2 (same as 3.42) on the C scale read the answer 84.5 on the D scale. The answer is known to be 84.5 rather than 8.45 or 845, because using the approximate simple numbers 2 × 30 gives 60, which is nearer 84.5 than 8.45 or 845.

FIG. 17.

Actually 2.47 × 34.2 is 84.474, but the slide rule can be used only to the first three significant figures, which in this case would be 84.5.

Division

Rule.—*To divide one number (the dividend) by another (the divisor), set the divisor on the C scale directly above the dividend on the D scale, and under the index of the C scale, read the answer (the quotient) on the D scale.*

The above rule is summarized in Formula 2 on page 97.

Example a: Divide 6 by 3.

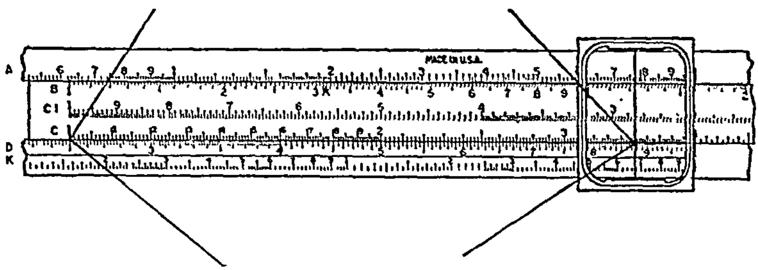


FIG. 18.

Since division is the reverse of multiplication, Fig. 15 can be used. The divisor 3 of the C scale is placed (with the aid of the indicator) directly above the dividend 6 of the D scale, and under the index of the C scale is the answer 2.

Example b: Divide 875 by 35.

Place the divisor 35 on the C scale directly above the dividend 875 on the D scale as in Fig. 18. Under the index of the C scale read the quotient 25 on the D scale. Using approximate numbers, $900 \div 30 = 30$, which shows that the answer is 25 rather than 2.5 or 250.

Multiplication and Division

In a problem involving several multiplications and divisions, first carry out a division, then a multiplication, then another division, then another multiplication, etc. It is not necessary to record the intermediate answers in such problems.

Squares and Square Roots

The A scale consists of two complete logarithmic scales each half as long as the logarithmic scales of C and D. The B scale is similar to the A scale, and multiplication and division can be carried out with the A and B scales. However, this is seldom done, as less accurate estimates can be made with shorter scales.

The principal use of the A scale is in obtaining squares and square roots when used in conjunction with the D scale.

Rule for Squares.—*Set the indicator line on any number on the D scale, and the square of that number will be found under the indicator line on the A scale.* See formula 4 on page 97.

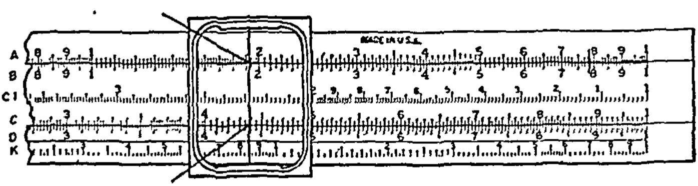


FIG. 19.

Example: Obtain the square of 43.8.

Set the indicator line on 43.8 on the D scale as in Fig. 19, and under the line on the A scale read the answer 1920. To determine the position of the decimal point, note that the square of the approximate simple number 40 is (by inspection) 1600, so that the answer had to be 1920 rather than 192 or 19200. Actually $(43.8)^2$ is 1918.44. However, the slide rule

gives only three significant figures, so if there are to be four figures in the answer one zero must be added to the three numbers given by slide rule. The square of 438 would be 192000, since $(400)^2$ is 160000.

Rule for Square Roots.

a. To find the square root of a number having an **odd** *number of figures before the decimal point or, in the case of a decimal fraction, having an* **odd** *number of zeros immediately to the right of the decimal point, set the indicator line on the number on the* **left** *half of the A scale and read the square root on the D scale under the indicator line.*

FIG. 20.

b. To find the square root of a number having an **even** *number of figures before the decimal point or, in the case of a decimal fraction, having an* **even**[1] *number of zeros immediately to the right of the decimal point, set the indicator line on the number on the* **right** *half of the A scale and read the square root on the D scale under the indicator line.*

See formula 5, on page 97.

Example a: Obtain the square root of 625.

Since there are three figures before the decimal point, Rule *a* applies. With the indicator line on 625 of the left A scale as in Fig. 20, the square root is found on the D scale under the indicator line to be 25.0. The answer is 25, rather than 2.5 or 250, since using approximate simple numbers gives $20 \times 20 = 400$.

Example b: Find the square root of 6250.

Since there are four figures before the decimal point, Rule *b* applies. Setting the indicator line on 6250 on the right A scale as in Fig. 21, the square root is found on the D scale

[1] A decimal fraction with no zeros such as .432 is equivalent to having an even number of zeros.

under the indicator line to be 79.1. The position of the decimal point is determined by noting that $80 \times 80 = 6400$.

Example c: Find the square root of .0506.

Since this is a decimal fraction with one zero immediately to the right of the decimal point, Rule *a* applies. Setting the indicator line on .0506 of the left A scale gives the square root

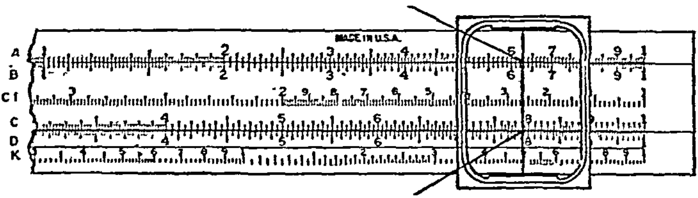


FIG. 21.

on the D scale to be .224. This is the correct position of the decimal point, since using approximate simple numbers gives $.2 \times .2 = .04$.

Proportion

Many problems may be solved by setting up a proportion. Thus if it is known that 8 castings weigh 23.2 lb., how much will 67 castings weigh? How much will 39 of these castings weigh?

$$\frac{23.2}{8} = \frac{x}{67}.$$

Set 8 on the C scale above 23.2 on the D scale, and under 67 on the C scale read the answer 194 on the D scale. Without moving the slide (the middle movable section of the slide rule), move the indicator to 39 on the C scale, and on the D scale under the indicator read the second answer 113.

Note that the above procedure is summarized in formula 3 on page 97.

Example: If 50 bolts from a bin weigh 113 lb., compute: the weight of 6000 such bolts; the weight of 10 gross (1440) such bolts.

$$\frac{113}{50} = \frac{x}{6000} = \frac{y}{1440}.$$

Set 50 on the C scale directly above 113 on the D scale.

Under 6000 on the C scale read on the D scale the answer for x (13560 lb.).

Since 1440 on the C scale is not above the D scale, the left index must be moved to the reading occupied by the right index (with the aid of the indicator). Then under 1440 on the C scale is the answer on the D scale for y (3254). Note that the last figure of the answer is very uncertain; however, the answer is obviously somewhat more than 3250 and less than 3260.

Problems Involving Sines, Tangents, Cosecants, Cotangents, Cosines, and Secants

This part of this chapter will be better understood after the student has studied the material given on trigonometric functions given in Chap. VIII.

If the slide is reversed, the S scale is adjacent to the A scale and the T scale to the D scale.

Rule for Obtaining Sine of an Angle.—*With the ends of the S and A scales coincident, place the indicator over the angle on the S scale and the sine of this angle is under the indicator on the A scale.* See formula 16 on page 98.

Note: (*a*) All sines read on the *right* half of the A scale have the decimal point just before the first figure. (*b*) All sines read on the *left* half of the A scale have a zero between the decimal point and the first significant figure.

Example a: Obtain the sine of 30°. Since 500 on the *right-hand* A scale is just above 30 on the S scale, sine 30° = .500.

Example b: Obtain the sine of 4°. Since 698 on the *left-hand* A scale is just above 4 on the S scale, sine 4° = .0698.

The procedure just discussed may be reversed to give an angle corresponding to a given value of the sine.

Example a: Obtain the angle when its sine is .0425.

Under 425 of the *left-hand* A scale, the angle is found to be 2° 26′.

Example b: Obtain the angle α when $\sin \alpha = .623$.

Under 623 of the *right-hand* A scale, the angle is found to be 38° 30′.

Rule for Obtaining Tangent of an Angle.—*With the ends of the T and D scales coincident, set the indicator over the angle*

on the T scale, and the tangent of this angle is under the indicator on the D scale.

Example a: Obtain the tangent of 30°.

Under 30° on the T scale, .577 is seen to be the tangent.

The angle corresponding to a given value of the tangent may be obtained by reversing the procedure just given.

Example b: Obtain the angle α when $\tan \alpha = .352$.

Just over .352 on the D scale is the answer 19° 23′ on the T scale.

The use of the T and D scales gives directly the tangents of angles from 5° 43′ to 45°. However, by using the relations, $\tan \alpha = \frac{1}{\cot \alpha}$, page 183 and $\cot \alpha = \tan (90° - \alpha)$, page 183, the tangents of angles from 45° to 84° 17′ can also be found. For example,

$$\tan 62° = \frac{1}{\cot 62°} = \frac{1}{\tan (90° - 62°)} = \frac{1}{\tan 28°}.$$

For angles less than 5° 43′, $\tan \alpha = \sin \alpha$ nearly, so the tangent is obtained on the slide rule by looking up the sine.

If the cosine of an angle is needed, it may be obtained from the relation, $\cos \alpha = \sin (90° - \alpha)$, page 183. Thus $\cos 40° = \sin (90° - 40°) = \sin 50°$, which is given on the S scale to be .766.

Cotangents for angles up to 45° may be obtained from the relation $\cot \alpha = \frac{1}{\tan \alpha}$, and for angles over 45°, $\cot \alpha = \tan (90° - \alpha)$.

Cosecants may be handled on the slide rule from the relation $\csc \alpha = \frac{1}{\sin \alpha}$, page 183.

Secants are obtained from the S scale by using the relations $\sec \alpha = \frac{1}{\cos \alpha} = \frac{1}{\sin (90° - \alpha)}$, page 183.

In problems in trigonometry, frequently a number must be multiplied or divided by the sine of an angle or by the tangent, by the cosecant, by the cotangent, by the cosine, or by the secant.

Example a: Multiply $12 \sin 28°$.

Set the index (beginning mark) of the S scale under 12 of the A scale. The answer is on the A scale directly above 28 on the S scale and is found to be 5.63.

Example b: Compute 17 cot 25°.

Since $\cot 25 = \dfrac{1}{\tan 25°}$, the problem is $\dfrac{17}{\tan 25°}$. Set 25 on the T scale over 17 on the D scale and read the answer 36.5 on the D scale under the index of the T scale.

Example c: Compute 44 sec 48°.

Since $\sec 48 = \dfrac{1}{\cos 48°} = \dfrac{1}{\sin\ (90° - 48°)}$, the problem is $\dfrac{44}{\sin 42°}$. Set 42° on the S scale under 44 of the right half of the A scale and read the answer 65.8 on the A scale over the index of the T scale.

SUMMARY OF SLIDE-RULE MANIPULATIONS

The following slide-rule "formulas" are summaries of preceding operations with the addition of certain other combinations.

1. $X = a \times b$	Set 1 on C to a on D; at b on C read X on D.
2. $X = a \div b$	Set b on C to a on D; at 1 on C read X on D.
3. $X = a \times b \div c$	Set c on C to a on D; at b on C read X on D.
4. $X = a^2$	Over a on D, read X on A.
5. $X = \sqrt{a}$	Under a on A, read X on D.
6. $X = a \times b^2$	Set 1 on B to a on A; over b on on C, read X on A.
7. $X = a \div b^2$	Set b on C under a on A; at 1 on B read X on A.
8. $X = a^2 \div b$	Set b on B over a on D; at 1 on B read X on A.
9. $X = a^2 \times b^2$	Set 1 on C to a on D; over b on C read X on A.
10. $X = a^2 \div b^2$	Set b on C to a on D; over 1 on C read X on A.

11. $X = a^2 \times b \div c$	Set c on B to b on A; over a on C read X on A.
12. $X = a \times b \div c^2$	Set c on C under a on A; over b on B read X on A.
13. $X = a^2 \times b^2 \div c$	Set c on B over a on D; over b on C read X on A.
14. $X = a^2 \times b \div c^2$	Set c on C to a on D; at b on B read X on A.
15. $X = a^2 \times b^2 \div c^2$	Set c on C to a on D; over b on C read X on A.
16. $X = \sin \alpha$	Set index of S to index of A; over α on S read X on A.
17. $X = \tan \alpha$	Set index of T to index of D; under α on T read X on D.
18. $\sin \alpha = b$	Set index of S to index of A; under b on A read α on S.
19. $\tan \alpha = b$	Set index of T to index of D; over b on D read α on T.
20. $X = b \sin \alpha$	Set index of S to b on A; over α on S read X on A.
21. $X = b \tan \alpha$	Set index of T to b on D; under α on T read X on D.
22. $X = b \csc \alpha$	Set α on S to b on A; over index of S read X on A.
23. $X = b \cot \alpha$	Set α on T to b on D; under index of T read X on D.
24. $\sin \alpha = a \div b$	Set index of S to b on A; under a on A read α on S.
25. $\tan \alpha = a \div b$	Set index of T to b on D; over a on D read α on T.
26. $\csc \alpha = a \div b$	Set index of S to a on A; under b on A read α on S.
27. $\cot \alpha = a \div b$	Set index of T to a on D; over a on D read α on T.

The last 12 formulas will be used by the student after he has studied the following chapter but are placed here to avoid splitting the discussion on the use of the slide rule.

PROBLEMS

Use a slide rule to solve:

1. $8.31 \times A = ?$ 2. $.325 \times B = ?$ 3. $\frac{59.3}{C} = ?$ 4. $\frac{7.12}{D} = ?$

5. $\frac{76.4 \times E}{28.3} = ?$ 6. $\frac{1.286 \times 94.5}{F} = ?$ 7. $G^2 = ?$ 8. $H^2 = ?$

9. $\sqrt{I} = ?$ 10. $\sqrt{J} = ?$ 11. $.918 \times K^2 = ?$ 12. $L \times (.416)^2 = ?$

13. $\frac{M}{(13.6)^2} = ?$ 14. $\frac{962}{N^2} = ?$ 15. $\frac{P^2}{4.18} = ?$ 16. $\frac{(13.7)^2}{Q} = ?$

17. $(32.1)^2 \times R^2 = ?$ 18. $S^2 \times (4.93)^2 = ?$ 19. $\frac{T^2}{(6.38)^2} = ?$

20. $\frac{(19.6)^2}{U^2} = ?$ 21. $\frac{(2.54)^2 \times V}{19.8} = ?$ 22. $\frac{W^2 \times 78.6}{624} = ?$

23. $\frac{7.31 \times A}{(62.4)^2} = ?$ 24. $\frac{29.3 \times 82.4}{B^2} = ?$ 25. $\frac{(22.7)^2 \times C^2}{13.2} = ?$

26. $\frac{(39.4)^2 \times (.132)^2}{D} = ?$ 27. $\frac{(1.56)^2 \times E}{(14.6)^2} = ?$ 28. $\frac{F^2 \times 16.5}{(19.4)^2} = ?$

29. $\frac{(21.8)^2 \times G^2}{(16.7)^2} = ?$ 30. $\frac{(3.17)^2 \times (13.8)^2}{H^2} = ?$ 31. $\sin J = ?$

32. $\sin K = ?$ 33. $\tan L = ?$ 34. $\tan M = ?$

35. $\sin \alpha = N, \alpha = ?$ 36. $\sin \alpha = P, \alpha = ?$ 37. $\tan \alpha = Q, \alpha = ?$

38. $\tan \alpha = R, \alpha = ?$ 39. $24.6 \sin S = ?$ 40. $T \sin 15° 37' = ?$

41. $642 \tan U = ?$ 42. $V \tan 6° 53' = ?$ 43. $68.3 \csc W = ?$

44. $A \csc 32° 14' = ?$ 45. $93.6 \cot B = ?$ 46. $C \cot 5° 55' = ?$

47. $\sin \alpha = \frac{D}{12.4}, \alpha = ?$ 48. $\sin \alpha = \frac{61.2}{E}, \alpha = ?$ 49. $\tan \alpha = \frac{F}{76.2}, \alpha = ?$

50. $\tan \alpha = \frac{2.84}{G}, \alpha = ?$ 51. $\csc \alpha = \frac{H}{39.4}, \alpha = ?$

52. $\csc \alpha = \frac{82.9}{J}, \alpha = ?$ 53. $\cot \alpha = \frac{K}{32.4}, \alpha = ?$

54. $\cot \alpha = \frac{78.6}{L}, \alpha = ?$

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	*A*	5.62	5.82	6.02	6.22	6.42	6.62
2	*B*	132	134	136	138	142	144
3	*C*	7.26	7.28	7.32	7.34	7.36	7.38
4	*D*	14 28	14.32	14.34	14.36	14.38	14.42
5	*E*	24.7	26.7	28.7	30.7	31.7	32.7
6	*F*	.305	.307	.309	.311	.313	.315
7	*G*	3.11	3.31	3.51	3.71	3.91	4.11
8	*H*	31.1	33.1	35.1	37.1	39.1	41.1
9	*I*	53.9	56.0	58.1	60.2	62.3	64.4
10	*J*	539	560	581	602	623	644

Prob	Sym	No 1	No 2	No 3	No 4	No 5	No 6
11	*K*	11 2	11 4	11 6	11 8	12 2	12 4
12	*L*	25 9	26 4	26 9	27 4	27 9	28 4
13	*M*	42 8	43 9	45 0	46 1	47 2	48 3
14	*N*	7 12	7 17	7 22	7 27	7 32	7 37
15	*P*	7 81	7 92	8 03	8 14	8 25	8 36
16	*Q*	1 88	1 93	1 98	2 03	2 08	2 13
17	*R*	198	187	176	165	154	143
18	*S*	6 38	6 49	6 60	6 71	6 32	6 93
19	*T*	9 02	9 13	9 24	9 35	9 46	9 57
20	*U*	40 2	39 2	38 2	37 2	36 2	35 2
21	*V*	5 65	5 76	5 87	5 98	6 09	6 20
22	*W*	5 58	5 62	5 64	5 66	5 68	5 70
23	*A*	22 6	23 6	24 6	25 6	26 6	27 6
24	*B*	37 7	36 7	35 7	34 7	33 7	32 7
25	*C*	20 8	20 6	20 4	20 2	19 8	19 6
26	*D*	88 6	89 6	90 6	91 6	92 6	93 6
27	*E*	396	407	418	429	440	451
28	*F*	31 7	31 2	30 7	30 2	29 7	29 2
29	*G*	27 9	28 3	28 9	29 3	29 8	30 4
30	*H*	40 2	40 7	41 2	41 7	42 2	42 7
31	*J*	3° 54′	3° 48′	3° 42′	3° 36′	3° 24′	3° 18′
32	*K*	40° 10′	39° 40′	39° 10′	38° 40′	38° 10′	37° 40′
33	*L*	8° 28′	8° 17′	8° 6′	7° 55′	7° 44′	7° 33′
34	*M*	30° 57′	30° 52′	30° 46′	30° 41′	30° 33′	30° 25′
35	*N*	0801	0806	0811	0816	0821	0826
36	*P*	694	698	702	706	710	714
37	*Q*	236	233	227	224	221	218
38	*R*	868	878	888	898	908	918
39	*S*	48° 10′	48° 30′	48° 50′	49° 10′	49° 30′	49° 50′
40	*T*	45 4	44 4	43 4	42 4	41 4	40 4
41	*U*	44° 12′	44° 2′	43° 52′	43° 42′	43° 32′	43° 22′
42	*V*	3 66	3 76	3 86	3 96	4 06	4 16
43	*W*	2° 43′	2° 38′	2° 33′	2° 28′	2° 23′	2° 18′
44	*A*	11 9	12 4	12 9	13 4	13 9	14 4
45	*B*	41° 48′	41° 43′	41° 38′	41° 33′	41° 28′	41° 23′
46	*C*	30 4	29 9	29 4	28 9	28 4	27 9
47	*D*	1 84	1 89	1 94	1 99	2 04	2 09
48	*E*	93 3	92 3	91 3	90 3	89 3	88 3
49	*F*	8 74	8 64	8 54	8 44	8 34	8 24
50	*G*	4 01	4 11	4 21	4 31	4 41	4 51
51	*H*	76 6	77 6	78 6	79 6	80 6	81 6
52	*J*	44 4	45 4	46 4	47 4	48 4	49 4
53	*K*	198	218	238	258	278	298
54	*L*	70 2	69 2	68 2	67 2	66 2	65 2

CHAPTER VII

GEOMETRY

PLANE GEOMETRY

1. **Plane geometry** is a study of points, lines, triangles, quadrilaterals, circles, and other common figures. For this study we assume the truth of a certain number of fundamental statements called **axioms.**

2. From these axioms and certain proved statements, we reason the proofs of other statements. These proved statements are called **propositions** or **theorems.**

3. A statement, the truth of which is seen to be a direct consequence of a proposition or axiom, is called a **corollary** (abbreviated cor.).

AXIOMS

The following axioms will be referred to frequently:

Axiom I.—*Things which are equal to the same thing, or to equal things, are equal to each other.*

Axiom II.—*Any quantity may be substituted for its equal in a mathematical expression.*

Axiom III.—*If equals are added to equals, the sums are equal.*

Axiom IV.—*If equals are subtracted from equals, the remainders are equal.*

Axiom V.—*If equals are multiplied by equals, the products are equal.*

Axiom VI.—*If equals are divided by equals, the quotients are equal.*

Axiom VII.—*The whole is greater than any of its parts.*

Axiom VIII.—*The whole is equal to the sum of its parts.*

Axiom IX.—*Only one straight line can be drawn from one point to another. That is to say, two points determine a straight line.*

Corollary to Axiom IX.—*Two straight lines can intersect in only one point.*

For, if two straight lines could intersect in two points, we should have two straight lines drawn between the two points.

Axiom X.—*Through a given point only one line can be drawn parallel to a given line.*

Corollary to Axiom X.—*If two lines are each parallel to a third line, they are parallel to each other.*

For, if the two are not parallel, they would intersect, which would give two lines through the same point, which is impossible by Axiom X.

DEFINITIONS

4. A **straight line** is the shortest line that can be drawn through two points. If any portion (or segment) of a straight line be placed with its extremities on another part of the straight line, the whole of the first part will lie along the second portion.

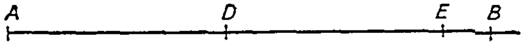


Fig. 22.

Thus, the line AB is the shortest line that can be drawn from A to B, and if AD is placed with its extremities at D and E, it coincides with DE throughout.

5. An **angle** is the figure formed by drawing two straight lines outward from a common point. The point is called the **vertex** of the angle, and the bounding straight lines are called the **sides** of the angle.

Fig. 23.

Thus the angle AVB (often written $\angle AVB$) has the vertex V and the sides VA and VB.

6. If the two sides of an angle extend in opposite directions from the vertex, the angle is called a **straight angle.**

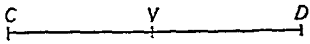


FIG. 24.

In Fig. 16, $\angle CVD$ is a straight angle.

7. Two angles are called **adjacent angles** if they have a common side. $\measuredangle EVF$ and FVG; having the common side VF are adjacent angles.

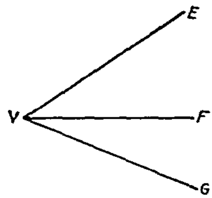


FIG. 25.

8. If two adjacent angles formed by the intersection of two straight lines are equal, each angle is a **right angle.** The equal adjacent angles HVI and IVJ are each right angles.

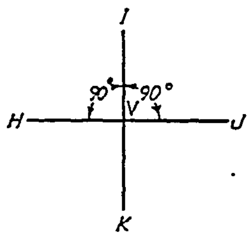


FIG. 26.

In the future, some of the right angles will be indicated by a small arc. Thus:

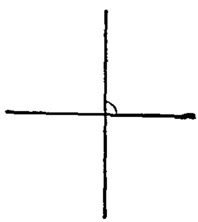


FIG. 27.

9. An angle is measured in degrees. A **degree** is $\frac{1}{360}$ part of a circle and is subdivided into 60 minutes, and a minute is subdivided into 60 seconds. Hence a **minute** is $\frac{1}{60}$ of a degree and a **second** is $\frac{1}{60}$ of a minute. The **symbols** used to indicate degrees, minutes, and seconds, placed at the upper right hand corner of a numeral, are as follows: 12° 15′ 45″, respectively.

10. Two angles are said to be **complementary** if their sum is equal to a right angle (90°). $\measuredangle LVM$ and MVN are complementary. Complements of the same angle or of equal angles are equal (Axiom IV).

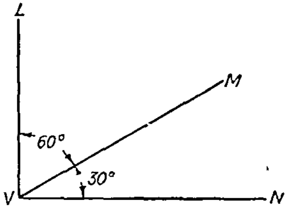


Fig. 28.

11. Two angles are said to be **supplementary** if their sum is equal to a straight angle (180°). Thus $\measuredangle OVP$ and PVQ are supplementary. Supplements of the same angle or of equal angles are equal (Axiom IV).

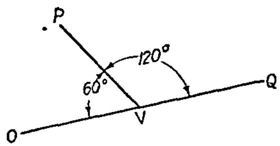


Fig. 29.

12. A **perpendicular** to a given line is a line which makes a right angle with the given line. In Figs. 26 and 28, IK is perpendicular to HJ (usually written $IK \perp HJ$), and $VN \perp VL$.

13. The point of intersection of the perpendicular with the given line is called the **foot of the perpendicular.** Point V is the foot of the perpendicular NV (Fig. 28).

14. Two straight lines are said to be **parallel** if they do not meet however far they are extended. Lines RS and TU are parallel (often written $RS \parallel TU$) (Fig. 30).

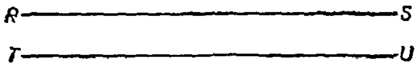


FIG. 30.

15. A **plane surface,** or **plane,** is a surface such that a straightedge will touch the surface at all points, no matter where the surface may be tested. The top of a table is a portion of a plane.

16. A **polygon** is a portion of a plane enclosed by three or more straight lines.

17. A **triangle** is a polygon of three sides.

FIG. 31.

18. A **quadrilateral** is a polygon of four sides.

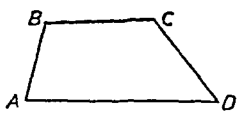


FIG. 32.

19. A **pentagon** is a polygon of five sides.

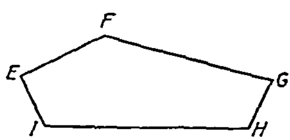


FIG. 33.

20. A **hexagon** is a polygon of six sides.

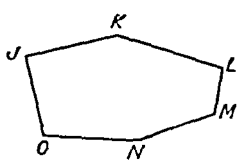


FIG. 34.

21. An **octagon** is a polygon of eight sides.

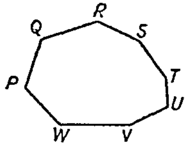


FIG. 35.

22. A **regular polygon** is one that is both equilateral and equiangular. Thus a square is a regular quadrilateral.

23. The **perimeter** of a polygon is the sum of the sides of the polygon.

24. A **parallelogram** is a quadrilateral having its opposite sides parallel. Thus $ABCD$ is a parallelogram if $AB \parallel DC$ and $BC \parallel AD$.

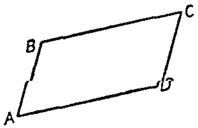


FIG. 36.

25. In equal figures, the points, lines, and angles of the two figures which coincide, when the one figure is superposed on the other, are called **homologous parts** (or corresponding parts), and homologous parts of equal figures are equal.

26. A **right-angled triangle** (or rt. $\triangle$) is a triangle one of whose angles is a right angle, as $\triangle EFG$, which has a right angle at F. The sides adjacent to the right angle are called the legs of the right triangle and the side opposite the right angle is called the **hypotenuse**.

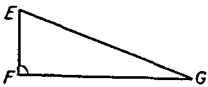


FIG. 37.

27. An **acute angle** is an angle less than 90°, as $\angle E$ or $\angle G$ (Fig. 37).

28. An **obtuse angle** is an angle greater than 90°, as $\angle HIJ$.

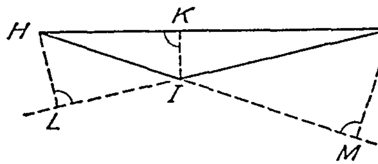


FIG. 38.

29. An **oblique triangle** is one not having any of its angles equal to a right angle as $\triangle HIJ$ (sometimes called an obtuse triangle) and $\triangle NOP$ (sometimes called an acute triangle).

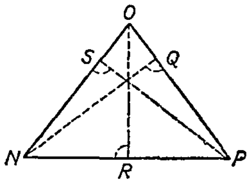


FIG. 39.

30. The three **altitudes** of an oblique triangle are the three perpendiculars from the three vertices to the opposite sides (extended if necessary) as IK, HL, and JM for $\triangle HIJ$ and NQ, OR, and PS for $\triangle NOP$.

31. An **isosceles triangle** is one having two of its sides equal, as $\triangle TUV$ (side TU = side VU).

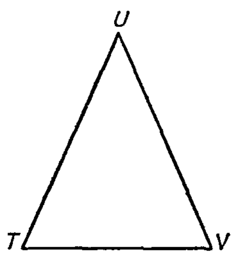


FIG. 40.

32. The **projection** of one line upon a second line is the segment of the second line included between the perpendiculars drawn to it from the extremities of the first line. Thus the

projection of AB on FG is HJ, and the projection of AB on CD is AE.

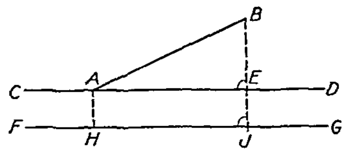


Fig. 41.

PROPOSITIONS

In the following work, axioms will be referred to as A-I, A-II, etc., definitions as D-1, D-2, etc., and propositions as P-1, P-2, etc. In the propositions to be proved, the given conditions will be referred to as the **hypothesis**, which will be abbreviated hyp.

PROPOSITION 1

If two straight lines intersect, the opposite or vertical angles are equal.

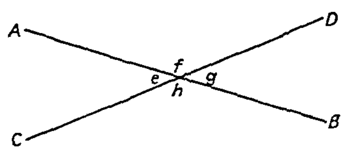


Fig. 42.

Given: The intersecting straight lines AB and CD, which *form the two pairs of vertical angles, e and g and f and h.*

To prove: $\angle e = \angle g$ and $\angle f = \angle h$.

$\angle f$ is the supplement of $\angle g$ (D-6 and D-11).

$\angle h$ is the supplement of $\angle g$ (D-6 and D-11).

$\therefore \angle f = \angle h$ (D-11).

Similarly it may be proved that $\angle e = \angle g$.

PROPOSITION 2

Two triangles are equal if two sides and the included angle of the one are equal, respectively, to two sides and the included angle of the other.

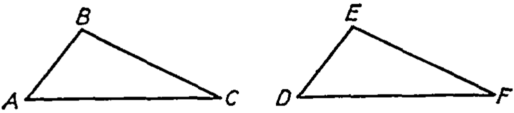


FIG. 43.

Given: $\triangle s\ ABC$ and DEF having $AB = DE$, $BC = EF$, and $\angle ABC = \angle DEF$.

To prove: $\triangle ABC = \triangle DEF$.

Place $\triangle DEF$ on $\triangle ABC$ so that vertex E falls on vertex B, side EF along BC and side ED along BA ($\angle E = \angle B$ by hyp.).

Then F will fall on C ($EF = BC$ by hyp.)
and D will fall on A ($ED = BA$ by hyp.).

Hence line DF coincides with line AC (A-IX).

Thus the triangles can be made to coincide throughout and are therefore equal.

PROPOSITION 3

Two triangles are equal if two angles and the included side of one are equal respectively to two angles and the included side of the other.

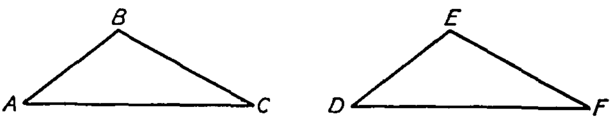


FIG. 44.

Given: $\triangle s\ ABC$ and DEF having $\angle A = \angle D$, $\angle B = \angle E$, and $AB = DE$.

To prove: $\triangle ABC = \triangle DEF$.

Place $\triangle DEF$ on $\triangle ABC$ so that DE falls on AB
($DE = AB$ by hyp.).

Then side EF will fall along BC ($\angle E = \angle B$ by hyp.).
And side DF will fall along AC ($\angle D = \angle A$ by hyp.).
Hence point F will fall on point C (cor. to A-IX).

Thus the triangles can be made to coincide throughout and are therefore equal.

Hence, since GH is perpendicular to EF, CD must be perpendicular to EF.

33. If two straight lines are cut by a third, the angles are named as follows:

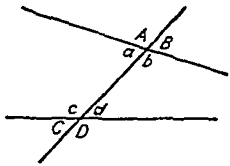


Fig. 48.

$\measuredangle A$, B, C, and D are exterior angles.

$\measuredangle a$, b, c, and d are interior angles.

$\measuredangle A$ and D, and B and C, are pairs of alternate-exterior angles.

$\measuredangle a$ and d, and b and c, are pairs of alternate-interior angles.

$\measuredangle A$ and c, B and d, C and a, D and b are pairs of exterior-interior angles (often called corresponding angles).

PROPOSITION 7

If two parallel lines are cut by a third line, the alternate-interior angles are equal.

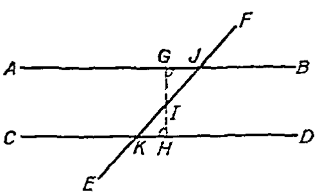


Fig. 49.

Given: Two parallel lines AB and CD cut by the third line EF.

To prove: $\angle GJI = \angle HKI$, and $\angle IJB = \angle IKC$.

Through I, midway between the two lines AB and CD and on the line EF, draw a line GH perpendicular to AB.

Then $GH \perp CD$ (P-6).

In the $\triangle IGJ$ and KHI,

$$\angle GIJ = \angle HIK \qquad \text{(P-1)},$$
$$\angle KHI = \angle JGI \qquad \text{(both rt. } \angle\text{s)}.$$
$$GI = IH \qquad (I \text{ taken as midway}).$$
$$\therefore \triangle IGJ = \triangle KHI \qquad \text{(P-3)}.$$
$$\text{Hence } \angle GJI = \angle HKI \qquad \text{(D-25)}.$$

Similarly, $\angle IJB$ may be proved equal to $\angle IKC$.

PROPOSITION 8

If two parallel lines are cut by a third line, the exterior-interior angles are equal.

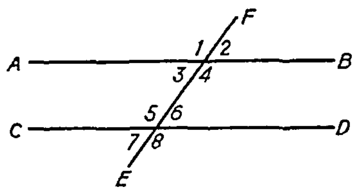


FIG. 50.

Given: Two parallel lines AB and CD cut by the third line EF.
To prove: $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 4 = \angle 8$.

$$\angle 1 = \angle 4 \qquad \text{(P-1)}.$$
$$\angle 5 = \angle 4 \qquad \text{(P-7)}.$$
$$\therefore \angle 1 = \angle 5 \qquad \text{(A-I)}.$$

Similarly, the other pairs may be proved equal.

PROPOSITION 9

If two lines in the same plane are intersected by a third line, and the exterior-interior angles are equal, the two lines are parallel.

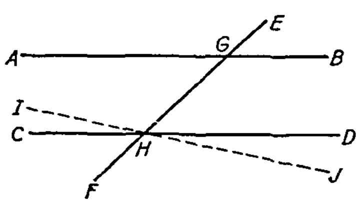


FIG. 51.

Given: Two lines AB and CD cut by the third line EF with $\angle EGB = \angle GHD$.

To prove: $CD \parallel AB$.

Assuming that CD is not parallel to AB, draw a line IJ through H parallel to AB.

Then $\angle EGB = \angle GHJ$ (P-8).

But $\angle EGB = \angle GHD$ (hyp.).

Hence $\angle GHJ = \angle GHD$ (A-I).

$\therefore$ IJ and CD must coincide since the vertices and other sides of the two equal angles coincide.

$\therefore$ $CD \parallel AB$ since CD coincides with IJ which was drawn parallel to AB.

PROPOSITION 10

The sum of the degrees of the three angles of any triangle is equal to 180°.

Given: $\triangle ABC$.

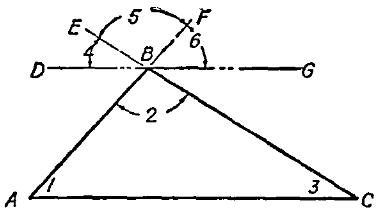


FIG. 52.

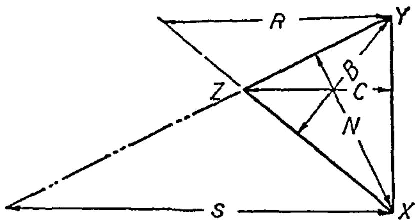
To prove: $\angle 1 + \angle 2 + \angle 3 = 180°$.

Extend AB to F, CB to E, and draw DG, through B, parallel to AC.

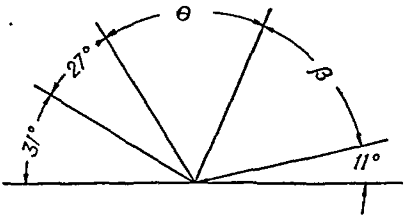
$$\angle 5 = \angle 2 \qquad \text{(P-1).}$$
$$\angle 4 = \angle 3 \qquad \text{(P-8).}$$
$$\angle 6 = \angle 1 \qquad \text{(P-8).}$$
$$\angle 4 + \angle 5 + \angle 6 = 180° \qquad \text{(D-6).}$$
$$\therefore \angle 1 + \angle 2 + \angle 3 = 180° \qquad \text{(A-II).}$$

Corollary to Proposition 10.—*The two acute angles of a right triangle are complementary.*

PROBLEMS



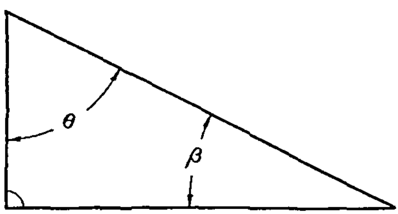
1. In the triangle XYZ, name the three altitudes. No Variable.



VARIABLE		
No.	Sym.	Value
1	θ	48°
2	θ	50°
3	θ	52°
4	θ	54°
5	θ	56°
6	θ	58°

$\theta = 60°$
Ans. $\beta = 51°$

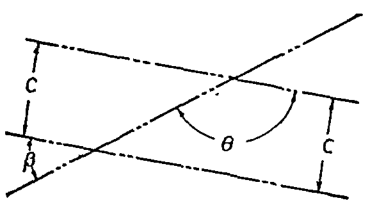
2. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	θ	51°
2	θ	52°
3	θ	53°
4	θ	54°
5	θ	55°
6	θ	56°

$\theta = 57°$
Ans. $\beta = 33°$

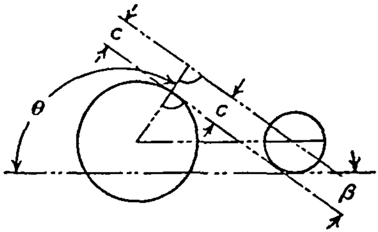
3. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	θ	131°
2	θ	132°
3	θ	133°
4	θ	134°
5	θ	135°
6	θ	136°

$\theta = 137°$
Ans. $\beta = 43°$

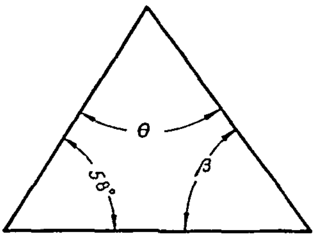
4. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	131°
2	θ	133°
3	θ	135°
4	θ	137°
5	θ	139°
6	θ	141°

$\theta = 143°$
Ans. $\beta = 53°$

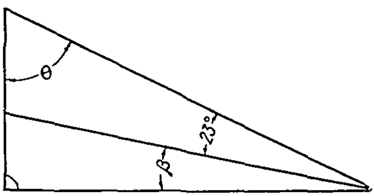
5. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	72°
2	θ	74°
3	θ	76°
4	θ	78°
5	θ	80°
6	θ	82°

$\theta = 84°$
Ans. $\beta = 38°$

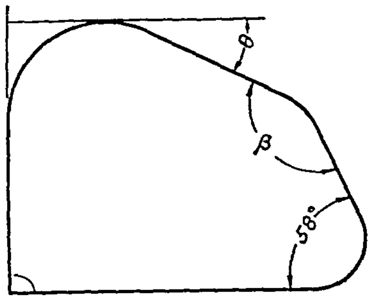
6. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	58°
2	θ	61°
3	θ	63°
4	θ	65°
5	θ	52°
6	θ	56°

$\theta = 54°$
Ans. $\beta = 13°$

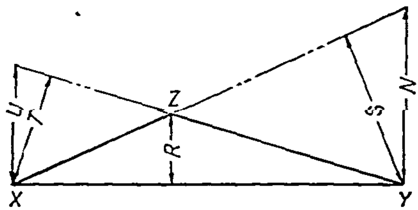
7. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	θ	26°
2	θ	28°
3	θ	30°
4	θ	32°
5	θ	34°
6	θ	36°

$\theta = 38°$
Ans. $\beta = 160°$

8. Determine the angle β.



9. In the triangle XYZ, name the three altitudes. No Variable.

PROPOSITION 11

When two angles of a triangle are equal to two angles of another triangle, the third angles are equal.

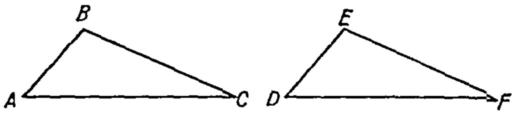


FIG. 53.

Given: Two $\triangle ABC$ and DEF with $\angle A = \angle D$ and $\angle C = \angle F$.
To prove: $\angle B = \angle E$.

$\angle A + \angle B + \angle C = 180°$ (P-10).
$\angle D + \angle E + \angle F = 180°$ (P-10).
$\angle A + \angle C = \angle D + \angle F$ (hyp. and A-III).
$\therefore \angle B = \angle E$ (A-IV).

PROPOSITION 12

The sum of the degrees of the interior angles of a polygon of N sides is (N − 2) times 180°.

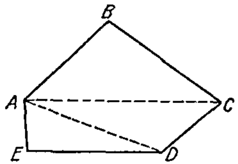


FIG. 54.

Given: Polygon $ABCDE$.

To prove: $\angle ABC + \angle BCD + \angle CDE + \angle DEA + \angle EAB = (N - 2)\ 180°$.

Draw diagonals AC and AD. This will divide the polygon into $(N - 2)$ triangles (one for each side except the adjacent sides AB and AE).

The sum of the angles of these triangles is the sum of the interior angles of the polygon.

The sum of the degrees in each triangle is 180° (P-10).

$\therefore$ the sum of the degrees of the interior angles of the polygon is $(N - 2)$ times 180°.

PROPOSITION 13

The exterior angle formed by prolonging one side of a triangle is equal to the sum of the two opposite interior angles.

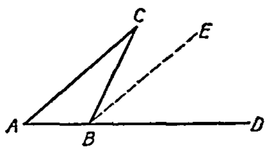


FIG. 55.

Given: $\triangle ABC$ with exterior $\angle CBD$.

To prove: $\angle CBD = \angle BAC + \angle BCA$.

Draw BE parallel to AC.

$\angle DBE = \angle BAC$ (P-8).

$\angle EBC = \angle BCA$ (P-7).

$\angle DBE + \angle EBC = \angle BAC + \angle BCA$ (A-III).
$\angle DBC = \angle DBE + \angle EBC$ (A-VIII).
$\therefore \angle DBC = \angle BAC + \angle BCA$ (A-I).

PROPOSITION 14

Two straight lines drawn from a point in a perpendicular to a given line, cutting on the given line equal segments from the foot of the perpendicular, are equal and make equal angles with the perpendicular.

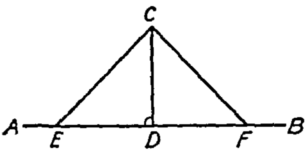


FIG. 56.

Given: CD perpendicular to line AB and oblique lines CE and CF cutting off equal segments ED and DF.

To prove: $CE = CF$ and $\angle ECD = \angle FCD$.

Fold over $\triangle CDE$ on CD as an axis until it falls on the plane to the right of CD.

ED will fall along DF ($\angle CDE = \angle CDF$, each being 90°).
Point E will fall on F ($DE = DF$ by hyp.).
$\therefore CE$ coincides with CF throughout (A-IX).
That is, $CE = CF$.
Also $\angle ECD = \angle FCD$ (vertices and sides coincide).

Corollary to Proposition 14.—*All points on the perpendicular bisector of a line are equidistant from the extremities of the line.*

In the foregoing figure, CD is the perpendicular bisector of EF, and EC has already been proved equal to CF.

PROPOSITION 15

Two angles are equal when their sides are parallel, right to right and left to left.

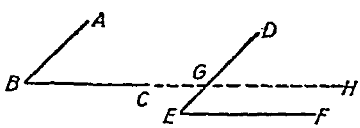
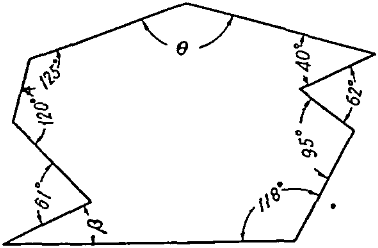


FIG. 57.

Given: $\measuredangle B$ and E with $EF \parallel BC$ and $ED \parallel BA$.
To prove: $\angle E = \angle B$.
Let BC (extended if necessary) meet ED at G.
Then $\angle E = \angle DGH$ (P-8).
And $\angle B = \angle DGH$ (P-8).
$\therefore \angle E = \angle B$ (A-I).

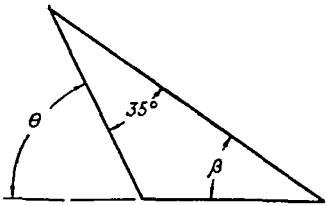
PROBLEMS



Variable		
No.	Sym.	Value
1	θ	123°
2	θ	125°
3	θ	127°
4	θ	129°
5	θ	131°
6	θ	133°

$\theta = 135°$
Ans. $\beta = 30°$

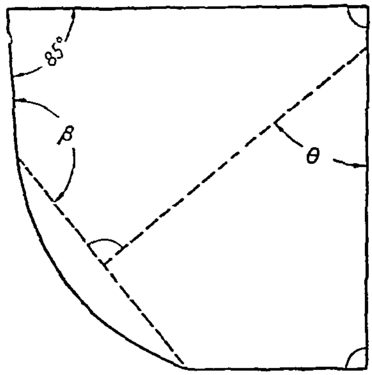
1. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	75°
2	θ	76°
3	θ	77°
4	θ	78°
5	θ	79°
6	θ	80°

$\theta = 81°$
Ans. $\beta = 46°$

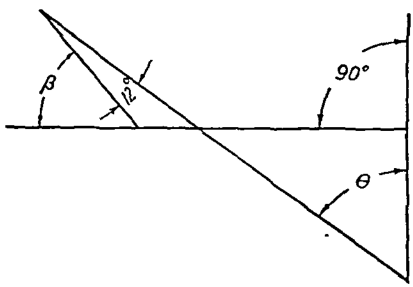
2. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	42°
2	θ	44°
3	θ	46°
4	θ	48°
5	θ	50°
6	θ	52°

$\theta = 54°$
Ans. $\beta = 149°$

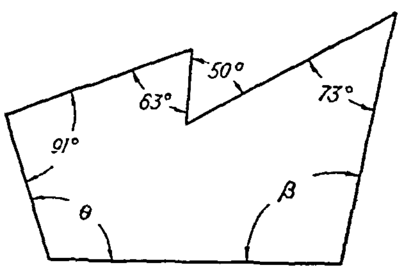
3. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	46°
2	θ	48°
3	θ	50°
4	θ	52°
5	θ	54°
6	θ	56°

$\theta = 58°$
Ans. $\beta = 44°$

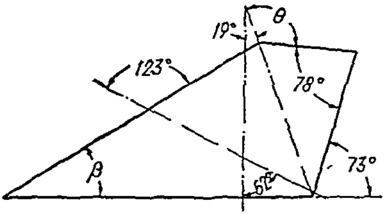
4. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	101°
2	θ	102°
3	θ	103°
4	θ	104°
5	θ	105°
6	θ	106°

$\theta = 107°$
Ans. $\beta = 76°$

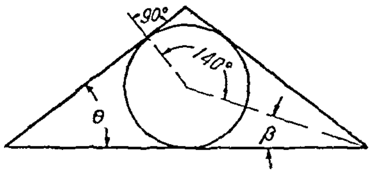
5. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	118°
2	θ	119°
3	θ	120°
4	θ	121°
5	θ	122°
6	θ	123°

$\theta = 124°$
Ans. $\beta = 39°$

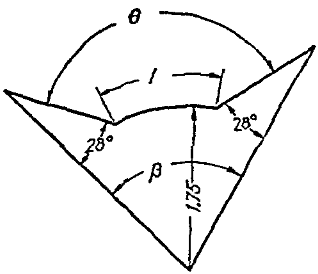
6. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	28°
2	θ	26°
3	θ	24°
4	θ	22°
5	θ	20°
6	θ	18°

$\theta = 16°$
Ans. $\beta = 34°$

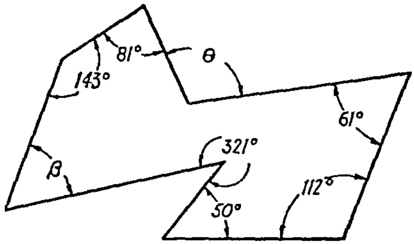
7. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	125°
2	θ	127°
3	θ	129°
4	θ	131°
5	θ	133°
6	θ	135°

$\theta = 137°$
Ans. $\beta = 81°$

8. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	θ	91°
2	θ	93°
3	θ	95°
4	θ	97°
5	θ	99°
6	θ	101°

$\theta = 103°$
Ans. $\beta = 55°$

9. Determine the angle β.

PROPOSITION 16

Two angles whose sides are perpendicular right to right and left to left are equal.

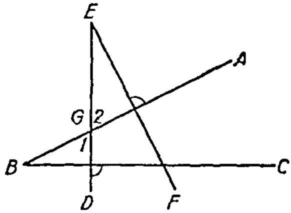


FIG. 58.

Given: ∡ABC and DEF with right side ED perpendicular to right side BC and left side EF perpendicular to left side BA.

To prove: $\angle E = \angle B$.

$\angle B$ is the complement of $\angle 1$ (cor. to P-10).
$\angle E$ is the complement of $\angle 2$ (cor. to P-10).
$\angle 1 = \angle 2$ (P-1).
$\therefore \angle B = \angle E$ (D-10).

PROPOSITION 17

The angles at the base of an isosceles triangle are equal.

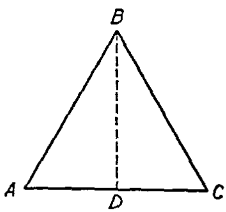


FIG. 59.

Given: Isosceles $\triangle ABC$ $(AB = BC)$.

To prove: $\angle A = \angle C$.

Draw BD so as to bisect $\angle ABC$.

In $\triangle ADB$ and CDB,

$\angle ABD = \angle CBD$ (by construction).

$AB = BC$ (hyp.).

$BD = BD$ (common).

$\therefore \triangle ADB = \triangle CDB$ (P-2).

and $\angle A = \angle C$ (D-25).

Corollary 1 to Proposition 17.—*The line from the vertex perpendicular to the base of an isosceles triangle bisects the base and the angle at the vertex.*

$\triangle ADB = \triangle CDB$ (Proof left to student).

$AD = DC$ (D-25).

and $\angle ABD = \angle CBD$ (D-25).

Corollary 2 to Proposition 17.—*If a triangle is equilateral, it is also equiangular.*

PROPOSITION 18

If equal lines are drawn from a point in a perpendicular to a given line, they cut off equal segments on that line from the foot of the perpendicular.

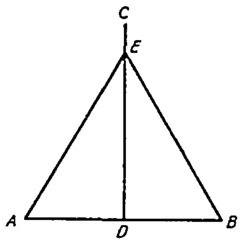


FIG. 60.

Given: $CD \perp AB$ and equal lines EA and EB drawn from point E to line AB.

To prove: $AD = DB$.

In $\triangle ADE$ and BDE,

$AE = BE$ (hyp.).

$ED = ED$ (common).

$\angle A = \angle B$ (P-17).

$$\angle ADE = \angle BDE \qquad \text{(both rt. } \angle\text{s).}$$
$$\therefore \angle AED = \angle DEB \qquad \text{(P-11).}$$
$$\text{Hence } \triangle ADE = \triangle BDE \qquad \text{(P-2).}$$
$$\therefore AD = DB \qquad \text{(D-25).}$$

PROPOSITION 19

If two angles of a triangle are equal, the sides opposite are equal (i.e., the triangle is isosceles).

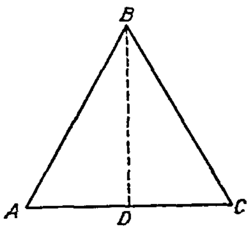


FIG. 61.

Given: $\angle A = \angle C$.
To prove: $AB = BC$.
Draw BD so as to bisect $\angle ABC$.

$$\text{Since } \angle A = \angle C \qquad \text{(hyp.),}$$
$$\text{and } \angle ABD = \angle DBC \qquad \text{(construction),}$$
$$\angle ADB = \angle CDB \qquad \text{(P-11).}$$
$$BD = BD \qquad \text{(common).}$$
$$\therefore \triangle ADB = \triangle CDB \qquad \text{(P-3).}$$
$$\text{Hence } AB = BC \qquad \text{(D-25).}$$

Corollary 1 to Proposition 19.—*The bisector of the vertical angle of an isosceles triangle is perpendicular to the base and bisects the base.*

$\angle ADB$ and $\angle CDB$ are both right angles (D-8).
$AD = DC$ (D-25).

Corollary 2 to Proposition 19.—*If a triangle is equiangular, it is also equilateral.*

PROPOSITION 20

Two triangles are equal if the three sides of the one are equal respectively to the three sides of the other.

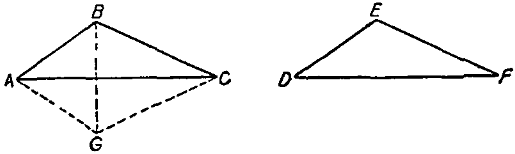


FIG. 62.

Given: $AB = DE$, $BC = EF$, and $AC = DF$.

To prove: $\triangle ABC = \triangle DEF$.

Place $\triangle DEF$ so that DF falls on its equal AC and point E falls on the opposite side from B, say at G. Draw BG.

In $\triangle BAG$, $\angle ABG = \angle AGB$ (P-17).

In $\triangle BCG$, $\angle GBC = \angle BGC$ (P-17).

$\angle ABG + \angle GBC = \angle AGB + \angle BGC$ (A-III).

That is, $\angle ABC = \angle AGC$ (A-VIII).

Hence, $\triangle ABC = \triangle AGC$ (P-2).

But $\triangle AGC = \triangle DEF$ (by construction).

$\therefore \triangle ABC = \triangle DEF$ (A-I).

PROPOSITION 21

Two right triangles are equal if the hypotenuse and a leg of one are equal respectively to the hypotenuse and a leg of the other.

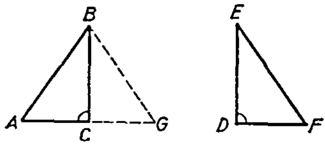


FIG. 63.

Given: Hypotenuse AB = hypotenuse EF and leg BC = leg ED.

To prove: $\triangle ABC = \triangle DEF$.

Place $\triangle DEF$ so that DE falls on its equal BC and point F falls on the opposite side from point A, say at G.

Side ACG is a straight line (D-6).
and $\triangle ABG$ is an isosceles triangle ($AB = EF$ by hyp.).
$AC = CG$ (cor. 1 to P-17).
$\therefore \triangle ABC = \triangle GCB$ (P-20).
But $\triangle GCB = \triangle DEF$ (by construction).
$\therefore \triangle ABC = \triangle DEF$ (A-I).

PROPOSITION 22

If two angles of a triangle are unequal, the sides opposite these angles are unequal and the longer side lies opposite the greater angle.

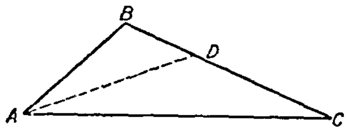


FIG. 64.

Given: $\triangle ABC$ with $\angle BAC$ greater than $\angle BCA$ (usually written $\angle BAC > \angle BCA$).
To prove: $BC > AB$.
Construct AD making $\angle DAC = \angle DCA$.
Then $AD = DC$ (P-19).
$AB < AD + DB$ (D-4).
$\therefore AB < DC + DB$ (A-II).
or $AB < BC$ (A-VIII).
That is, $BC > AB$.

PROPOSITION 23

The opposite sides of a parallelogram are equal.

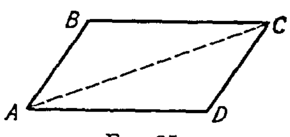


FIG. 65.

Given: Parallelogram $ABCD$ with opposite sides parallel, *i.e.*, $AB \parallel CD$ and $BC \parallel AD$.
To prove: $AB = CD$ and $BC = AD$.
Draw the diagonal AC.

In $\triangle$ ABC and ADC,

$\angle DAC = \angle BCA$, (P-7).

$\angle BAC = \angle DCA$ (P-7).

$AC = AC$ (common).

$\therefore \triangle ABC = \triangle ADC$ (P-3).

Hence $AB = CD$ and $BC = AD$ (D-25).

Corollary 1 to Proposition 23.—*Segments of parallel lines intercepted by parallel lines are equal.*

Corollary 2 to Proposition 23.—*A diagonal divides a parallelogram into two equal triangles.*

PROPOSITION 24

If three or more parallel lines intercept equal segments on one intersecting line (often called a transversal), they intercept equal segments on all intersecting lines.

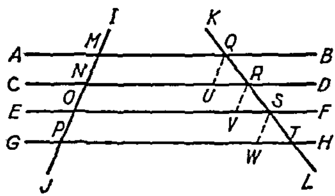


FIG. 66.

Given: Parallel lines AB, CD, EF, and GH cut by the intersecting lines IJ and KL with $MN = NO = OP$.

To prove: $QR = RS = ST$.

Through the points Q, R, and S, draw the lines QU, RV, and $SW \parallel IJ$. Then $QU \parallel RV \parallel SW$ (cor. to A-X).

$\angle QRU = \angle RSV = \angle STW$ (P-8).

$\angle RQU = \angle SRV = \angle TSW$ (P-8).

$\therefore \angle QUR = \angle RVS = \angle SWT$ (P-11).

$QU = RV = SW$ (cor. 1 to P-23).

Hence $\triangle QUR = \triangle RVS = \triangle SWT$ (P-3).

$\therefore QR = RS = ST$ (D-25).

PROPOSITION 25

If a line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally.

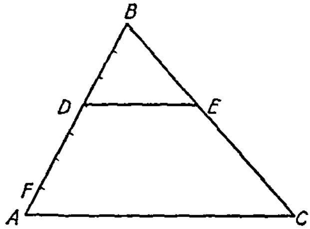


FIG. 67.

Given: $\triangle ABC$ with $DE \parallel AC$.

To prove: $\dfrac{AD}{DB} = \dfrac{CE}{EB}$.

Assume that AF is a small unit of length that fits an exact number of times into both AD and DB, say X ($X = 4$ in the figure) times in AD and Y ($Y = 3$ in the figure) times in DB.

Then $\dfrac{AD}{DB} = \dfrac{X}{Y}$.

Through the points of division of AD and DB draw lines parallel to AC. These lines will divide line BC into $X + Y$ equal parts, of which X will be in CE and Y in EB. (P-24).

Then $\dfrac{CE}{EB} = \dfrac{X}{Y}$.

Hence $\dfrac{AD}{DB} = \dfrac{CE}{EB}$ (A-I).

Note: In case no common unit can be found for the lengths AD and DB, the proposition may still be proved by using the method of limits.

Corollary 1 to Proposition 25.—*One side of a triangle is to either of its segments cut off by a line parallel to the base as the other side is to its corresponding segment.*

Since $\dfrac{AD}{DB} = \dfrac{CE}{EB}$, (P-25).

$\dfrac{AD + DB}{DB} = \dfrac{CE + EB}{EB}$ (Theorem V of Chap. V).

$$\text{That is, } \frac{AB}{DB} = \frac{BC}{EB}.$$

34. Two polygons are said to be similar if their homologous (corresponding) angles are equal and their homologous sides are proportional.

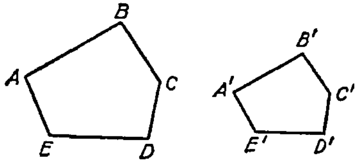


FIG. 68.

Thus polygons $ABCDE$ and $A'B'C'D'E'$ are similar (often written $ABCDE \sim A'B'C'D'E'$) if $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$, $\angle D = \angle D'$, $\angle E = \angle E'$, and $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{DE}{D'E'} = \frac{EA}{E'A'}$.

PROPOSITION 26

If two triangles are mutually equiangular, their corresponding sides are proportional and hence the triangles are similar.

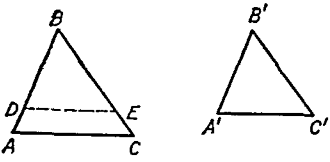


FIG. 69.

Given: $\triangle ABC$ and $A'B'C'$ with $\angle A = \angle A'$, $\angle B = \angle B'$, and $\angle C = \angle C'$.

To prove: $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$.

and $$\triangle ABC \sim \triangle A'B'C'.$$

Place $\angle A'B'C'$ on $\angle ABC$ so that $\angle B'$ coincides with $\angle B$ (vertex B' on vertex B and sides $B'A'$ and $B'C'$ falling along corresponding sides BA and BC).

A' will fall at some point D and C' at some point E. Thus the $\triangle A'B'C'$ takes the position BDE.

$$\angle BDE = \angle A \qquad \text{(hyp.)}.$$

$$\text{Hence } DE \parallel AC \qquad \text{(P-9)}.$$

$$\therefore \frac{AB}{DB} = \frac{BC}{BE} \qquad \text{(cor. 1 to P-25)}.$$

$$\text{That is, } \frac{AB}{A'B'} = \frac{BC}{B'C'} \qquad \text{(A-II)}.$$

Similarly by placing $\triangle A'B'C'$ on $\triangle ABC$ so that $\angle A'$ falls on $\angle A$, it may be shown that

$$\frac{AB}{A'B'} = \frac{AC}{A'C'}$$

$$\therefore \frac{BC}{B'C'} = \frac{AC}{A'C'} \qquad \text{(A-I)};$$

$$\text{and we have } \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}.$$

$$\therefore \triangle ABC \sim \triangle A'B'C' \qquad \text{(D-34)}.$$

Corollary 1 to Proposition 26.—*Two triangles are similar if two angles of one are equal to two angles of the other* (P-11 and P-26).

Corollary 2 to Proposition 26.—*Two right triangles are similar if they have an acute angle of one equal to an acute angle of the other.* (The right angles are also equal so Corollary 1 to Proposition 26 applies.)

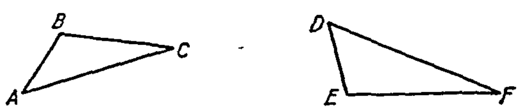


FIG. 70.

Note: It follows from Proposition 22 that the largest angle of a triangle is opposite its longest side, the next largest angle is opposite the next longest side, etc. Hence in two similar triangles as ABC and DEF, the angles that are equal are the angles opposite the sides corresponding in length. Thus $\angle C$, which is opposite AB, the shortest side of $\triangle ABC$, is equal to $\angle F$, which is opposite to DE, the shortest side of $\triangle DEF$, etc.

The student should be able to recognize corresponding angles at a glance by this method.

PROPOSITION 27

Two triangles are similar if their sides are respectively perpendicular.

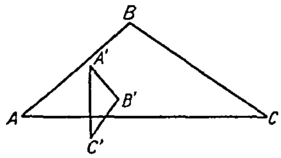


FIG. 71.

Given: $\triangle ABC$ and $A'B'C'$ with $A'B' \perp AB$, $B'C' \perp BC$, and $A'C' \perp AC$.

To prove: $\triangle ABC \sim \triangle A'B'C'$.

$\angle A = \angle A'$, $\angle B = \angle B'$, and $\angle C = \angle C'$ (P-16).

$\therefore \triangle ABC \sim \triangle A'B'C'$ (P-26).

PROPOSITION 28

Two triangles are similar if their sides are respectively parallel.

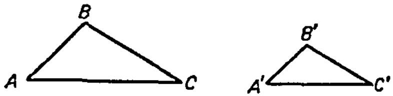


FIG. 72.

Given: $\triangle ABC$ and $A'B'C'$ with $A'B' \parallel AB$, $B'C' \parallel BC$, and $A'C' \parallel AC$.

To prove: $\triangle ABC \sim \triangle A'B'C'$.

$\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$ (P-15).

$\therefore \triangle ABC \sim \triangle A'B'C'$ (P-26).

PROPOSITION 29

If perpendiculars are drawn from two points on one side of an angle to the other side of the angle, the triangles formed are similar.

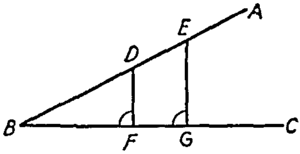


FIG. 73.

Given: $\angle ABC$ and the $\perp$s DF and EG drawn from the points D and E to the line BC.

To prove: $\triangle DBF \sim \triangle EBG$.

$\angle B$ is common and both triangles are right triangles (D-26).

$\therefore \triangle DBF \sim \triangle EBG$ (cor. 2 to P-26).

PROPOSITION 30

If, in a right triangle, a perpendicular is drawn from the vertex of the right angle to the hypotenuse:

a. The triangles formed on either side of the perpendicular are similar to the whole triangle and to each other.

b. The perpendicular is the mean proportional between the segments of the hypotenuse.

c. Each side adjacent to the right angle is a mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to that side.

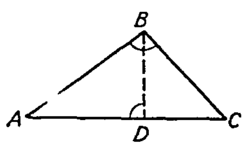


FIG. 74.

Given: $\triangle ABC$ with right angle at B and BD drawn from B perpendicular to the hypotenuse.

a. *To prove:* $\triangle$s ABC, ADB, and CDB, all similar.

Rt. $\triangle ADB \sim$ rt. $\triangle ABC$ since $\angle A$ is common (cor. 2 to P-26).

Rt. $\triangle CDB \sim$ rt. $\triangle ABC$ since $\angle C$ is common (cor. 2 to P-26).

$\triangle$s ADB and CDB, being both similar to $\triangle ABC$, have their angles equal to those of $\triangle ABC$ (D-34).

$\therefore$ Angles of $\triangle ADB$ = angles of $\triangle CDB$, respectively (A-I).

$\therefore$ $\triangle ADB \sim \triangle CDB$ (P-26).

b. To prove: $\dfrac{AD}{BD} = \dfrac{BD}{DC}$

Since $\triangle ADB \sim \triangle BDC$ (P-30*a*),

$$\frac{AD}{BD} = \frac{BD}{DC} \qquad \text{(D-34).}$$

c. To prove: $\dfrac{AC}{AB} = \dfrac{AB}{AD}$ and $\dfrac{AC}{BC} = \dfrac{BC}{DC}$.

Since $\triangle ABC \sim \triangle ADB$ (P-30*a*),

$$\frac{AC}{AB} = \frac{AB}{AD} \qquad \text{(D-34).}$$

Since $\triangle ABC \sim \triangle CDB$ (P-30*a*),

$$\frac{AC}{BC} = \frac{BC}{DC} \qquad \text{(D-34).}$$

Note: The solutions of many problems depend on Proposition 30*a* and hence it is very important that the student recognize at once the equal angles of the similar triangles. The following statement will assist in recognizing the equal angles:

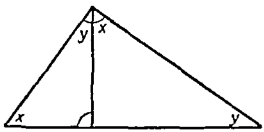
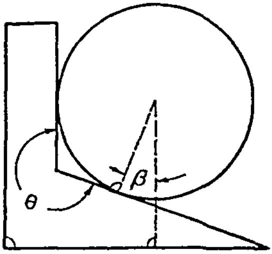


FIG. 75.

Corollary to Proposition 30.—*If a perpendicular is dropped from the vertex of the right angle to the hypotenuse, the angle opposite the perpendicular in one triangle is equal to the angle adjacent to this perpendicular in the other triangle.*

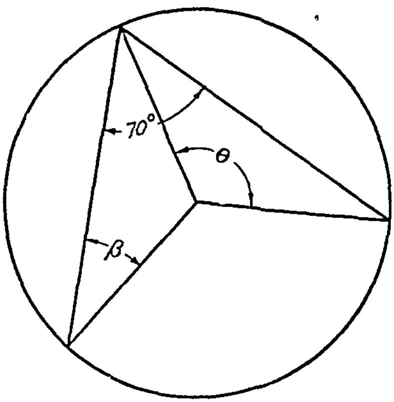
PROBLEMS



VARIABLE		
No.	Sym.	Value
1	θ	231°
2	θ	233°
3	θ	235°
4	θ	237°
5	θ	239°
6	θ	241°

$\theta = 243°$
Ans. $\beta = 27°$

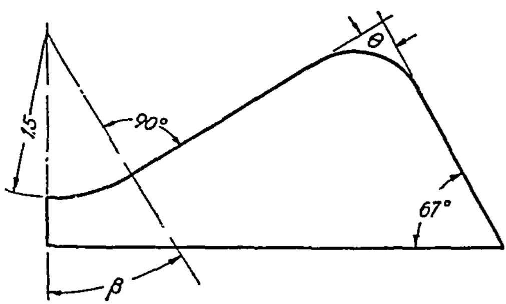
1. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	θ	116°
2	θ	118°
3	θ	120°
4	θ	122°
5	θ	124°
6	θ	126°

$\theta = 128°$
Ans. $\beta = 44°$

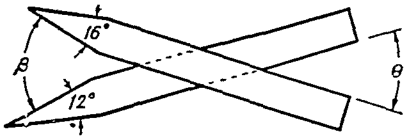
2. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	θ	75°
2	θ	78°
3	θ	80°
4	θ	83°
5	θ	85°
6	θ	88°

$\theta = 73°$
Ans. $\beta = 40°$

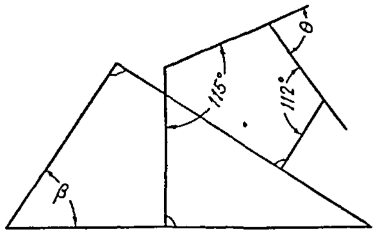
3. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	30°
2	θ	32°
3	θ	34°
4	θ	36°
5	θ	38°
6	θ	40°

$\theta = 42°$
Ans. $\beta = 56°$

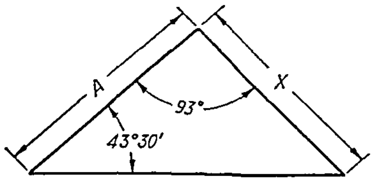
4. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	71°
2	θ	72°
3	θ	73°
4	θ	74°
5	θ	75°
6	θ	76°

$\theta = 77°$
Ans. $\beta = 60°$

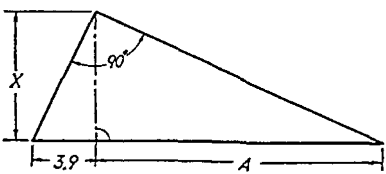
5. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	3.446
2	A	3.853
3	A	4.118
4	A	4.561
5	A	4.876
6	A	5.113

$A = 5.234$
Ans. $x = 5.234$

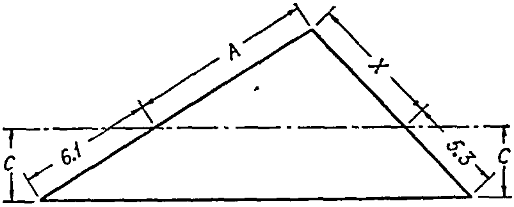
6. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	6.51
2	A	6.62
3	A	6.73
4	A	6.84
5	A	6.95
6	A	7.06

$A = 7.17$
Ans. $x = 5.2880$

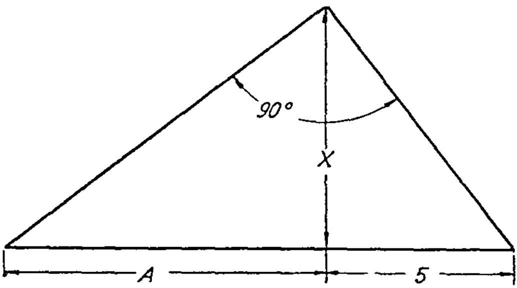
7. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	10.2
2	A	10.5
3	A	10.8
4	A	11.1
5	A	11.4
6	A	11.7

$A = 11.9$
Ans. $x = 10.339$

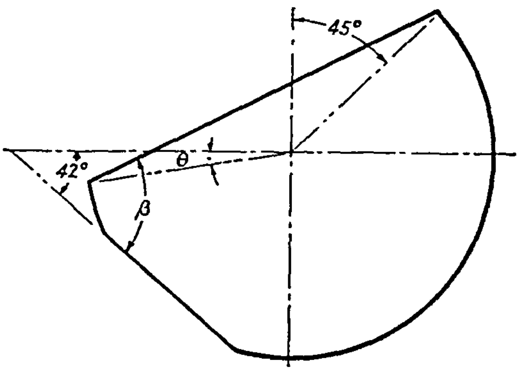
8. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	9.1
2	A	9.2
3	A	9.3
4	A	9.4
5	A	9.5
6	A	9.6

$A = 9.7$
Ans. $x = 6.9641$

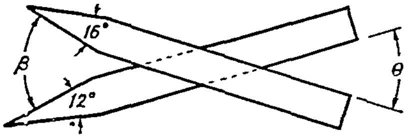
9. Determine the distance x.



Variable		
No.	Sym.	Value
1	θ	5°
2	θ	7°
3	θ	9°
4	θ	11°
5	θ	13°
6	θ	15°

$\theta = 17°$
Ans. $\beta = 73°$

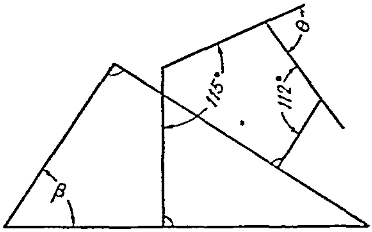
10. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	30°
2	θ	32°
3	θ	34°
4	θ	36°
5	θ	38°
6	θ	40°

$\theta = 42°$
Ans. $\beta = 56°$

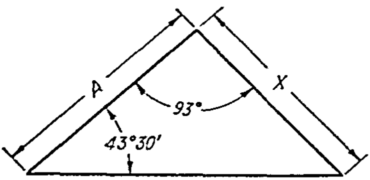
4. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	71°
2	θ	72°
3	θ	73°
4	θ	74°
5	θ	75°
6	θ	76°

$\theta = 77°$
Ans. $\beta = 60°$

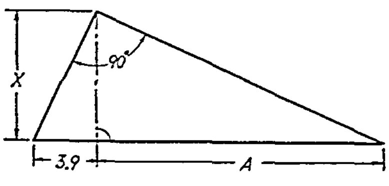
5. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	3 446
2	A	3.853
3	A	4.118
4	A	4.561
5	A	4.876
6	A	5.113

$A = 5.234$
Ans. $x = 5.234$

6. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	6.51
2	A	6.62
3	A	6.73
4	A	6.84
5	A	6.95
6	A	7.06

$A = 7.17$
Ans. $x = 5.2880$

7. Determine the distance x.

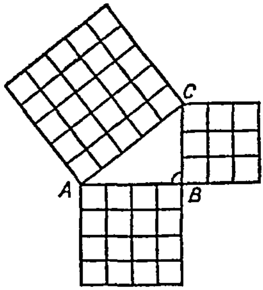
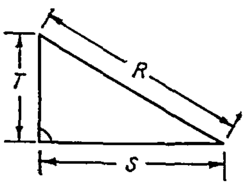


FIG. 77.

to be 5 units long. As seen from the figure the square on the hypotenuse contains 25 square units and those on the legs 16 and 9 square units, respectively.

$$25 = 16 + 9.$$

PROBLEMS

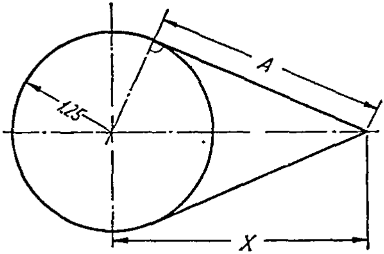


Prob.	R	S	T
1	A	?	5.76
2	B	7.53	?
3	?	12.95	C
4	20.53	?	D
5	?	E	5.876
6	17.32	F	?
7	?	8.95	G

Substitute the given values for the letters in the diagram above and solve for the unknown side. *A*, *B*, *C*, etc., are the variables.

VARIABLES

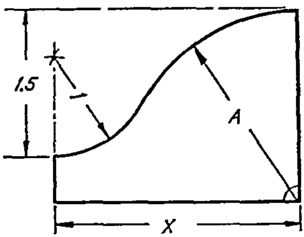
Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	A	8.425	8.752	7.754	6.793	9.913	9.375
2	B	10.52	9.625	8.461	9.453	11.25	10.88
3	C	7.755	8.252	9.748	9.275	8.644	11.58
4	D	16.28	15.93	15.25	14.48	12.85	13.75
5	E	9.252	8.925	3.975	10.28	11.45	10.75
6	F	13.75	12.96	12.83	14.55	15.25	11.82
7	G	2.875	3.812	4.125	4.775	5.237	5.375



VARIABLE		
No.	Sym.	Value
1	A	2.5
2	A	2.75
3	A	3.0
4	A	3.25
5	A	3.5
6	A	3.75

$A = 4$
Ans. $x = 4.1908$

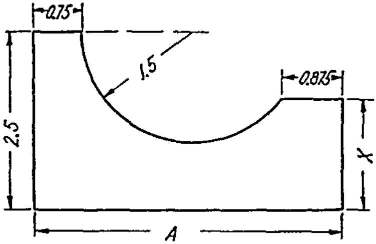
8. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	1.75
2	A	1.875
3	A	2.0
4	A	2.125
5	A	2.25
6	A	2.375

$A = 2.5$
Ans. $x = 2.8723$

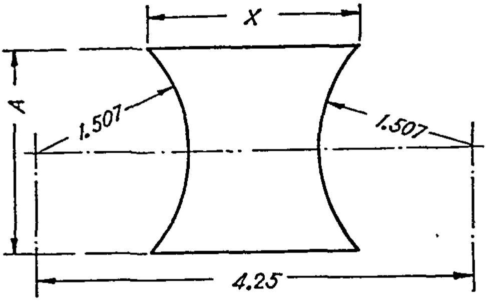
9. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	3.75
2	A	3.875
3	A	4.0
4	A	4.125
5	A	4.25
6	A	4.375

$A = 4.5$
Ans. $x = 1.9005$

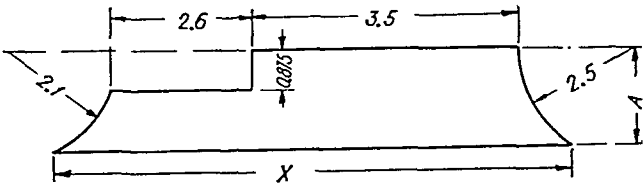
10. Determine the distance x.



$A = 2.5$
Ans. $x = 2.5665$

Variable		
No.	Sym.	Value
1	A	1.75
2	A	1.875
3	A	2.0
4	A	2.125
5	A	2.25
6	A	2.375

11. Determine the distance x.



$A = 1.963$
Ans. $x = 8.2148$

Variable

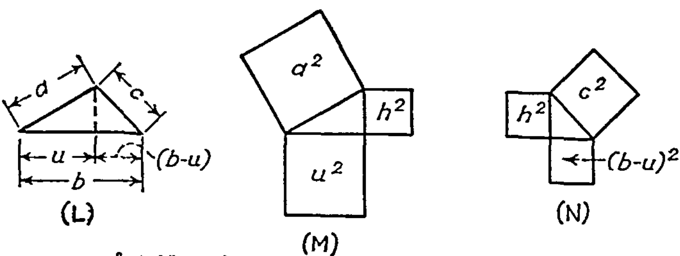
1. $A = 1.041$ 2. $A = 1.122$ 3. $A = 1.253$
4. $A = 1.624$ 5. $A = 1.755$ 6. $A = 1.886$

12. Determine the distance x.

PROPOSITION 32

The projection of a side of a triangle upon the base is equal to the square of this side plus the square of the base minus the square of the third side, divided by two times the base.

Note: For students who prefer the graphic method, the figures are used to lead directly to the required result.



To prove: $u = \dfrac{a^2 + b^2 - c^2}{2b}$.

Fig. 78.

$c^2 = h^2 + (b - u)^2$

$a^2 = u^2 + h^2$,

(P-31)

The expression $a^2 + b^2 - c^2$ may be evaluated diagrammatically as follows: The order of the expression $a^2 + b^2 - c^2$ may be arranged thus: $b^2 + a^2 - c^2$. b^2 is the area of a square erected upon the base of the original triangle and is shown in figure R. $a^2 - c^2$ as shown in figures M and N is equivalent to u^2 minus $(b - u)^2$ since the h^2 in M minus the h^2 in N is zero.

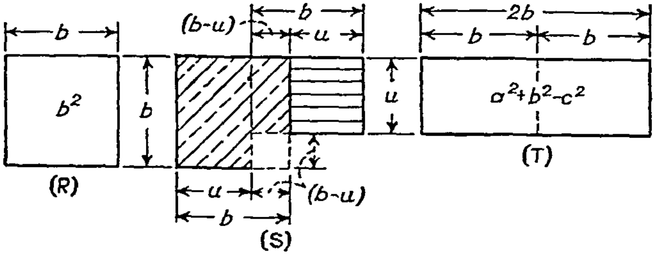


Fig. 79.

Figure S shows the combined areas expressed by b^2 plus u^2 minus $(b - u)^2$ or $(b^2 + a^2 - c^2)$. This area may be rearranged as shown in figure T into two rectangular pieces each of which has a length b and a width u. The area of T which is $a^2 + b^2 - c^2$ is thus seen to be equal to $2bu$.

Hence $2bu = a^2 + b^2 - c^2$

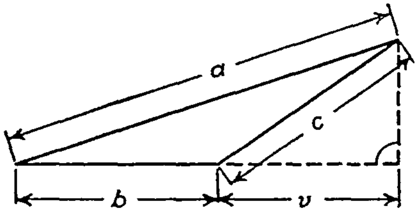
and
$$u = \frac{a^2 + b^2 - c^2}{2b}.$$

For students who prefer the algebraic method, the following may be used:

$$\begin{aligned} a^2 + b^2 - c^2 &= u^2 + \cancel{h^2} + b^2 - \cancel{h^2} - (b - u)^2 \\ &= \cancel{u^2} + \cancel{b^2} - \cancel{b^2} + 2bu - \cancel{u^2} \text{ [expanding } (b - u)^2] \\ &= 2bu \end{aligned}$$

or
$$u = \frac{a^2 + b^2 - c^2}{2b}.$$

Similarly, for an obtuse triangle as shown in Fig. 80, the following expression may be worked out for the projection v:

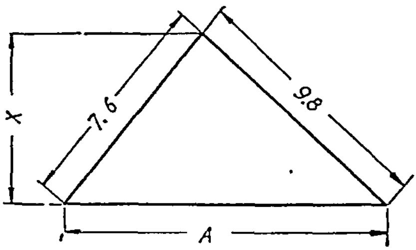


$$v = \frac{a^2 - b^2 - c^2}{2b}.$$

FIG. 80.

These two formulas for u and v will be used in trigonometry and will be referred to as the projection formulas.

PROBLEMS

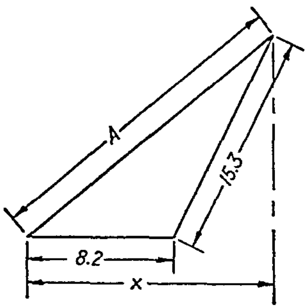


$A = 12.4$

Ans. $x = 6.0065$

VARIABLE		
No.	Sym.	Value
1	A	10.6
2	A	10.9
3	A	11.2
4	A	11.5
5	A	11.8
6	A	12.1

1. Determine the altitude x.

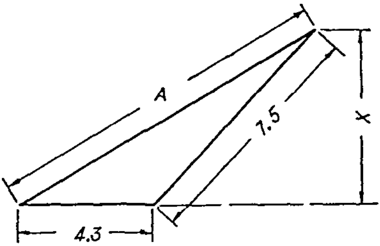


$A = 18.917$

Ans. $x = 11.648$

VARIABLE		
No.	Sym.	Value
1	A	21.611
2	A	20.998
3	A	20.469
4	A	20.008
5	A	19.601
6	A	19.240

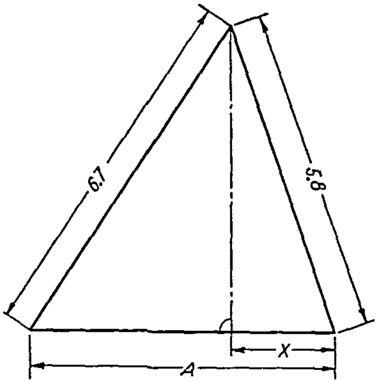
2. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	8.8
2	A	9.1
3	A	9.3
4	A	9.5
5	A	9.8
6	A	10.3

$A = 10.5$
Ans. $x = 6.2611$

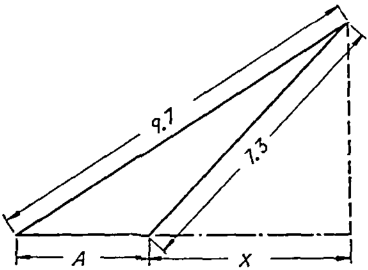
3. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	4.1
2	A	4.0
3	A	3.9
4	A	3.8
5	A	3.7
6	A	3.6

$A = 4.7$
Ans. $x = 1.1531$

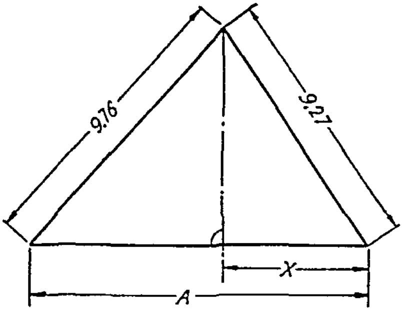
4. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	5.5
2	A	5.3
3	A	5.1
4	A	4.9
5	A	4.7
6	A	4.5

$A = 4.3$
Ans. $x = 2.5941$

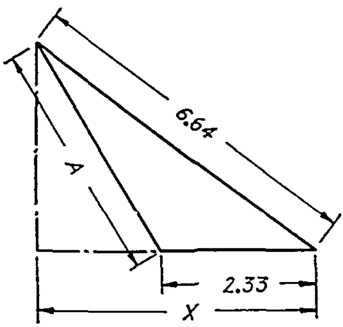
5. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	12.2
2	A	12.4
3	A	12.6
4	A	12.8
5	A	13.0
6	A	13.2

$A = 13.4$
Ans. $x = 6.3520$

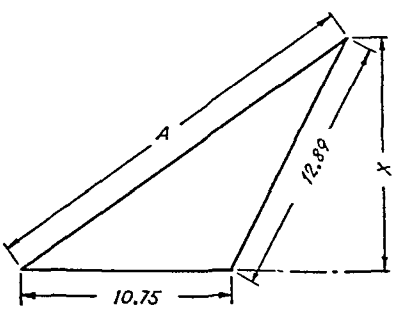
6. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	4.87
2	A	4.93
3	A	4.99
4	A	5.05
5	A	5.11
6	A	5.17

$A = 5.23$
Ans. $x = 4.7565$

7. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	18.5
2	A	18.7
3	A	18.9
4	A	19.1
5	A	19.3
6	A	19.5

$A = 17.3$
Ans. $x = 12.864$

8. Determine the distance x.

CIRCLES

Definitions

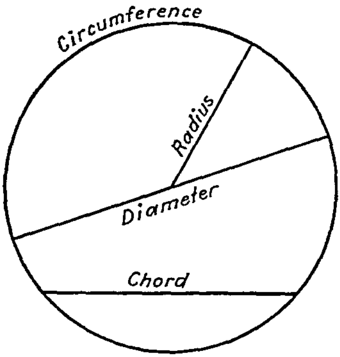


FIG. 81.

35. A **circle** is a plane figure bounded by a line called the **circumference**, all points of which are equidistant from a point within called the center.

36. A **radius** of a circle is a straight line from the center to a point on the circumference.

37. A **diameter** of a circle is a straight line through the center, having its ends in the circumference.

38. A **chord** of a circle is a straight line having its ends in the circumference.

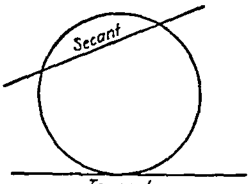


FIG. 82.

39. A **secant** is any straight line intersecting a circle.

40. A **tangent** to a circle is a straight line which touches the circumference in only one point.

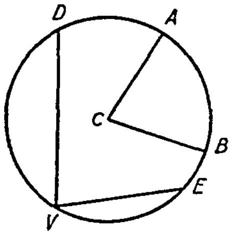


FIG. 83.

41. **A central angle** is an angle having its vertex at the center of the circle and radii of the circle for its sides, as $\angle ACB$.

42. An angle **inscribed in a circle** (usually called an *inscribed angle*) is an angle whose vertex lies on the circumference and whose sides are chords of the circle, as $\angle DVE$.

43. An angle is said to **intercept** the arc included within its sides. Thus $\angle ACB$ intercepts arc AB (often written $\widehat{AB}$) and $\angle DVE$ intercepts arc DAE ($\widehat{DAE}$). The arc is said to be subtended by the angle.

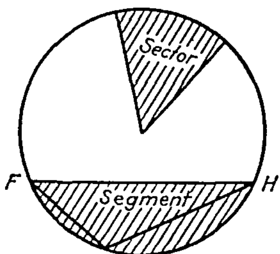


FIG. 84.

44. A **sector** of a circle is that portion bounded by two radii and the intercepted arc.

45. A **segment** of a circle is that portion bounded by a chord and the intercepted arc.

46. An angle is said to be **inscribed in a segment** if the vertex is on the circumference and the sides pass through the ends of the arc of the segment. Thus $\angle FGH$ is inscribed in the shaded segment.

47. A **regular circumscribed polygon** is a regular polygon having all of its sides tangent to a circle, as $EFGH$ (Fig. 85).

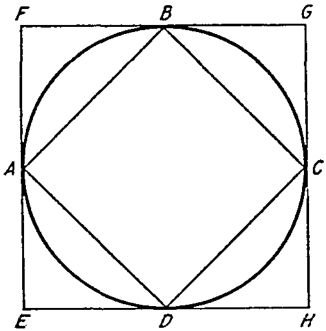


FIG. 85.

48. **A regular inscribed polygon** is a regular polygon having all of its vertices on the circumference of a circle, as *ABCD*.

49. The length of the circumference of a circle divided by its diameter is equal to the number 3.1416— which is called π (Pī), *i.e.*, the circumference is equal to the diameter multiplied by π.

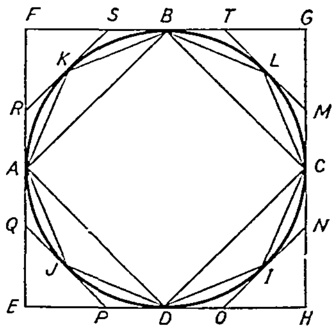


FIG. 86.

50. Informal discussion of the determination of π.

It can be seen from the accompanying figure that the perimeter of the inscribed regular polygon *ABCD* is less than the circumference of the circle in which it is inscribed, and that the perimeter of the circumscribed regular polygon *EFGH* is greater than the circumference of the circle around which it is circumscribed. It is also readily seen that, as the number of sides of the regular inscribed and circumscribed polygons

is increased, their perimeters more nearly equal the circumference of the circle. Thus octagon $IDJAKBLCI >$ square $ABCD$ and octagon $MNOPQRSTM <$ square $EFGH$.

Hence, if the perimeter of an inscribed (or circumscribed) polygon of a large number of sides be divided by the diameter of the circle in which it is inscribed (or about which it is circumscribed) the quotient will closely approximate π as defined in D-49.

The following data will bring out this point:

Number of sides	Perimeter of inscribed regular polygon ÷ diameter	Perimeter of circumscribed regular polygon ÷ diameter	Difference in these perimeters
4	2.82843	4.00000	1.17157
8	3.06147	3.31371	0.25224
64	3.14033	3.14412	0.00379
512	3.14157	3.14163	0.00006

Thus π, which must be between the two ratios given in the second and third columns, is seen to be approximately 3.1416.

PROPOSITION 33

In the same circle or in equal circles, if two central angles are equal, they subtend equal arcs; conversely, if two arcs of the same circle, or of equal circles, are equal, they are subtended by equal central angles.

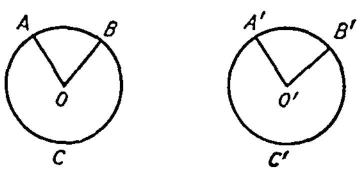


FIG. 87.

a. Given: Equal circles ABC and $A'B'C'$ with equal central angles AOB and $A'O'B'$.

To prove: $\widehat{AB} = \widehat{A'B'}$.

Place circle $A'B'C'$ on circle ABC with center O' falling on O and line $O'B'$ falling along line OB.

Point B' will fall on B

($O'B'$ and OB being radii of equal circles).

Line $O'A'$ will fall along line OA

($\angle A'O'B' = \angle AOB$ by hyp.).

Point A' will fall on A

($O'A' = OA$, being radii of equal circles).

Thus $\widehat{A'B'}$ is made to coincide with $\widehat{AB}$ and is equal to it.

b. Given: Equal circles with $\widehat{A'B'} = \widehat{AB}$.

To prove: $\angle A'O'B' = \angle AOB$.

Since both circles and the arcs are equal, circle $A'B'C'$ may be placed on circle ABC so that center O' falls on center O and $\widehat{A'B'}$ falls on its equal, $\widehat{AB}$.

Thus line $O'A'$ coincides with line OA (A-IX).

and line $O'B'$ coincides with line OB (A-IX).

$\therefore$ $\angle A'O'B'$ coincides with $\angle AOB$ and is equal to it.

PROPOSITION 34

In the same circle, or in equal circles, two central angles have the same ratio as their subtended arcs.

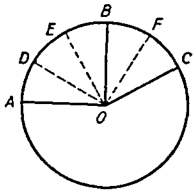


FIG. 88.

Given: Central $\measuredangle AOB$ and BOC.

To prove: $\dfrac{\angle AOB}{\angle BOC} = \dfrac{\widehat{AB}}{\widehat{BC}}$.

Assume that there is a certain small angle, such as $\angle AOD$, which is contained a whole number of times in both $\angle AOB$ and $\angle BOC$, say three times in $\angle AOB$ and twice in $\angle BOC$;

Then $\dfrac{\angle AOB}{\angle BOC} = \dfrac{3}{2}$

The radii drawn from O in applying the $\angle AOD$ to $\measuredangle AOB$ and BOC will divide the arcs AB and BC into 3 and 2 equal arcs, respectively (P-33a).

$$\therefore \frac{\widehat{AB}}{\widehat{BC}} = \frac{3}{2}.$$

Hence $\dfrac{\angle AOB}{\angle BOC} = \dfrac{\widehat{AB}}{\widehat{BC}}$ (A-I).

Note: In case no common unit of angle can be found for $\measuredangle AOB$ and BOC, the proposition may still be proved by using the method of limits.

Corollary to Proposition 34.—The relation expressed in Proposition 34 is usually stated as follows: *A central angle is measured by its subtended arc.*

PROPOSITION 35

Construction

To circumscribe a circle about a given triangle.

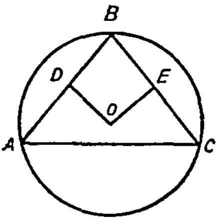


Fig. 89.

Given: $\triangle ABC$.

Required: To circumscribe a circle about $\triangle ABC$, *i.e.*, to draw a circle which will pass through the vertices A, B, and C.

At D, the mid-point of AB, erect a perpendicular to the line AB.

At E, the mid-point of BC, erect a perpendicular to the line BC.

Point O, the intersection of these two perpendiculars, is equidistant from points A and B (cor. to P-14).

That is, $AO = OB$.

Similarly, point O is equidistant from points B and C (cor. to P-14).

That is, $OB = OC$ and thus $OA = OB = OC$ (A-1).

A circle having O as a center and OA as a radius will pass through B and C (since $OA = OB = OC$) (D-35).

This is the only circle that can be drawn through the three points A, B, and C, for any other circle passing through these points must have its center on both of the perpendicular bisectors DO and OE. But there can be but one intersection of these two perpendiculars (cor. to A-IX).

Note: The foregoing relation may be stated as follows: *Through three given points, one, and only one, circle may be drawn; i.e., three points determine a circle.*

PROPOSITION 36

A straight line perpendicular to a radius at its extremity is tangent to the circle; conversely, the tangent at the extremity of a radius is perpendicular to the radius.

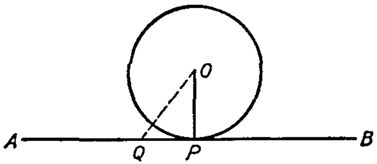


FIG. 90.

a. Given: $AB \perp OP$ (*i.e.*, $OP \perp AB$).

To prove: AB is tangent to the circle.

Draw any other line from O to the line AB, as OQ.

Then $OQ > OP$ (P-4).

$\therefore$ Q lies outside the circle (D-35).

Thus all points on AB except P lie outside the circle and hence AB is tangent to the circle (D-40).

b. Given: AB tangent to the circle at P.

To prove: $AB \perp OP$.

Since OP is the shortest line from O to the line AB (all points but the point of tangency lie outside the circle), $OP \perp AB$ or $AB \perp OP$ (P-4).

Corollary to Proposition 36. *A line perpendicular to a tangent at the point of contact passes through the center of the circle.*

PROPOSITION 37

Two lines drawn tangent to a circle from a given external point are equal and make equal angles with the line joining the point to the center of the circle.

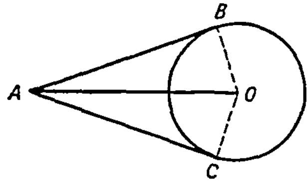


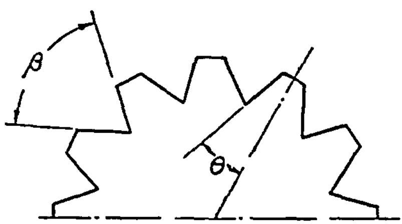
Fig. 91.

Given: Lines AB and AC drawn tangent to the circle.
To prove: $AB = AC$ and $\angle BAO = \angle CAO$.
Draw the radii OB and OC to the points of tangency.
$\angle ABO$ and $\angle ACO$ are both right angles (P-36).

$$OB = OC \quad \text{(D-35)}.$$
$$AO = AO \quad \text{(common)}.$$

$\therefore \triangle ABO = \triangle ACO$ (P-21).
Hence $AB = AC$ and $\angle BAO = \angle CAO$ (D-25).
Note: From the foregoing proposition, the following statement is true: *A line drawn from an external point through the center of a circle bisects the angle formed by the tangents drawn from that point.*

PROBLEMS

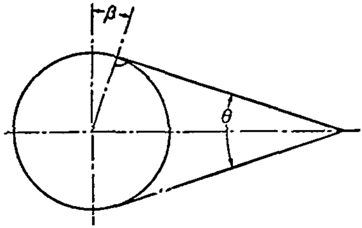


VARIABLE

No.	Sym.	Value
1	θ	20°
2	θ	22°
3	θ	25°
4	θ	27°
5	θ	29°
6	θ	31°

$\theta = 33°$
Ans. $\beta = 96°$
The broach above has 12 teeth.

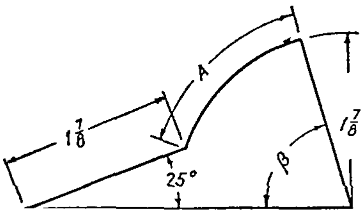
1. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	31°
2	θ	33°
3	θ	35°
4	θ	37°
5	θ	39°
6	θ	41°

$\theta = 43°$
Ans. $\beta = 21° \, 30'$

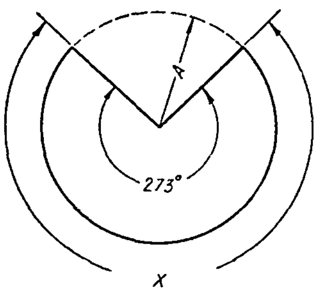
2. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	.875
2	A	1.0
3	A	1.125
4	A	1.25
5	A	1.375
6	A	1.5

$A = 1.625$
Ans. $\beta = 74° \, 39' \, 22''$

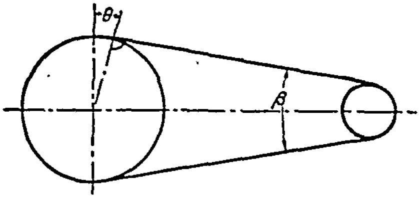
3. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	.75
2	A	.85
3	A	.97
4	A	1.5
5	A	1.8
6	A	2.3

$A = 2.5$
Ans. $x = 11.912$

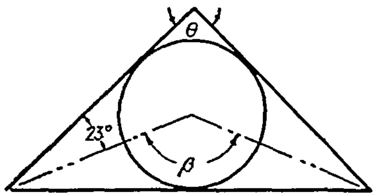
4. Determine the value of x.



VARIABLE		
No.	Sym.	Value
1	θ	16°
2	θ	18°
3	θ	20°
4	θ	22°
5	θ	24°
6	θ	26°

$\theta = 28°$
Ans. $\beta = 56°$

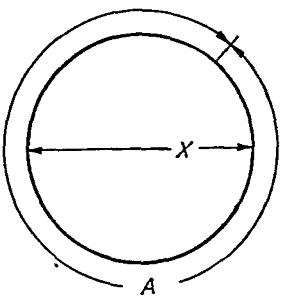
5. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	θ	83°
2	θ	84°
3	θ	85°
4	θ	86°
5	θ	87°
6	θ	88°

$\theta = 89°$
Ans. $\beta = 134° \ 30'$

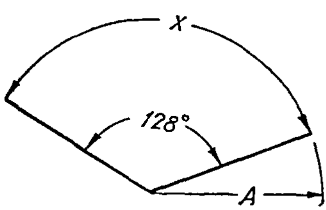
6. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	A	1.25
2	A	3.62
3	A	2.7
4	A	3.1
5	A	3.6
6	A	9.66

$A = 4.5$
Ans. $x = 1.4323$

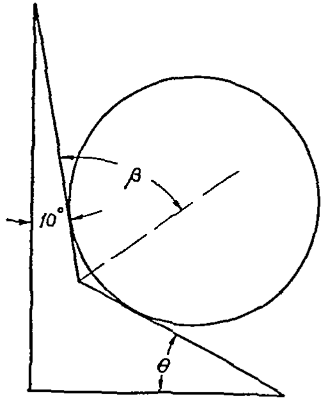
7. Determine the diameter x.



VARIABLE		
No.	Sym.	Value
1	A	2.92
2	A	2.87
3	A	2.82
4	A	2.77
5	A	2.72
6	A	2.67

$A = 2.62$
Ans. $x = 5.8531$

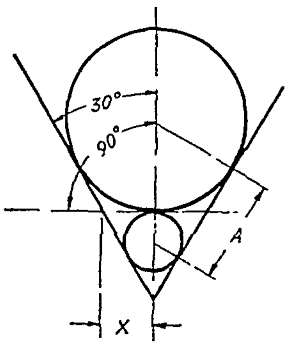
8. Determine the length of arc x.



Variable		
No.	Sym.	Value
1	θ	28°
2	θ	30°
3	θ	32°
4	θ	34°
5	θ	36°
6	θ	38°

$\theta = 40°$
Ans. $\beta = 70°$

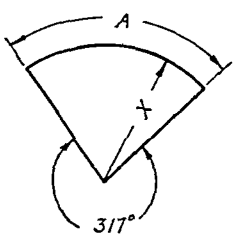
9. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	10 3
2	A	10 8
3	A	11.2
4	A	11.7
5	A	12.4
6	A	13.2

$A = 13.8$
Ans. $x = 6.9$

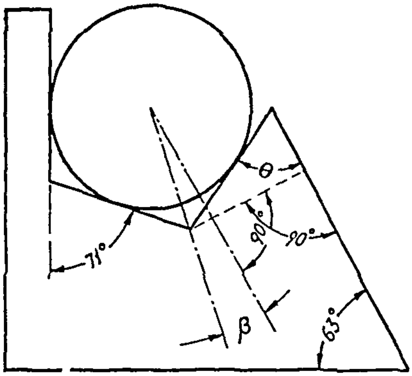
10. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	3 11
2	A	4.22
3	A	4.63
4	A	5.24
5	A	5.45
6	A	6.66

$A = 6.87$
Ans. $x = 9.1535$

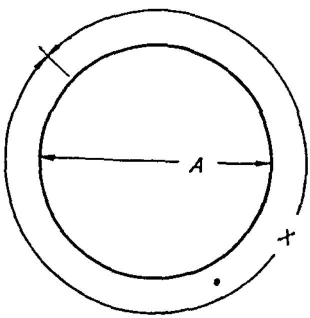
11. Determine the radius x.



VARIABLE		
No.	Sym	Value
1	θ	70°
2	θ	72°
3	θ	74°
4	θ	76°
5	θ	78°
6	θ	80°

$\theta = 82°$
Ans. $\beta = 19°$

12. Determine the angle β.



VARIABLE		
No.	Sym.	Value.
1	A	3.25
2	A	3.62
3	A	3.75
4	A	3.87
5	A	4.25
6	A	4.85

$A = 4.93$
Ans. $x = 15.488$

13. Determine the value of x.

PROPOSITION 38

The diameter of a circle inscribed in a right triangle is equal to the difference between the sum of the two legs and the hypotenuse.

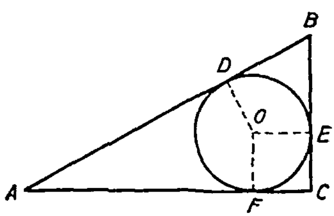


FIG. 92.

Given: Rt. $\triangle ABC$ and inscribed circle DEF.
To prove: $AC + BC - AB =$ diameter.
Draw radii OD, OE, and OF to points of tangency,
Then $AD = AF$, $BD = BE$, and $CF = CE$ (P-37).
EC and OF are parallel and FC and OE are parallel (P-36 and P-5).
$\therefore CE = OF$ and $FC = OE$ (cor. 1 to P-23).
$\therefore FC + CE = OF + OE$ (A-III).
But $OF + OE =$ diameter.
$\therefore FC + CE =$ diameter (A-1).
$AC - AD = FC$
and $BC - BD = EC$.
Adding these two equations:
$AC + BC - (AD + BD) = FC + EC$ (A-III).
$AC + BC - AB = FC + EC$ (A-VIII).
$\therefore AC + BC - AB =$ diameter (A-II).

PROPOSITION 39

Any diameter of a circle, which is perpendicular to a chord, bisects the chord and the arc subtended by it.

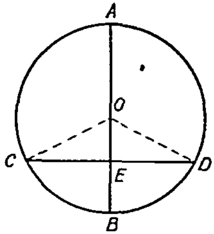


FIG. 93.

Given: Diameter $AB \perp$ chord CD.

To prove: $CE = ED$, $\widehat{CB} = \widehat{BD}$, and $\widehat{AC} = \widehat{AD}$.
Draw radii OC and OD.
Rt. $\triangle CEO =$ rt. $\triangle DEO$ (P-21).
$\therefore CE = ED$ (D-25).
$\angle COE = \angle DOE$ (D-25).
$\therefore \widehat{CB} = \widehat{DB}$ (P-33)
and $\widehat{AC} = \widehat{AD}$ (A-IV).

Corollary to Proposition 39.—*The perpendicular bisector of a chord passes through the center of the circle.*

There can be only one perpendicular bisector of a chord, and from (P-39) the diameter is a perpendicular bisector of the chord.

PROPOSITION 40

An inscribed angle is measured by one-half the intercepted arc.

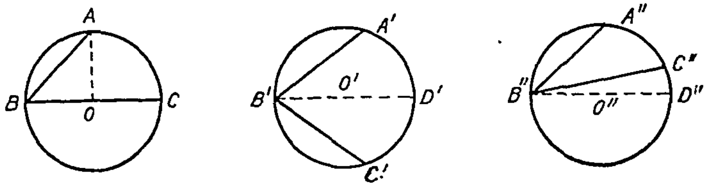


FIG. 94.

Given: Inscribed $\angle ABC$ (or $\angle A'B'C'$ or $\angle A''B''C''$).

To prove: That the inscribed angle is measured by one-half the intercepted arc AC (or $A'C'$ or $A''C''$).

Case I.—One side of the angle is a diameter.

Draw OA.

$\angle AOC$ is measured by $\widehat{AC}$ (cor. to P-34).
$\angle AOC = \angle OAB + \angle CBA$ (P-13).
$\angle OAB = \angle CBA$ (P-17).
$\therefore \angle AOC = 2\angle CBA$ (A-II)
or $\angle CBA = \frac{1}{2}\angle AOC$.

$\therefore \angle CBA$ is measured by $\frac{1}{2}\widehat{AC}$.

Case II.—Sides of the angle are on opposite sides of the center of the circle.

Draw diameter $B'D'$.

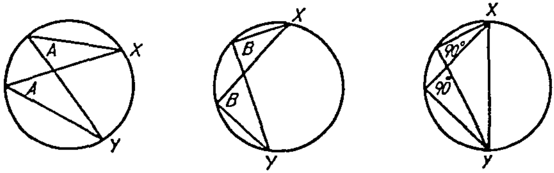
Then from Case I, $\angle C'B'D'$ is measured by $\frac{1}{2}\widehat{C'D'}$.

$\angle D'B'A'$ is measured by $\frac{1}{2}\widehat{A'D'}$.

Hence $\angle A'B'C'$ is measured by $\frac{1}{2}\widehat{A'C'}$. (A-III).

Case III.—Sides of the angle are on the same side of the center of the circle.

The proof in this case is left to the student.
(*Hint:* Draw diameter $B''D''$.)



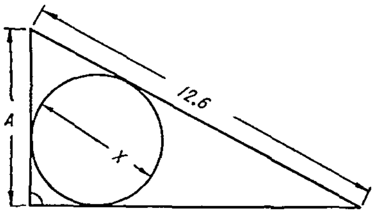
(*a*) (*b*) (*c*)

FIG. 95.

Corollary 1 to Proposition 40.—*All inscribed angles subtending the same arc are equal* (see Figs. 95*a* and *b*).

Corollary 2 to Proposition 40.—*An inscribed angle in a semicircle is a right angle* (see Fig. 95*c*).

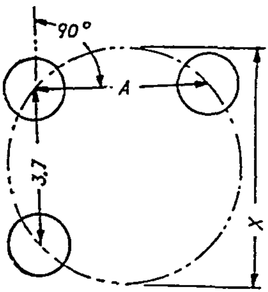
PROBLEMS



VARIABLE		
No.	Sym.	Value
1	A	4.9
2	A	5.1
3	A	5.3
4	A	5.5
5	A	7.1
6	A	6.9

$A = 6.7$
Ans. $x = 4.7709$

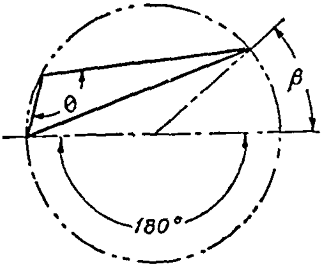
1. Determine the diameter x.



VARIABLE		
No.	Sym.	Value
1	A	2.5
2	A	2.8
3	A	3.1
4	A	3.3
5	A	3.5
6	A	3.9

$A = 4.1$
Ans. $x = 5.5226$

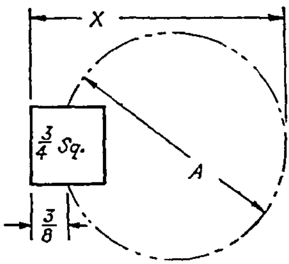
2. Determine the distance x.



Variable		
No.	Sym.	Value
1	θ	102°
2	θ	106°
3	θ	111°
4	θ	116°
5	θ	120°
6	θ	127°

$\theta = 130°$
Ans. $\beta = 80°$

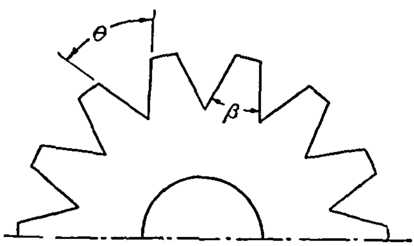
3. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	2.75
2	A	2.93
3	A	3.44
4	A	3.72
5	A	5.13
6	A	6.55

$A = 5.75$
Ans. $x = 6.1004$

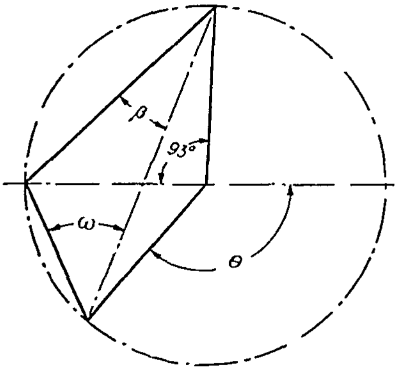
4. Determine the distance x.



Variable		
No.	Sym.	Value
1	θ	44°
2	θ	46°
3	θ	48°
4	θ	50°
5	θ	52°
6	θ	54°

$\theta = 56°$
Ans. $\beta = 30° \, 17' \, 10''$
Circular broach has 14 teeth.

5. Determine the angle β.

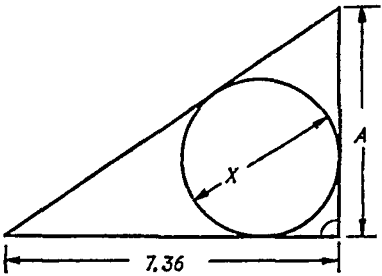


VARIABLE

No.	Sym.	Value
1	θ	128°
2	θ	130°
3	θ	132°
4	θ	134°
5	θ	136°
6	θ	138°

$\theta = 140°$
Ans. $\beta = 20°$

6. Determine the angle β.
7. Determine the angle ω.

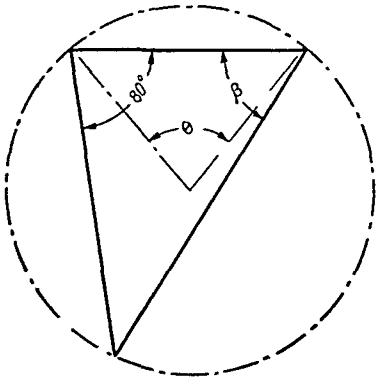


VARIABLE

No.	Sym.	Value
1	A	3 31
2	A	3 52
3	A	3.73
4	A	3 94
5	A	4.15
6	A	4.36

$A = 4.57$
Ans. $x = 3.2666$

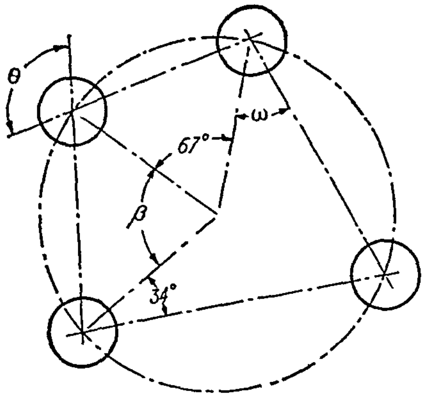
8. Determine the diameter x.



VARIABLE

No.	Sym.	Value
1	θ	80°
2	θ	82°
3	θ	84°
4	θ	86°
5	θ	88°
6	θ	90°

$\theta = 92°$



VARIABLE		
No.	Sym.	Value
1	θ	102°
2	θ	104°
3	θ	106°
4	θ	108°
5	θ	110°
6	θ	112°

$\theta = 114°$

Ans. $\begin{cases} \beta = 65° \\ \omega = 32° \end{cases}$

10. Determine the angle β.
11. Determine the angle ω.

PROPOSITION 41

Construction

To inscribe a square in a given circle.

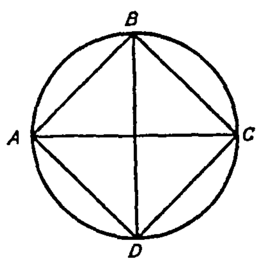


FIG. 96.

Given: Circle $ABCD$.

Required: To inscribe a square, *i.e.*, to draw a square whose vertices lie in the circumference of the circle.

Draw any diameter AC and another diameter BD, perpendicular to AC.

Each of $\measuredangle$ ABC, BCD, CDA, and DAB is inscribed in a semicircle and therefore each is a right angle (cor. 2 to P-40).

Hence, $ABCD$ is a square (D-22).

PROPOSITION 42

Construction

To inscribe a regular hexagon in a given circle.

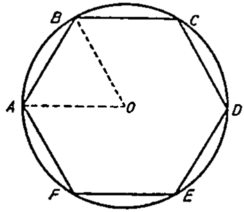


Fig. 97.

Given: Circle $ABCDEFA$.

Required: To inscribe a hexagon in the circle.

From any point A draw an arc using the radius of the circle OA as a radius. This arc will cut the circle at some point B.

Draw OB.

$\triangle AOB$ is equilateral (AB = OA by hyp.).

$\therefore \triangle AOB$ is equiangular (cor. 2 to P-17).

Hence $\angle AOB = 60°$ (P-10).

$60°$ is $\frac{1}{6}$ of $360°$ so arc AB is one-sixth of the circumference (P-34).

Hence AB fits into the circumference just six times.

$\therefore ABCDEFA$ is a regular hexagon (D-20 and D-22).

PROPOSITION 43

If two circles are tangent to each other externally or internally, the line of centers passes through the point of tangency.

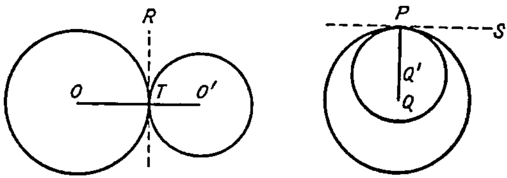


Fig. 98.

Case I. *Given:* Circles O and O' tangent externally at T.

To prove: Line OO' passes through T.

Draw the line of tangency TR and the radii OT and $O'T$.
$\measuredangle$ OTR and $O'TR$ are right angles (P-36).
Hence $\measuredangle$ OTR and $O'TR$ are supplementary (D-11).
$\therefore OTO'$ is a straight line.
That is, OO' passes through T.

Case II. *Given:* Circles Q and Q' tangent internally at P.
To prove: Line QQ' passes through P.
Draw the line of tangency PS and the radii QP and $Q'P$.
$PQ \perp PS$ and $PQ' \perp PS$ (P-36).
Hence PQ and PQ' coincide.
That is, QQ' passes through P.

PROPOSITION 44

The angle between two secants intersecting outside a circumference, the angle between an intersecting tangent and a secant, and the angle between two intersecting tangents are each measured by one-half the difference of the intercepted arcs.

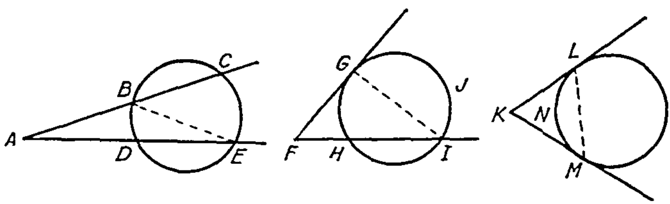


FIG. 99.

Case I. *Given:* Two secants AC and AE intersecting at point A outside the circumference.

To prove: $\angle A$ is measured by $\frac{1}{2}(\widehat{CE} - \widehat{BD})$.
Draw BE.

$\angle CBE$ is measured by $\frac{1}{2}\widehat{CE}$ (P-40).

$\angle DEB$ is measured by $\frac{1}{2}\widehat{BD}$ (P-40).
$\angle A = \angle CBE - \angle DEB$ (P-13).

$\therefore \angle A$ is measured by $\frac{1}{2}\widehat{CE} - \frac{1}{2}\widehat{BD}$ (A-II).

That is, $\angle A$ is measured by $\frac{1}{2}(\widehat{CE} - \widehat{BD})$.

Case II. *Given:* Tangent FG and secant FI intersecting at F.

To prove: $\angle F$ is measured by $\frac{1}{2}(\widehat{GJI} - \widehat{GH})$.

The proof is exactly like that of Case I, so it is left to the student.

Case III. Let the student tell what is given, what is to be proved, and supply the proof.

PROPOSITION 45

If two chords of a circle intersect within the circle, the product of the two segments of the one is equal to the product of the two segments of the other.

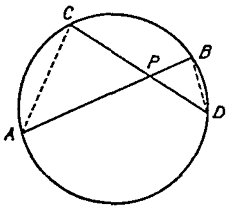


FIG. 100.

Given: Two chords AB and CD intersecting each other within the circle at point P.

To prove: $PA \times PB = PC \times PD$.

Draw AC and BD.

In the $\triangle APC$ and BPD,

$\angle CPA = \angle BPD$ (P-1).

$\angle ACD = \angle ABD$ (cor. 1 to P-40).

$\therefore \triangle APC \sim \triangle BPD$ (cor. 1 to P-26).

Hence $\dfrac{PC}{PB} = \dfrac{PA}{PD}$ (D-34).

$\therefore PA \times PB = PC \times PD$ (Theorem 1 of Chap. V).

PROPOSITION 46

The product of two sides of a triangle is equal to the product of the diameter of a circumscribed circle and the altitude upon the third side.

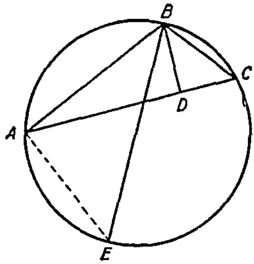


FIG. 101.

Given: $\triangle ABC$ with $BD \perp AC$ and the circumscribed circle $ABCE$ with diameter BE.

To prove: $AB \times BC = BE \times BD$.

Draw AE.

In $\triangle\triangle ABE$ and BCD,

$\angle BAE$ is a right angle (cor. 2 to P-40).

and $\therefore \angle BAE = \angle BDC$ (both rt. $\angle$s).

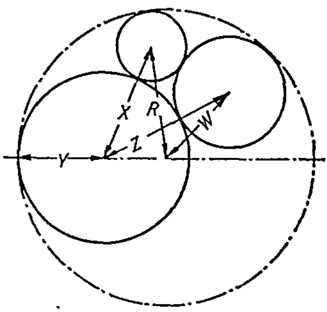
$\angle BCA = \angle BEA$ (cor. 1 to P-40).

$\therefore \triangle ABE \sim \triangle BCD$ (cor. 2 to P-26).

$\therefore \dfrac{AB}{BD} = \dfrac{BE}{BC}$ (D-34)

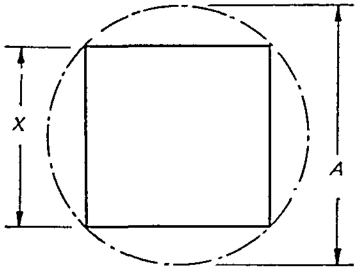
and $AB \times BC = BE \times BD$ (Theorem I of Chap. V).

PROBLEMS



NO VARIABLE

1. State which of the foregoing lines pass through points of contact of the large circle.

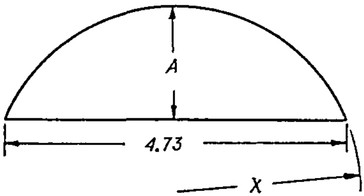


Variable		
No.	Sym.	Value
1	A	2.17
2	A	2.33
3	A	2.66
4	A	3.15
5	A	4.25
6	A	4.75

$A = 4.875$ $\qquad x = .70711A$

Ans. $x = 3.4471$ $\qquad A = 1.4142X$

2. Determine the value of x.

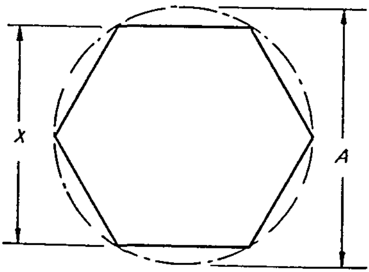


Variable		
No.	Sym.	Value
1	A	.13
2	A	35
3	A	.42
4	A	.47
5	A	.53
6	A	.58

$A = .68$

Ans. $x = 4.4526$

3. Determine the distance x.

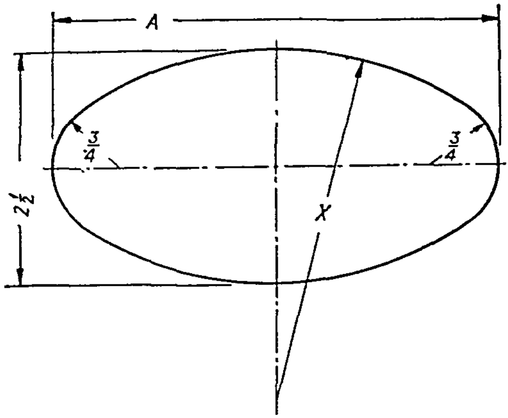


Variable		
No.	Sym.	Value
1	A	2.17
2	A	2.33
3	A	2.66
4	A	3.15
5	A	4.25
6	A	4.75

$A = 4.875$ $\qquad x = .86603A$

Ans. $x = 4.2219$ $\qquad A = 1.1547X$

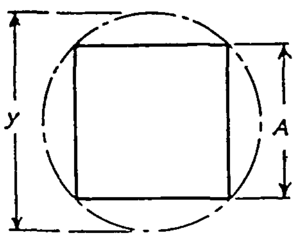
4. Determine the value of x.



VARIABLE No.	Sym.	Value
1	A	4.25
2	A	4.375
3	A	4.5
4	A	4.625
5	A	4.75
6	A	4.875

$A = 5$
Ans. $x = 4.0625$

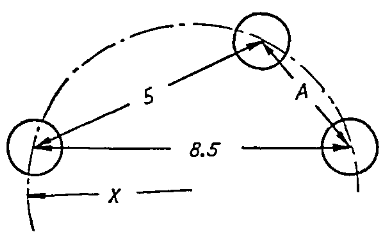
5. Determine the radius x



VARIABLE No.	Sym.	Value
1	A	.783
2	A	.763
3	A	.743
4	A	.723
5	A	.703
6	A	.683

$A = .663$
Ans. $y = .93761$

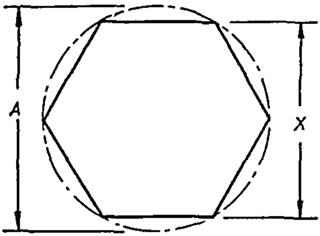
6. Determine the value of y.



VARIABLE No.	Sym.	Value
1	A	4.3
2	A	4.8
3	A	5.1
4	A	5.4
5	A	5.7
6	A	6.2

$A = 5.6$
Ans. $x = 4.4321$

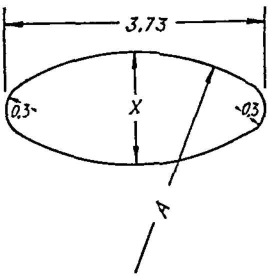
7. Determine the radius x.



VARIABLE		
No.	Sym.	Value
1	A	1.625
2	A	1.687
3	A	1.75
4	A	1.8125
5	A	1.875
6	A	1.9375

$A = 2.0625$
Ans. $x = 1.7861$

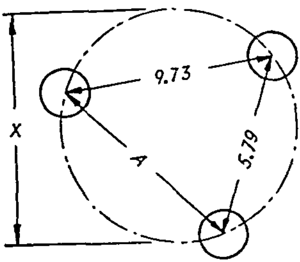
8. Determine the value of x.



VARIABLE		
No.	Sym.	Value
1	A	2.2
2	A	2.4
3	A	2.6
4	A	2.8
5	A	3 1
6	A	3.3

$A = 3.2$
Ans. $x = 1.5171$

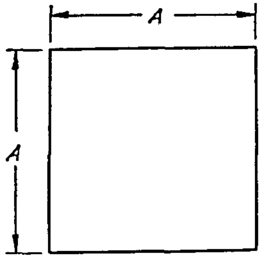
9. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	9.81
2	A	9.92
3	A	10 3
4	A	10 44
5	A	11 5
6	A	11.66

$A = 11.82$
Ans. $x = 11.882$

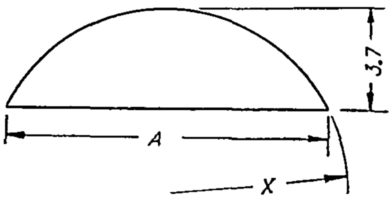
10. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	2.2
2	A	2.7
3	A	2.9
4	A	3.2
5	A	3.6
6	A	4.2

$A = 4.6$
Ans. Diam. $= 6.5053$

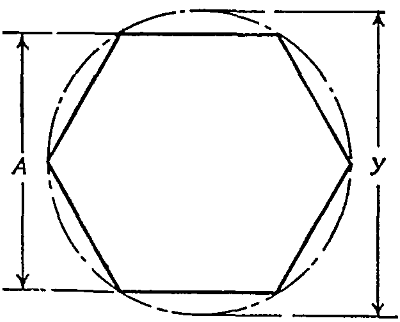
11. Determine the diameter of a circle that will circumscribe the given square.



Variable		
No.	Sym.	Value
1	A	5.5
2	A	5.7
3	A	5.9
4	A	6.1
5	A	6.3
6	A	6.5

$A = 6.7$
Ans. $x = 3.3665$

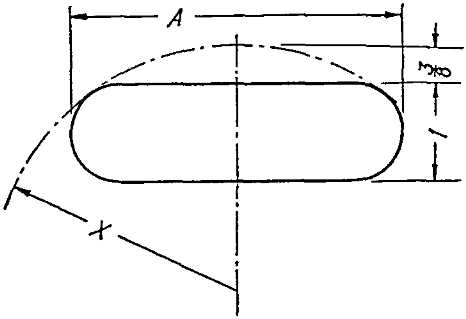
12. Determine the radius x.



Variable		
No.	Sym.	Value
1	A	1.75
2	A	2.12
3	A	2.37
4	A	2.87
5	A	3.18
6	A	3.62

$A = 3.87$
Ans. $y = 4.4686$

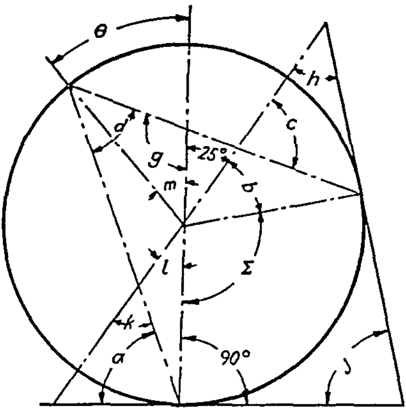
13. Determine the value of y.



Variable		
No.	Sym.	Value
1	A	3.125
2	A	3.25
3	A	3.375
4	A	3.5
5	A	3.625
6	A	3.75

$A = 3.875$
Ans. $x = 3.4427$

14. Determine the radius x.



Variable		
No.	Sym.	Value
1	θ and Σ	36°–95°
2	θ and Σ	38°–97°
3	θ and Σ	40°–99°
4	θ and Σ	42°–101°
5	θ and Σ	44°–103°
6	θ and Σ	46°–105°

15. Determine the following angles.

$a,\quad b,\quad c,\quad d,\quad g,\quad h,\quad j,\quad k,\quad l,\quad m.$

FORMULAS FOR THE AREAS OF VARIOUS PLANE FIGURES

Rectangle

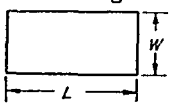


Fig. 102.

Area $= L \times W$.

Parallelogram

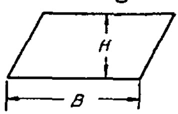


Fig. 103.

Area $= B \times H$.

Triangle

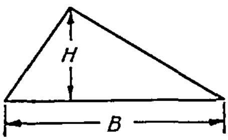


FIG. 104.

$$\text{Area} = \tfrac{1}{2}B \times H.$$

Trapezoid

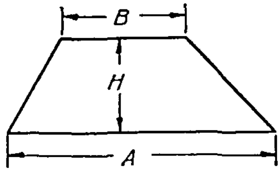


FIG. 105.

$$\text{Area} = \tfrac{1}{2}(A + B)H.$$

Circle

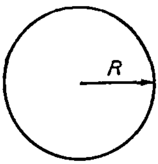


FIG. 106.

$$\begin{aligned}\text{Area} &= \pi R^2 \\ &= \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4} \\ &= .7854D^2.\end{aligned}$$

Circular Sector

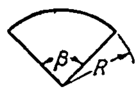


FIG. 107.

$$\begin{aligned}\text{Area} &= \frac{\beta}{360} \times \pi R^2 \\ &= .008727\beta R^2.\end{aligned}$$

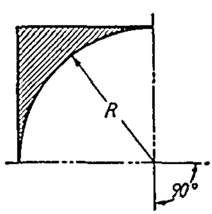


FIG. 108.

$$\begin{aligned}\text{Area (of shaded portion)} &= R^2 - \tfrac{1}{4}\pi R^2 \\ &= (1 - \tfrac{1}{4}\pi)R^2 \\ &= .2146R^2.\end{aligned}$$

FORMULAS FOR THE VOLUMES OF VARIOUS SOLID FIGURES

Rectangular Parallelepiped

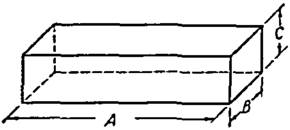


FIG. 109.

Volume = base × height
= ABC.

Oblique Parallelepiped

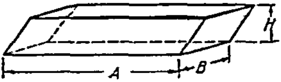


FIG. 110.

Volume = base × height
= ABH.

Right Circular Cylinder

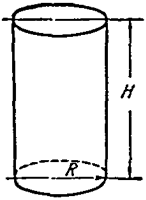


FIG. 111.

Lateral surface area = $2\pi RH$.

Total surface area $= 2\pi R^2 + 2\pi RH$
$= 2\pi R(R + H)$.

Volume = base × altitude
$= \pi R^2H$.

Oblique Circular Cylinder

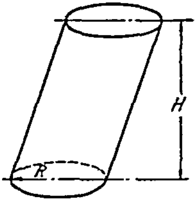


FIG. 112.

Volume = base × altitude
$= \pi R^2H$.

Note: The volume of any solid having the lateral elements parallel is equal to the product of the area of the base and the altitude. This applies to all types of rods.

Right Circular Cone

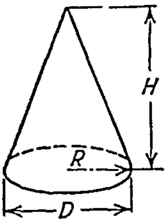


Fig. 113.

$$\text{Vol.} = \tfrac{1}{3}\text{ base} \times \text{altitude}$$
$$= \tfrac{1}{3}\pi R^2 H$$
$$= .2618D^2H.$$

Frustum of a Right Circular Cone

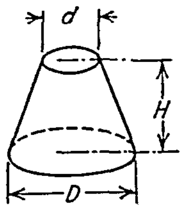


Fig. 114.

$$\text{Vol.} = .2618H(D^2 + d^2 + Dd).$$

Sphere

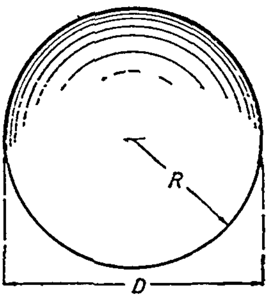


Fig. 115.

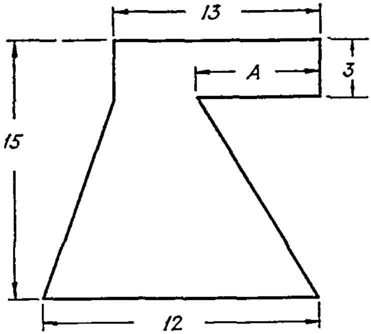
$$\text{Surface area} = 4\pi R^2 = 12.566R^2$$
$$\text{Volume} = \tfrac{4}{3}\pi R^3 = 4.1888R^3$$
$$= \tfrac{1}{6}\pi D^3 = .5236D^3.$$

The number of gallons in a given volume is expressed by the following formula.

$$\text{Volume (in gallons)} = \frac{\text{Volume (in cubic inches)}}{231}.$$

PROBLEMS

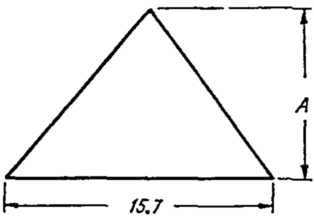
Some of the following problems are direct applications of the formulas for areas and volumes, but some of the figures are seen to consist of combinations of the single figures, and the total area or volume must be obtained by solving for areas or volumes of the separate simple units and adding.



Variable		
No.	Sym.	Value
1	A	6.3
2	A	6.5
3	A	6.8
4	A	7.1
5	A	7.3
6	A	7.5

$A = 5.4$
Ans. Area $= 156.6$

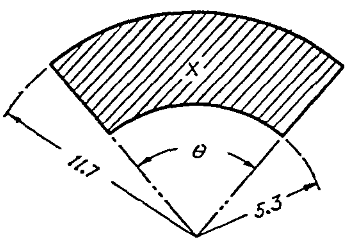
1. Determine the area.



Variable		
No.	Sym.	Value
1	A	7 52
2	A	7.65
3	A	7 76
4	A	7.89
5	A	7 95
6	A	8.12

$A = 8.25$
Ans. Area $= 64.762$

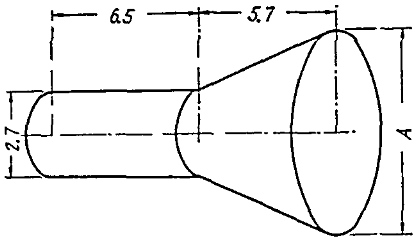
2. Determine the area.



Variable		
No.	Sym.	Value
1	θ	95°
2	θ	98°
3	θ	101°
4	θ	104°
5	θ	107°
6	θ	110°

$\theta = 113°$
Ans. $x = 107.29$

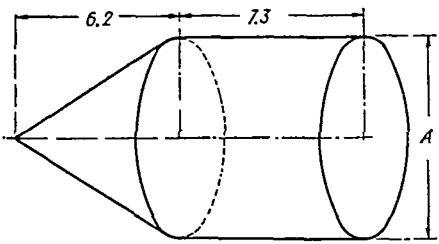
3. Determine the area x.



VARIABLE		
No.	Sym.	Value
1	A	8.4
2	A	7.9
3	A	7.5
4	A	6.8
5	A	6.3
6	A	5.9

$A = 5.6$
Ans. Volume = 117.451

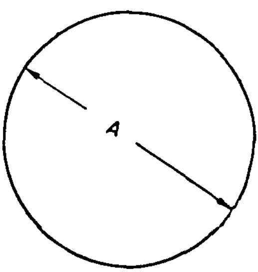
4. Determine the volume.



VARIABLE		
No.	Sym.	Value
1	A	8.4
2	A	7.9
3	A	7.5
4	A	6.8
5	A	6.3
6	A	5.9

$A = 5.6$
Ans. Volume = 230.69

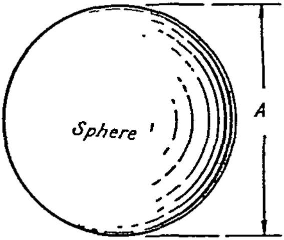
5. Determine the volume.



VARIABLE		
No.	Sym.	Value
1	A	10.6
2	A	10.9
3	A	11.3
4	A	11.8
5	A	12.4
6	A	12.9

$A = 13.4$
Ans. Area = 141.02

6. Determine the area.

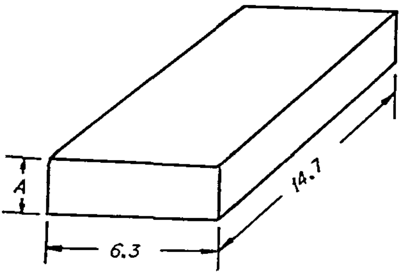


VARIABLE

No.	Sym.	Value
1	A	8.2
2	A	8.8
3	A	9.1
4	A	9.5
5	A	9.8
6	A	10.4

$A = 10.9$
Ans. Volume $= 678.07$

7. Determine the volume.

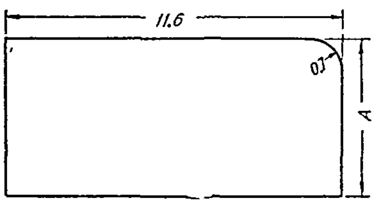


VARIABLE

No.	Sym.	Value
1	A	3.1
2	A	3.3
3	A	3.5
4	A	3.7
5	A	3.9
6	A	4.2

$A = 1.2$
Ans. Volume $= 111.13$

8. Determine the volume.

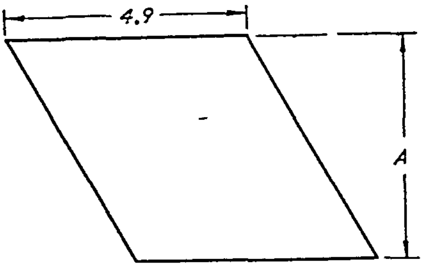


VARIABLE

No.	Sym.	Value
1	A	10.2
2	A	10.5
3	A	10.8
4	A	11.2
5	A	11.5
6	A	11.8

$A = 8.4$
Ans. Area $= 97.334$

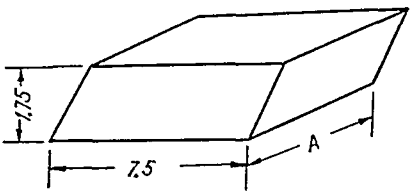
9. Determine the area.



VARIABLE		
No.	Sym.	Value
1	A	2.71
2	A	2.92
3	A	3.23
4	A	3.44
5	A	3.65
6	A	3.86

$A = 4.07$
Ans. Area $= 19.943$

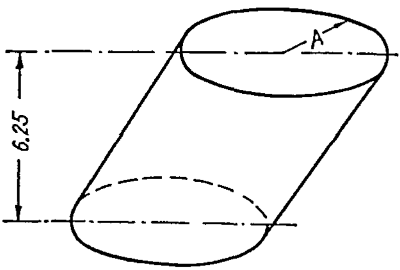
10. Determine the area.



VARIABLE		
No.	Sym.	Value
1	A	3.125
2	A	3.25
3	A	3.375
4	A	3.5
5	A	3.625
6	A	3.75

$A = 3.875$
Ans. Volume $= 50.859$

11. Determine the volume.



VARIABLE		
No.	Sym.	Value
1	A	2.5
2	A	2.625
3	A	2.75
4	A	2.875
5	A	3.125
6	A	3.25

$A = 3.5$
Ans. Volume $= 240.53$

12. Determine the volume.

13. A tank having the size and shape of the figure of Problem 8 can contain how many gallons of water?

The geometrical propositions given in this chapter are the basis for the solution of most practical shop problems. In

this chapter, problems have been given to enable the student to acquire the ability to use each theorem separately. In the problems of the next chapter, the solution of any one problem will usually involve the use of a combination of several geometrical theorems. The student will gradually, through practice, acquire the ability to recognize and apply the proper combination of geometrical theorems involved in a solution.

CHAPTER VIII

TRIGONOMETRY

In geometry a triangle is said to be determined when sufficient sides and angles are given so that the triangle may be constructed. For example, a triangle is determined if two sides and the included angle are known, or if two angles and the included side are given. Frequently problems arise which require that the other parts (sides and angles) of a determined triangle shall be computed. This computation often cannot be carried out by geometry.

Trigonometry is a branch of mathematics which enables one to compute the remaining sides and angles of any triangle which has sufficient parts given. In order to do this, use must be made of what are called the trigonometric functions, *viz.*, sine, cosine, tangent, cotangent, secant, and cosecant. These functions, applied to any angle α, are usually written $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, $\cot \alpha$, $\sec \alpha$, and $\csc \alpha$.

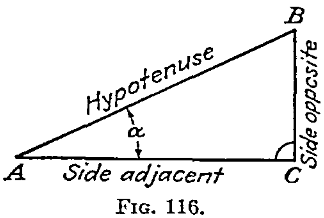


Fig. 116.

If one considers the angle α of the right triangle ABC, having the hypotenuse AB, AC is said to be the side adjacent, and BC the side opposite. The small quarter arc at C is used to denote the right angle and will be used to designate right angles throughout the book. It is very important that the student learn to recognize at a glance which side is the side opposite an angle and which is the side adjacent regardless of the position of the angle. For example, in the triangle DEF,

having the hypotenuse DF, DE is the side opposite angle θ, and EF is the side adjacent to angle θ.

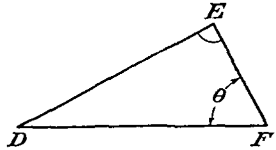


FIG. 117.

DEFINITIONS OF TRIGONOMETRIC FUNCTIONS

There are two methods of defining trigonometric functions the "ratio method" and the "unity method." The two methods are, of course, equivalent.

RATIO METHOD

Referring to the angle α of the right triangle ABC, the six trigonometric functions are defined as follows:

$$\sin \alpha = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{BC}{AB}. \qquad \csc \alpha = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{AB}{BC}$$

$$\cos \alpha = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{AC}{AB}. \qquad \sec \alpha = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{AB}{AC}$$

$$\tan \alpha = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{BC}{AC}. \qquad \cot \alpha = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{AC}{BC}$$

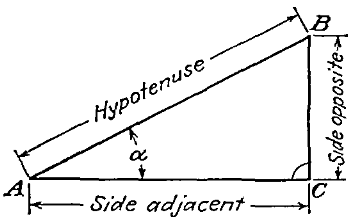


FIG. 118.

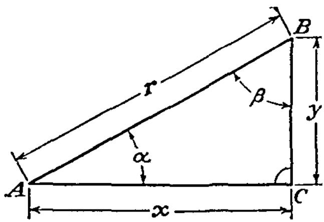


FIG. 119.

RECIPROCAL RELATIONS OF TRIGONOMETRIC FUNCTIONS

By the "ratio" definitions, in Fig. 119, $\sin \alpha = \frac{y}{r}$ and $\csc \alpha = \frac{r}{y}$. Since $\frac{y}{r} = \frac{1}{\frac{r}{y}}$, it follows that

$$\sin \alpha = \frac{1}{\csc \alpha}.$$

Similarly, $$\cos \alpha = \frac{1}{\sec \alpha}$$

and $$\tan \alpha = \frac{1}{\cot \alpha}.$$

FUNCTIONS OF COMPLEMENTARY ANGLES

By definition, in right triangle ABC of Fig. 119, $\sin \alpha = \frac{y}{r}$ and $\cos \beta = \frac{y}{r}$. Hence $\sin \alpha = \cos \beta$. However, α and β are complementary angles (*i.e.*, $\alpha + \beta = 90°$). Hence $\beta = 90° - \alpha$.

$$\therefore \sin \alpha = \cos (90° - \alpha).$$

Similarly, $$\cos \alpha = \frac{x}{r} = \sin \beta$$

or $$\cos \alpha = \sin (90° - \alpha).$$

Also $$\tan \alpha = \frac{y}{x} = \cot \beta$$

or $$\tan \alpha = \cot (90° - \alpha).$$

The above relations mean that

$$\sin 30° = \cos (90° - 30°) = \cos 60°$$
$$\tan 40° = \cot (90° - 40°) = \cot 50°$$
$$\csc 20° = \sec (90° - 20°) = \sec 70°, \text{ etc.}$$

FUNDAMENTAL RELATIONS BETWEEN THE TRIGONOMETRIC FUNCTIONS

In Fig. 120 let α be any acute angle in the right triangle ABC.

$a^2 + b^2 = c^2$ by the Pythagorean theorem.

$$\sin \alpha = \frac{a}{c}, \cos \alpha = \frac{b}{c}.$$

Squaring each equation and adding;

$$\sin^2 \alpha + \cos^2 \alpha = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1$$

or
$$\sin^2 \alpha + \cos^2 \alpha = 1.$$

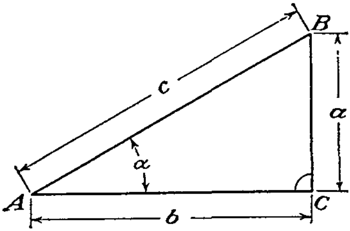


FIG. 120.

Also in Fig. 120,

$$\sec \alpha = \frac{c}{b}, \tan \alpha = \frac{a}{b}.$$

Squaring each equation and subtracting,

$$\sec^2 \alpha - \tan^2 \alpha = \frac{c^2}{b^2} - \frac{a^2}{b^2} = \frac{c^2 - a^2}{b^2} = \frac{b^2}{b^2} = 1$$

or
$$\sec^2 \alpha = 1 + \tan^2 \alpha.$$

Similarly,
$$\csc \alpha = \frac{c}{a}, \cot \alpha = \frac{b}{a}.$$

Squaring each equation and subtracting;

$$\csc^2 \alpha - \cot^2 \alpha = \frac{c^2}{a^2} - \frac{b^2}{a^2} = \frac{c^2 - b^2}{a^2} = \frac{a^2}{a^2} = 1$$

$$\csc^2 \alpha = 1 + \cot^2 \alpha.$$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{\cancel{c}} \cdot \frac{\cancel{c}}{b} = \frac{a}{b} = \tan \alpha$$

or
$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}.$$

Similarly,
$$\frac{\cos \alpha}{\sin \alpha} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{b}{\cancel{c}} \cdot \frac{\cancel{c}}{a} = \frac{b}{a} = \cot \alpha.$$

or
$$\cot \alpha = \frac{\cos \alpha}{\sin \alpha}.$$

UNITY METHOD OF THE TRIGONOMETRIC FUNCTIONS

From the three trigonometric relations, $\sin^2 \alpha + \cos^2 \alpha = 1$, $\sec^2 \alpha - \tan^2 \alpha = 1$, and $\csc^2 \alpha - \cot^2 \alpha = 1$, derived in the preceding section, the following diagrammatic relations of the trigonometric functions may be shown in their respective places in each of the triangles in Figs. 121, 122, and 123. The trigonometric functions now shown diagrammatically may be stated in terms of the unity method as follows:

In Fig. 121, $\sin \alpha$ is numerically equal to the length of the side opposite (HI) when the hypotenuse (GI) is unity. Likewise, $\cos \alpha$ is numerically equal to the length of the side adjacent (GH) when the hypotenuse (GI) is unity.

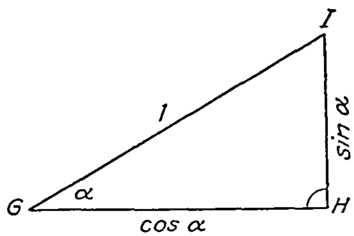


FIG. 121.

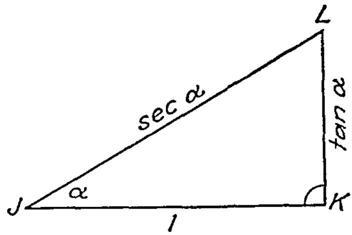


FIG. 122.

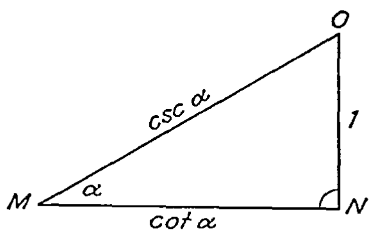


FIG. 123.

In Fig. 122, $\tan \alpha$ is numerically equal to the length of the side opposite (KL) when the side adjacent (JK) is unity. Likewise, $\sec \alpha$ is numerically equal to the length of the hypotenuse (JL) when the side adjacent (JK) is unity.

In Fig. 123, cot α is numerically equal to the length of the side adjacent (MN) when the side opposite (NO) is unity. Likewise, csc α is numerically equal to the length of the hypotenuse (MO) when the side opposite (NO) is unity.

Drills on Trigonometric Functions

A thorough understanding of the meaning of the trigonometric functions as given above is very essential and to attain this the following drill is beneficial:

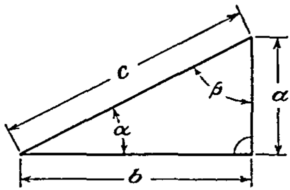


FIG. 124.

When a is 1, b is what function of the angle α?
Ans.: b is the cotangent of angle α.
When c is 1, a is what function of the angle β?
Ans.: a is the cosine of angle β.
When b is 1, c is what function of the angle β?
Ans.: csc β.
When c is 1, a is what function of the angle α?
Ans.: sin α.
When a is 1, c is what function of the angle α?
Ans.: csc α.
When b is 1, c is what function of the angle β?
Ans.: csc β.
When a is 1, c is what function of the angle β?
Ans.: sec β.
When c is 1, b is what function of the angle α?
Ans.: cos α.

Drills similar to the foregoing should be practiced frequently until the student is so familiar with this work that, instead of thinking in terms of a rule, he will think in terms of a triangle and will, considering one side to be unity, immediately recognize the other sides as the proper functions of a given angle.

EACH TRIGONOMETRIC FUNCTION EXPRESSED IN TERMS OF THE OTHER FIVE FUNCTIONS

From the reciprocal relations and the fundamental relations derived in the preceding section (all of which should be memorized by the student), each of the six functions may be expressed in terms of each of the other five.

To illustrate this, sin α can be expressed in terms of the others as follows:

From Fig. 121,

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha}$$

Using Fig. 122 and the relation $\sec^2 \alpha = 1 + \tan^2 \alpha$,

$$\sin \alpha = \frac{\text{opp. side}}{\text{hyp.}} = \frac{KL}{JL} = \frac{\tan \alpha}{\sec \alpha} = \frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}.$$

From Fig. 123 and the relation $\csc^2 \alpha = 1 + \cot^2 \alpha$,

$$\sin \alpha = \frac{\text{opp. side}}{\text{hyp.}} = \frac{NO}{MO} = \frac{1}{\csc \alpha} = \frac{1}{\sqrt{1 + \cot^2 \alpha}}.$$

From Fig. 122,

$$\sin \alpha = \frac{KL}{JL} = \frac{\tan \alpha}{\sec \alpha} = \frac{\sqrt{\sec^2 \alpha - 1}}{\sec \alpha}.$$

By the reciprocal relation,

$$\sin \alpha = \frac{1}{\csc \alpha}.$$

The following chart gives the value of each of the trigonometric functions in terms of the other five. The expressions given in the first horizontal row are the values just developed for sin α. The other expressions should be verified by the student and worked out in a manner similar to that given above, if a thorough understanding of the subject is desired.

TRIGONOMETRIC FUNCTIONS AND THEIR RELATIONS SHOWN IN CHART FORM

Function	In terms of					
	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$	$\sec \alpha$	$\csc \alpha$
$\sin \alpha$		$\sqrt{1 - \cos^2 \alpha}$	$\frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}$	$\frac{1}{\sqrt{\cot^2 \alpha + 1}}$	$\frac{\sqrt{\sec^2 \alpha - 1}}{\sec \alpha}$	$\frac{1}{\csc \alpha}$
$\cos \alpha$	$\sqrt{1 - \sin^2 \alpha}$		$\frac{1}{\sqrt{1 + \tan^2 \alpha}}$	$\frac{\cot \alpha}{\sqrt{1 + \cot^2 \alpha}}$	$\frac{1}{\sec \alpha}$	$\frac{\sqrt{\csc^2 \alpha - 1}}{\csc \alpha}$
$\tan \alpha$	$\frac{\sin \alpha}{\sqrt{1 - \sin^2 \alpha}}$	$\frac{\sqrt{1 - \cos^2 \alpha}}{\cos \alpha}$		$\frac{1}{\cot \alpha}$	$\sqrt{\sec^2 \alpha - 1}$	$\frac{1}{\sqrt{\csc^2 \alpha - 1}}$
$\cot \alpha$	$\frac{\sqrt{1 - \sin^2 \alpha}}{\sin \alpha}$	$\frac{\cos \alpha}{\sqrt{1 - \cos^2 \alpha}}$	$\frac{1}{\tan \alpha}$		$\frac{1}{\sqrt{\sec^2 \alpha - 1}}$	$\sqrt{\csc^2 \alpha - 1}$
$\sec \alpha$	$\frac{1}{\sqrt{1 - \sin^2 \alpha}}$	$\frac{1}{\cos \alpha}$	$\sqrt{1 + \tan^2 \alpha}$	$\frac{\sqrt{\cot^2 \alpha + 1}}{\cot \alpha}$		$\frac{\csc \alpha}{\sqrt{\csc^2 \alpha - 1}}$
$\csc \alpha$	$\frac{1}{\sin \alpha}$	$\frac{1}{\sqrt{1 - \cos^2 \alpha}}$	$\frac{\sqrt{1 + \tan^2 \alpha}}{\tan \alpha}$	$\sqrt{1 + \cot^2 \alpha}$	$\frac{\sec \alpha}{\sqrt{\sec^2 \alpha - 1}}$	

The relations given in the preceding table, together with the simple relations, enable one to determine the value of any of the remaining functions when one of them is given.

Example: Determine each of the other functions of the angle α if $\sin \alpha = .8$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - .64} = \sqrt{.36} = .6.$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{.8}{.6} = 1.3333$$

$$\cot \alpha = \frac{\cos \alpha}{\sin \alpha} = \frac{.6}{.8} = .75$$

$$\sec \alpha = \frac{1}{\cos \alpha} = \frac{1}{.6} = 1.6667$$

$$\csc \alpha = \frac{1}{\sin \alpha} = \frac{1}{.8} = 1.25$$

VARIATION OF THE TRIGONOMETRIC FUNCTIONS FROM 0 TO 90°

In Fig. 125, let DE be a quadrant of a circle of unit radius.

$$\sin \alpha = \frac{BC}{AB} = \frac{BC}{1} = BC$$

As the angle α decreases and approaches 0 the line BC which represents the $\sin \alpha$ approaches 0.

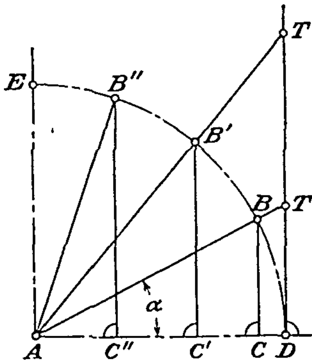


FIG. 125.

As the angle α increases, the $\sin \alpha$ increases to $B'C'$, to $B''C''$, and finally to AE, which is 1. Thus the sine of an angle increases from 0 to 1 as the angle increases from 0 to 90°.

The $\cos \alpha = \dfrac{AC}{AB}$ is represented by the line AC which, for $\alpha = 0$, has the value $AD = 1$ and successively takes the smaller values AC, AC', and AC'' as α gets larger. As α approaches 90°, the line AC approaches 0.

Thus the cosine varies from 1 to 0 as the angle varies from 0 to 90°.

$\tan \alpha = \dfrac{TD}{AD}$ is represented by the line DT, which for $\alpha = 0°$ is 0, and successively takes larger values DT, DT', etc., as the angle α increases. As α approaches 90°, DT gets longer and finally approaches infinity (∞).

Thus the tangent varies from 0 to ∞ as the angle varies from 0 to 90°.

From the reciprocal relation, $\cot \alpha = \dfrac{1}{\tan \alpha}$, the cotangent varies from $\dfrac{1}{0} \to \infty$ to $\dfrac{1}{\infty} \to 0$ as the angle varies from 0 to 90°.

From the reciprocal relation, $\sec \alpha = \dfrac{1}{\cos \alpha}$, the secant

varies from $\frac{1}{1} = 1$ to $\frac{1}{0} \rightarrow \infty$ as the angle varies from 0 to 90°.

From the reciprocal relation, $\csc \infty = \dfrac{1}{\sin \alpha}$, the cosecant varies from $\frac{1}{0} \rightarrow \infty$ to $\frac{1}{1} = 1$ as the angle varies from 0 to 90°.

The variations of the functions may be summarized as follows:

As angle	increases	from	0°	to'	90°
sine	increases	from	0	to	1
cosine	decreases	from	1	to	0
tangent	increases	from	0	to	∞
cotangent	decreases	from	∞	to	0
secant	increases	from	1	to	∞
cosecant	decreases	from	∞	to	1

TO FIND THE TRIGONOMETRIC FUNCTIONS OF A GIVEN ANGLE

The numerical values of the six simple trigonometric functions have been accurately worked out for all angles. These values are given to five figures in the table* for all angles in degrees and minutes from 0° to 90°. This table is used as follows: For any angle up to 45°, the degree of the angle is at the top of the page and the minutes of the degree are in the vertical column at the left. The functions for any given angle from 0° to 45° are given in the horizontal rows to the right of the given minute, the names of the functions for each column being read at the top.

Example a: The tangent of 37° 21′ is found on the 37° page (page 333) in the column labeled tangent, in the horizontal row opposite 21′, the value being .76318. For angles from 45° to 90°, the degree of the angle is at the bottom of the page, the minutes are in the vertical column at the right, reading from bottom to top, and the names of the functions for each column are read at the bottom.

Example b: The cosine of 64° 51′ is found on the 64° page (page 342) in the column labeled cosine (at the bottom) in the horizontal row opposite 51′ (in the *right* vertical column), the value being .42499.

* See table of trigonometric functions at end of book (pp. 324–358).

Determination of an Unknown Side

Consider the type of problem of determining the length of one side of a right-angled triangle when another side and an acute angle are given:

Procedure: Assume the given side to be unity. Then the side in question will be some function of the given angle according to the definitions of the trigonometric functions. For any problem, the side in question will be as many times as great as the function as the given side is of unity.

Example a: In the accompanying figure, determine the length of BC.

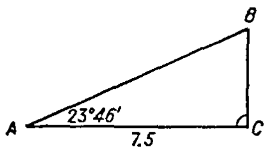


FIG. 126.

Solution: If AC were unity, BC would be, by definition, tan 23° 46′.

Since AC is 7.5 (which is 7.5 times unity), BC will be 7.5 times the tan 23° 46′, or $7.5 \times .44036 = 3.3027$.

Example b: In the figure DEF, determine DF.

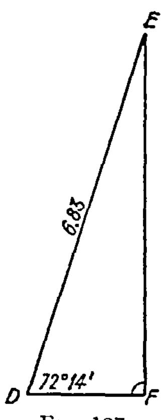


FIG. 127.

Solution: If DE were unity, DF by definition would be cos 72° 14′. But DE is 6.83 and therefore $DF = 6.83 \times \cos$ 72° 14′ or $6.83 \times .30514 = 2.0841$.

Example c: In the figure GHJ, determine X.

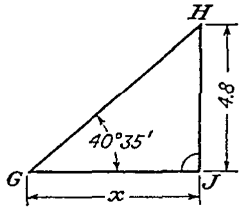
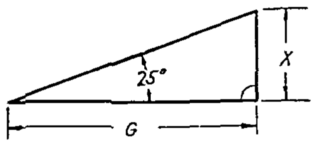


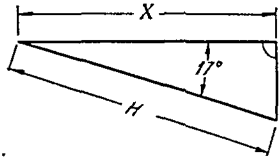
FIG. 128.

Solution: If HJ were unity, X, by definition, would be cot $40°\ 35'$. But HJ is 4.8, and therefore X is $4.8 \times \cot 40°\ 35' = 4.8 \times 1.1674 = 5.6035$.

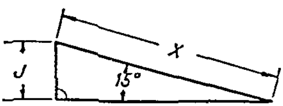
PROBLEMS



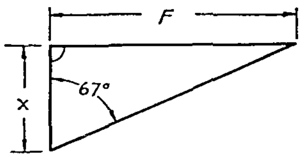
1. Determine the distance x.



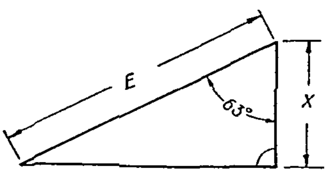
2. Determine the distance x.



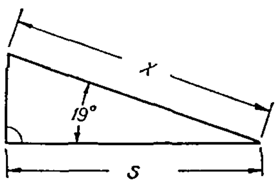
3. Determine the distance x.



4. Determine the distance x.



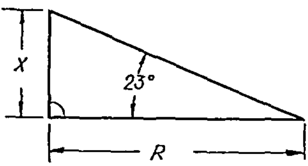
5. Determine the distance x.



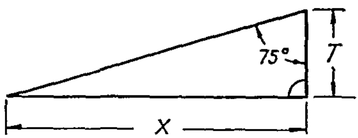
6. Determine the distance x.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	G	10.106	10.218	10.342	10.412	10.818	10.937
2	H	12.135	12.248	12.375	12.492	12.625	12.75
3	J	2.225	2.312	2.386	2.468	2.591	2.724
4	F	8.875	8.955	9.128	9.322	9.462	9.575
5	E	15.25	15.41	15.54	15.63	15.76	15.88
6	S	17.32	17.46	17.59	17.72	17.83	17.97



7. Determine the distance x.



8. Determine the distance x.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
7	R	14.46	14.52	14.89	15.08	15.23	15.39
8	T	2.751	2.812	2.933	3.104	3.225	3.376

TO FIND THE ANGLE CORRESPONDING TO A GIVEN TRIGONOMETRIC FUNCTION OR COFUNCTION

It should be noticed in the table of the trigonometric functions that the degrees from 0 to 45 are given at the top of the page and the degrees from 45 to 90 are given at the bottom. It should also be noticed that in the same column of the trigonometric tables a column headed at the top of the page by the function of the angle is headed at the bottom of the page by the cofunction of the complement of the angle, and *vice versa*. Hence, to locate the value of a given $\begin{pmatrix}\text{function}\\ \text{cofunction}\end{pmatrix}$ in the trigonometric table, proceed to find the nearest number $\begin{pmatrix}\text{smaller}\\ \text{larger}\end{pmatrix}$ than the given $\begin{pmatrix}\text{function}\\ \text{cofunction}\end{pmatrix}$ in either column

headed by the $\begin{pmatrix}\text{function}\\ \text{cofunction}\end{pmatrix}$ or its $\begin{pmatrix}\text{cofunction}\\ \text{function}\end{pmatrix}$. If this nearest $\begin{pmatrix}\text{smaller}\\ \text{larger}\end{pmatrix}$ number is found in the vertical column headed at the top by that $\begin{pmatrix}\text{function}\\ \text{cofunction}\end{pmatrix}$, the degree of the angle is taken from the top of the page and the minutes from the left-hand column horizontally opposite the nearest $\begin{pmatrix}\text{smaller}\\ \text{larger}\end{pmatrix}$ number to the given $\begin{pmatrix}\text{function}\\ \text{cofunction}\end{pmatrix}$; but if this nearest $\begin{pmatrix}\text{smaller}\\ \text{larger}\end{pmatrix}$ number is found in the vertical column headed at the bottom by that $\begin{pmatrix}\text{function}\\ \text{cofunction}\end{pmatrix}$, the degree of the angle is taken from the bottom of the page and the minutes from the right-hand column horizontally opposite the nearest $\begin{pmatrix}\text{smaller}\\ \text{larger}\end{pmatrix}$ number to the given $\begin{pmatrix}\text{function}\\ \text{cofunction}\end{pmatrix}$.

Example a: Find the angle whose sine is .36442.

Solution: First, locate the nearest smaller number in the column headed by either sin or cos. In this case the nearest smaller number to .36442 is .36434, which is found on page 341 in the column headed sin at the top. Hence, the degree of the angle is at the top of the page, and the minute is found in the left-hand column horizontally opposite .36434. Thus, in this case, the angle in degrees and minutes is 21° 22′.

Example b: Find the angle whose tangent is 4.0908.

Solution: Locate the nearest smaller number in the column headed by either tan or cot, which is 4.08666 on page 327 in the column headed cot at the top. Since this column is headed tan at the bottom (and since the angle desired is the angle whose tan is 4.0908), the degree of the angle is read at the bottom of the page and the minute is found in the right-hand column horizontally opposite the value 4.08666. The angle in degrees and minutes is found to be 76° 15′.

Note: In the foregoing problem the nearest degree and minute corresponding to the tangent of 4.0908 is 76° 16′.

However, the student is cautioned to follow the rule of using the nearest smaller number in the case of finding an angle corresponding to a *function*, and the nearest larger number in the case of finding an angle corresponding to a *cofunction*, in order to maintain a definite procedure for the coming work of interpolating for seconds.

Example c: Find the angle whose cosine is .53155.

Solution: Locate the nearest *larger* number (since this is a cofunction) in the column headed either cos or sin, which is .53164 on page 343 in the column headed sin at the top and cos at the bottom. Since the angle desired is the angle whose cos is .53155, the degree of the angle is read at the bottom of the page and the minute is found in the right-hand column horizontally opposite .53164. The angle in degrees and minutes is found to be 57° 53′.

TO DETERMINE AN ANGLE WHEN TWO SIDES OF A RIGHT TRIANGLE ARE GIVEN

In actual practice it is often necessary to obtain an angle of a right triangle when two of its sides are given.

Example: Determine the angle α when the two sides AC and BC are given.

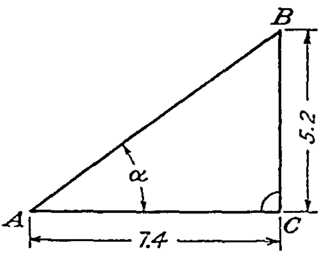


FIG. 129.

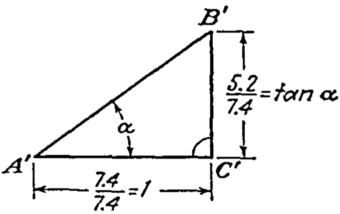


FIG. 130.

Solution: If AC is made unity, BC becomes the tangent of α. To make AC unity, it must be divided by itself (7.4). However, if AC is divided by 7.4, BC must also be divided by 7.4 in order that the triangle retain a similar shape as in triangle $A'B'C'$. $B'C'$ is now the tangent of α. Hence,

$\tan \alpha = \frac{5.2}{7.4} = .70270$. Find the angle having a tangent of .70270 as in the foregoing example *b*. This angle is 35° 5′.

RULE FOR FINDING FUNCTION OF AN ANGLE

From the foregoing procedure a **general rule** may be formulated for finding a function of an unknown angle: Divide one side of the right-angled triangle by another. The side which is the denominator of the fraction thus formed may be considered as one, and the side which is the numerator of the fraction then represents the function of the angle, and the value of the fraction is equal to the function of the angle.

Note: When one of the sides given is the hypotenuse, always divide a side by the hypotenuse (*not* the hypotenuse by a side). The reason for this is that dividing the hypotenuse by a side gives the secant or the cosecant. These functions, for a large range of angles, have very small differences, which thus makes it difficult to compute the seconds accurately.

Example: Determine the angle α in Fig. 131.

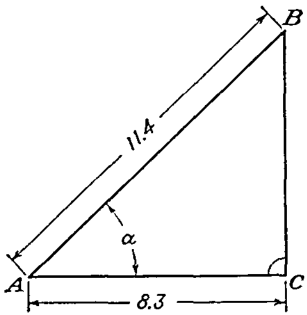
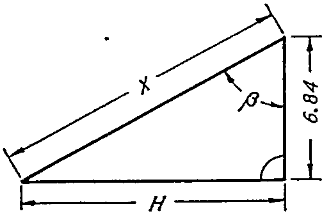


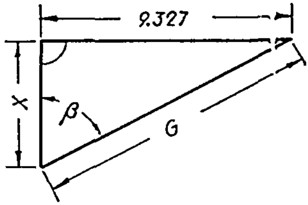
FIG. 131.

Solution: Following the general rule, divide 8.3 by 11.4. Hence 11.4 is the side to be considered unity and 8.3 is the side which represents the function, which in this case is the $\cos \alpha$, and the value of which is $\frac{8.3}{11.4} = .72807$. Find the angle having this cosine as in the foregoing example *c*.

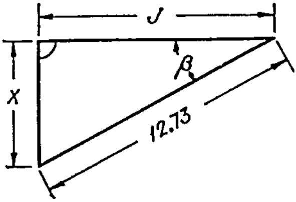
$$\alpha = 43° \; 16'.$$

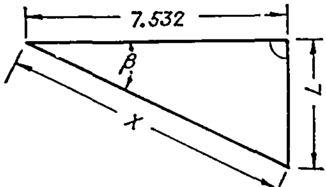
PROBLEMS



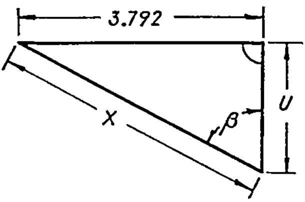


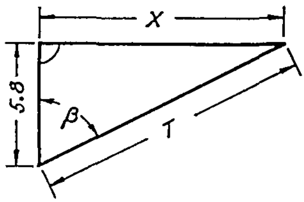
1. Determine the angle β
2. Determine the distance x.
3. Determine the angle β.
4. Determine the distance x.





5. Determine the angle β.
6. Determine the distance x.
7. Determine the angle β.
8. Determine the distance x.

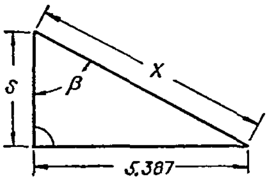


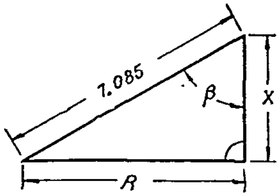


9. Determine the angle β.
10. Determine the distance x.
11. Determine the angle β.
12. Determine the distance x.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	H	9.12	9.25	9.37	9.48	9.56	9.75
2	H	9.12	9.25	9.37	9.48	9.56	9.75
3	G	11.431	11.572	11.613	11.724	11.815	11.976
4	G	11.431	11.572	11.613	11.724	11.815	11.976
5	J	8.55	8.71	8.82	8.96	9.25	9.42
6	J	8.55	8.71	8.82	8.96	9.25	9.42
7	L	4.28	4.37	4.46	4.65	4.84	4.93
8	L	4.28	4.37	4.46	4.65	4.84	4.93
9	U	1.54	1.66	1.78	1.82	1.85	1.95
10	U	1.54	1.66	1.78	1.82	1.85	1.95
11	T	10.3	10.8	11.1	11.5	11.9	12.2
12	T	10 3	10.8	11.1	11.5	11.9	12.2





13. Determine the angle β.
14. Determine the distance x.
15. Determine the angle β.
16. Determine the distance x.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
13	S	3.12	3.25	3.37	3.52	3.65	3.75
14	S	3.12	3.25	3.37	3.52	3.65	3.75
15	R	4.91	5.12	5.23	5.44	5.65	5.86
16	R	4.91	5.12	5.23	5.44	5.65	5.86

INTERPOLATION

General Method

The tables of natural trigonometric functions of angles are usually given for degrees and minutes only. If an angle is required in degrees, minutes, and seconds corresponding to a given function or cofunction, or if the value of a function or cofunction is required for an angle in degrees, minutes, and seconds, a process called **interpolation** must be used.

The process of interpolating the number of seconds, when a function or cofunction of an angle is given, can be best explained by an illustrative problem.

Illustrative Problem: Let it be required to find the angle in degrees, minutes, and seconds corresponding to a tangent of .27038. From the method previously given, the required angle is found to be between 15° 7′ and 15° 8′. The accompanying figure is distorted in order to bring out the procedure more clearly. When $AE = 1$, $DE = \tan 15° \ 7' = .27013$ and $BE = \tan 15° \ 8' = .27044$ and $BD = BE - DE$, or $BD = .27044 - .27013$ which, disregarding the decimal point is 31.

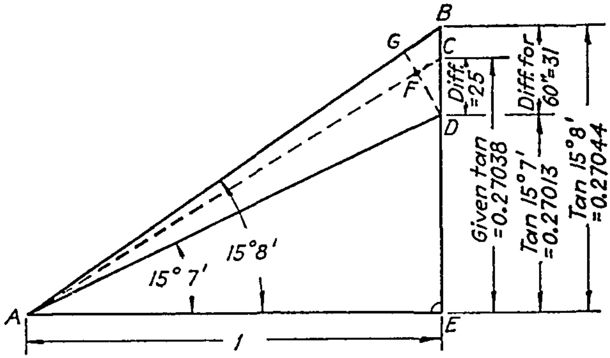


FIG. 132.

The difference between the tangent CE of the required ngle and the tangent DE of 15° 7′ is $CD = .27038 - .27013$, hich, disregarding the decimal point, is 25. Draw an arc FG with A as a center and radius AD. $\frac{\text{Angle } DAF}{\text{Angle } DAG} = \frac{\widehat{DF}}{\widehat{DG}}$ P-34), Arc DFG is nearly a straight line, so that the gures DFC and DGB approximate two similar triangles and $\frac{\widehat{F}}{\widehat{G}} = \frac{DC}{DB}$ nearly. Hence, $\frac{\text{angle } DAF}{\text{angle } DAG} = \frac{DC}{DB}$ nearly where DAF is the required angle (θ) and $\angle DAG = 1'$ or $60''$. Ience, $\frac{\theta}{60''} = \frac{25}{31}$ or $\theta = \frac{25}{31} \times 60'' = 48''$ with no appreciable rror.

ROCEDURE FOR FINDING AN ANGLE IN DEGREES, MINUTES, AND SECONDS BY INTERPOLATION

1 *Find the number of degrees and minutes by the method reviously outlined.*

2. *Find the difference between the next smaller and the next arger values of the function (Disregard the decimal point.)*

3. *Find the difference between the* $\left(\begin{matrix}\textit{function}\\ \textit{cofunction}\end{matrix}\right)$ *corresponding o the next* $\left(\begin{matrix}\textit{smaller}\\ \textit{larger}\end{matrix}\right)$ *in value and the given value. (Disregard he decimal point.)*

4. *To obtain the number of seconds, multiply* 60 *by a common fraction, the numerator of which is the number obtained in* 3, *and the denominator of which is the number obtained in* 2.

The following examples will illustrate this procedure:

Example a: Find the angle whose sine is .19758.

Solution: By (2), $\sin 11°\ 24' = .19766$

$\sin 11°\ 23' = \underline{.19737}$

The difference is 29

By (3), given value = .19758

$\sin 11°\ 23' = \underline{.19737}$

The difference is 21

By (4), $60'' \times \frac{21}{29} = 43''$

Hence the required angle is 11° 23′ 43″.

Example b: Find the angle whose cotangent is 1.9096.

Solution: By (2), $\cot 27°\ 38' = 1.9101$

$\cot 27°\ 39' = \underline{1.9088}$

The difference is 13

By (3), $\cot 27°\ 38' = 1.9101$

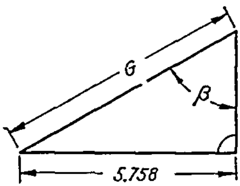
given value = $\underline{1.9096}$

The difference is 5

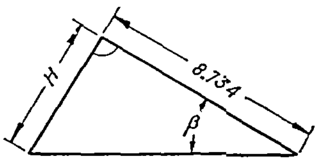
By (4), $60'' \times \frac{5}{13} = 23''$

Hence, the required angle is 27° 38′ 23″.

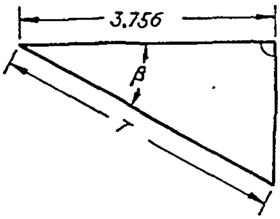
PROBLEMS



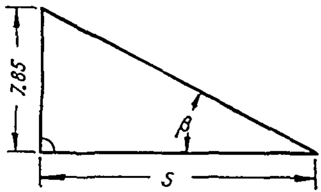
1. Determine the angle β.



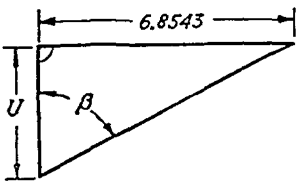
2. Determine the angle β.



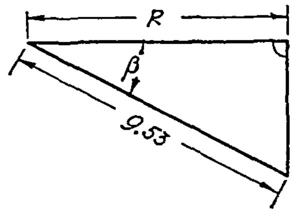
3. Determine the angle β.



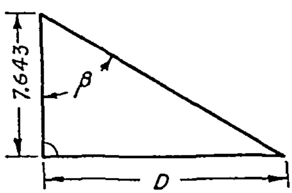
4. Determine the angle β.



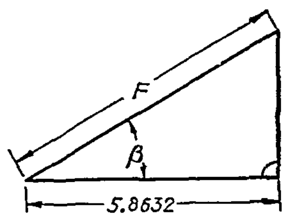
5. Determine the angle β.



6. Determine the angle β.



7. Determine the angle β.



8. Determine the angle β.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	*G*	8.543	8.628	8.764	8.775	8.848	8.965
2	*H*	3.223	3.394	3.430	3.568	3.672	3.828
3	*T*	5.128	5.237	5.382	5.446	5.529	5.644
4	*S*	9.108	9.266	9.391	9.473	9.582	9.712
5	*U*	3.516	3.728	3.935	4.069	4.222	4.475
6	*R*	6.112	6.329	6.546	6.762	6.876	6.989
7	*D*	9.344	9.575	9.773	9.919	10.101	10.362
8	*F*	7.231	7.462	7.663	7.824	7.985	8.226

PROCESS OF FINDING THE FUNCTION OR COFUNCTION OF AN ANGLE GIVEN IN DEGREES, MINUTES AND SECONDS BY INTERPOLATION

1. *Find the value of the* $\begin{pmatrix}\textit{function}\\ \textit{cofunction}\end{pmatrix}$ *of the angle in degrees and minutes by the method previously given.*

2. *Find the value of the* $\begin{pmatrix}\textit{function}\\ \textit{cofunction}\end{pmatrix}$ *for an angle* 1′ *greater.*

3. *Obtain the difference between these two* $\begin{pmatrix}\textit{functions}\\ \textit{cofunctions}\end{pmatrix}$. *Disregard the decimal point.*

4. *Multiply this difference by a fraction, the numerator of which is the number of seconds given and the denominator of which is* 60″.

5. $\begin{pmatrix}\textit{Add}\\ \textit{Subtract}\end{pmatrix}$ *the result obtained in* 4 *to the last digits of the value of the* $\begin{pmatrix}\textit{function}\\ \textit{cofunction}\end{pmatrix}$ *obtained in* 1.

The following problems will illustrate this procedure:

Example a: Find the tangent of 27° 16′ 38″.

Solution: 1. The tangent of 27° 16′ is .51540
2. The tangent of 27° 17′ is .51577
3. The difference is 37
4. $37 \times \frac{38}{60} = 23.4$ or 23.
5. Since the tangent is a *function,* the 23 must be added to the last digits of the figure of step 1.

$$\begin{array}{r} .51540 \\ 23 \\ \hline .51563 \end{array}$$

Thus the tangent of 27° 16′ 38″ is .51563.

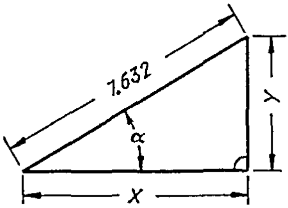
Example b: Find the cosecant of 7° 48′ 18″.

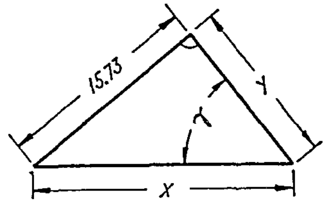
Solution: 1. The cosecant of 7° 48′ is 7.3683
2. The cosecant of 7° 49′ is 7.3527
3. The difference is 156
4. $156 \times \frac{18}{60} = 46.8$ or 47.
5. Since the cosecant is a *cofunction*, the 47 must be subtracted from the last digits of the figure of step 1.

$$\begin{array}{r} 7.3683 \\ 47 \\ \hline 7.3636 \end{array}$$

Thus the cosecant of 7° 48′ 18″ is 7.3636.

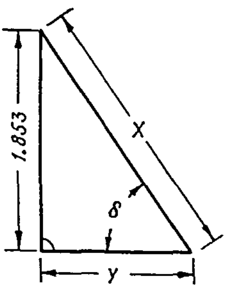
PROBLEMS

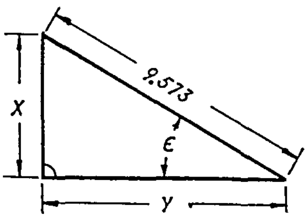




1. Determine the distance x.
2. Determine the distance y.

3. Determine the distance x.
4. Determine the distance y.



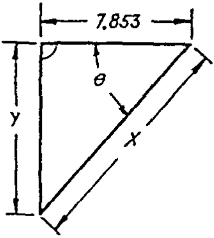


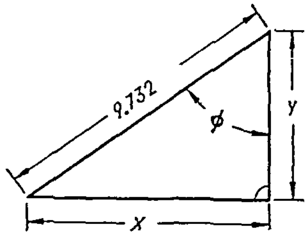
5. Determine the distance x.
6. Determine the distance y.

7. Determine the distance x.
8. Determine the distance y.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	α	32° 12′ 28″	33° 16′ 19″	34° 12′ 33″	35° 42′ 12″	36° 48′ 46″	37° 52′ 48″
2	α	32° 12′ 28″	33° 16′ 19″	34° 12′ 33″	35° 42′ 12″	36° 48′ 46″	37° 52′ 48″
3	γ	46° 42′ 28″	47° 18′ 32″	48° 36′ 10″	49° 15′ 15″	50° 11′ 56″	51° 24′ 14″
4	γ	46° 42′ 28″	47° 18′ 32″	48° 36′ 10″	49° 15′ 15″	50° 11′ 56″	51° 24′ 14″
5	δ	55° 34′ 31″	56° 53′ 46″	57° 18′ 45″	57° 36′ 56″	53° 24′ 18″	48° 13′ 18″
6	δ	55° 34′ 31″	56° 53′ 46″	57° 18′ 45″	57° 36′ 56″	53° 24′ 18″	48° 13′ 18″
7	ϵ	32° 18′ 36″	56° 25′ 13″	30° 46′ 48″	29° 21′ 27″	28° 56′ 11″	27° 34′ 50″
8	ϵ	32° 18′ 36″	56° 25′ 13″	30° 46′ 48″	29° 21′ 27″	28° 56′ 11″	27° 34′ 50″





9. Determine the distance x.
10. Determine the distance y.
11. Determine the distance x.
12. Determine the distance y.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
9	θ	47° 22′ 15″	48° 34′ 40″	49° 16′ 10″	50° 58′ 56″	51° 12′ 14″	52° 45′ 11″
10	θ	47° 22′ 15″	48° 34′ 40″	49° 16′ 10″	50° 58′ 56″	51° 12′ 14″	52° 45′ 11″
11	ϕ	41° 11′ 31″	42° 19′ 42″	43° 32′ 13″	44° 33′ 24″	45° 35′ 15″	46° 52′ 6″
12	ϕ	41° 11′ 31″	42° 19′ 42″	43° 32′ 13″	44° 33′ 24″	45° 35′ 15″	46° 52′ 6″

INTERPOLATION

Special Method

The authors of this text are introducing a new system of interpolating trigonometric functions analogous to that frequently used in connection with logarithms. The advantage of this system is that it eliminates many arithmetical computations, thus resulting in a saving of time and an increase in accuracy.

In especially prepared trigonometric tables,* the differences of the values of the functions for successive minutes have been computed and placed in columns to the right of the functions.

On the page opposite the functions or on the margin below, each of these differences has been divided into 12 equal parts, which may be referred to as proportional parts, corresponding to twelfths of minutes (*i.e.*, 5″, 10″, 15″, etc.). The proportional parts for 1″, 2″, 3″, and 4″ may be obtained by dividing the proportional parts for 10″, 20″, 30″, and 40″ by 10. This

* "Tables of Natural Trigonometric Functions with Differences and Rapid and Easy Method of Interpolation" by J. H. Wolfe and E. R. Phelps.

may be done by moving the decimal point one place to the left in the table. Then the proportional part for 10″ becomes the value for 1″, the proportional part for 20″ becomes the value for 2″, etc.

The use of this system may be best explained by two illustrative examples.

Example a: Find the sine of 33° 29′ 34″.

Solution: 1. Find the sine of 33° 29′ as previously explained (.55169).

2. Note the tabulated difference between the sin 33° 29′ and sin 33° 30′, which in this case is 25.

3. On the page opposite the functions, locate this difference of 25 in the vertical column headed 60″.

4. Since 34″ is between 30″ and 35″, note the difference corresponding to 30″, which is 12.5.

5. To this difference of 12.5, add the difference corresponding to 4″. This value may be obtained by moving the decimal point for 40″ and its proportional part one place to the left. This gives 1.7 as the proportional part for 4″. Thus the difference for 34″ is 12.5 + 1.7 = 14.2 or 14.

6. Since the sine is a *function*, the result of (5) is added to the value obtained from (1). Thus sin 33° 29′ 34″ is .55169 + 14 = .55183.

Example b: Find the angle in degrees, minutes, and seconds when its cotangent is 1.6395.

Solution: 1. Find this angle in degrees and minutes as previously explained (31° 22′).

2. Note the tabulated difference between the cot 31° 22′ and cot 31° 23′, which in this case is 11.

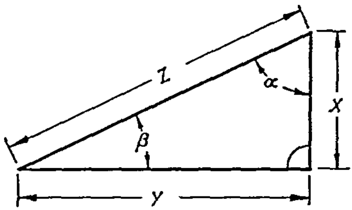
3. Obtain the difference between 1.6395 and the next larger value, (since this is a *cofunction*) 1.6404, which is 9.

4. Referring to the table of proportional parts, in the horizontal row opposite 11, the next number smaller than 9 is 8.3 which corresponds to 45″.

5. Subtracting 8.3 from 9 leaves .7. To find the number of seconds corresponding to this .7, move the decimal point one place to the left for both the angles and the differences. The nearest number to .7 which corresponds to a whole number of

seconds is then 7.3, which corresponds to 4″. Thus the total number of seconds is $45'' + 4'' = 49''$. Hence, the angle is 31° 22′ 49″.

PROBLEMS



Insert the values given in the following tabular form in their proper places according to the foregoing diagram and solve for the distance, or angle in question. When solving for a distance, the result must be correct to five significant figures. When solving for an angle, the result must be correct to degrees, minutes, and seconds.

Prob.	X	Y	Z	β	α	Determine
1	3 567		E			Angle β
2		F		36° 17′ 18″		Distance Z
3	5 763	G				Angle α
4	5 783	D				Distance Z
5			H		67° 27′ 38″	Distance Y
6	J		7 892			Angle β
7		K	8 291			Angle α

VARIABLFS

Prob	Sym	No 1	No 2	No 3	No 4	No 5	No 6
1	E	6 258	6 879	7 138	7 386	7.897	8 207
2	F	2 789	3 569	3 896	4 689	4 973	5.289
3	G	12 87	13 24	13 59	13 96	14 42	14 78
4	D	8 875	8 923	9 134	9 356	9 785	9 982
5	H	4 679	4 876	5 136	5 297	5 587	5 956
6	J	1 876	2 196	2 375	2 869	3.158	3 621
7	K	5 347	5 682	5 913	6 147	6 258	6 873

PROBLEMS (*Continued*)

Prob.	X	Y	Z	β	α	Determine
8	L			25° 31′ 42″		Distance Y
9		M			53° 52′ 43″	Distance Z
10	N		12.87			Angle β
11			P	32° 18′ 25″		Distance Y
12	3.56	Q				Angle α
13		6.789	R			Angle β
14	S				72° 38′ 21″	Distance Z

VARIABLES

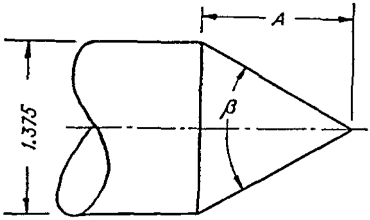
Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
8	L	14.34	14.89	15.13	15.67	15.89	16.14
9	M	3 675	3.875	3.146	4.472	4.735	4.963
10	N	6.137	6.478	6.783	6.924	7.246	7.473
11	P	18.75	18.93	19.46	19.87	20.25	20.79
12	Q	7.783	7.984	8.147	8.356	8.689	8.953
13	R	9.134	9.375	9.687	9.783	9.872	9.962
14	S	2.341	2.472	2.683	2.874	2.965	3.176

PRACTICAL PROBLEMS INVOLVING ONLY RIGHT TRIANGLES

The previous problems in trigonometry were of the simplest type in which the student was asked to solve for an unknown angle or side of a right triangle. In the following problems, which were all taken from the tool, die, and drawing rooms, the student must become familiar with a method of locating right triangles by drawing construction lines wherever necessary.

The first 45 problems are relatively simple and may all be reduced to right triangles by drawing only a few construction lines. The partial solutions will be given for several of these problems in order to teach the student how to construct properly the necessary auxiliary lines. On others, the construction lines have been drawn to assist the student in solving them, and the rest are left entirely to the student for solution.

Lathe Center



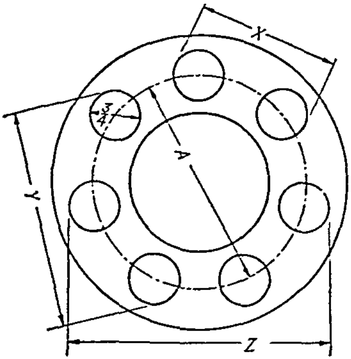
Variable		
No.	Sym.	Value
1	A	1.25
2	A	1.375
3	A	1.5
4	A	1.625
5	A	1.75
6	A	1.875

$A = 2$

Ans. $\beta = 37° \ 56' \ 30''$

1. Determine the angle β.

Flange



Variable		
No.	Sym.	Value
1	A	3.125
2	A	3.25
3	A	3.375
4	A	3.5
5	A	3.625
6	A	3.75

$A = 3.875$

$$\textit{Ans.} \begin{cases} x = 2.4312 \\ y = 3.7795 \\ z = 4.5278 \end{cases}$$

Seven holes equally spaced.

2. Determine the distance x.

3. Determine the distance y.

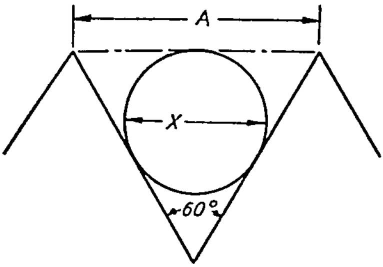
4. Determine the distance z.

Note. To express decimal degrees in degrees, minutes, and seconds, multiply the fractional part of the degree by 60 to obtain the minutes and multiply the resulting fractional part of a minute by 60 to obtain the seconds.

Thus: $25.3673° = 25° \ 22' \ 2''$

$$\begin{array}{r} 60 \\ \hline 22.0380' \\ 60 \\ \hline 2.2800'' \end{array}$$

Gage

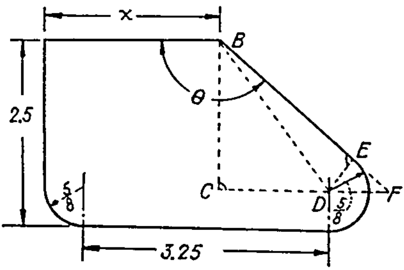


VARIABLE		
No.	Sym.	Value
1	A	2.25
2	A	2.375
3	A	2.5
4	A	2.625
5	A	2.75
6	A	2.875

$A = 3$
Ans. $x = 1.7320$

5. Determine the diameter x.

Gage



VARIABLE		
No.	Sym.	Value
1	θ	132°
2	θ	134°
3	θ	136°
4	θ	138°
5	θ	140°
6	θ	142°

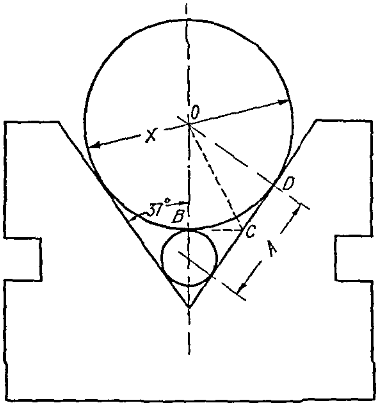
$\theta = 144°$
Ans. $x = 2.3575$

6. Determine the distance x.

Solution:

$BC = 2.5 - .625.$
$\angle CBF = \theta - 90°.$
In $\triangle CFB$, solve for CF.
$\angle EDF = \theta - 90°.$ Why?
In $\triangle DFE$, solve for DF.
$CD = CF - DF.$

V-Block



VARIABLE		
No.	Sym.	Value
1	A	1.29
2	A	1.33
3	A	1.38
4	A	1.45
5	A	1.57
6	A	1.64

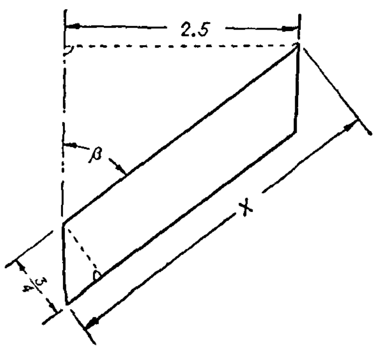
$A = 1.24$
Ans. $x = 2.487$

7. Determine the diameter x.

Solution:

$\angle DOB = 90° - 37°$.
The tangents BC and CD are equal.
$\therefore BC = A \div 2$.
$\angle BOC = \angle DOB \div 2$. Why?
In $\triangle BOC$, solve for OB.
$OB = x \div 2$. Why?

End View of Gib

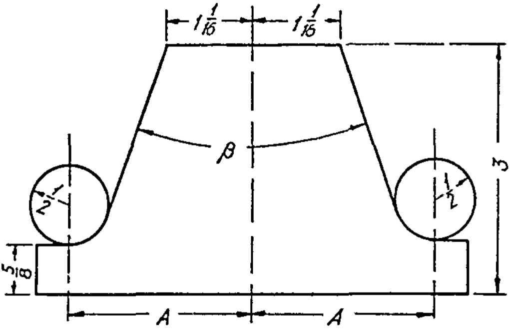


VARIABLE		
No.	Sym.	Value
1	β	48°
2	β	50°
3	β	52°
4	β	54°
5	β	56°
6	β	58°

$\beta = 60°$
Ans. $x = 3.3197$

8. Determine the distance x.

Gage

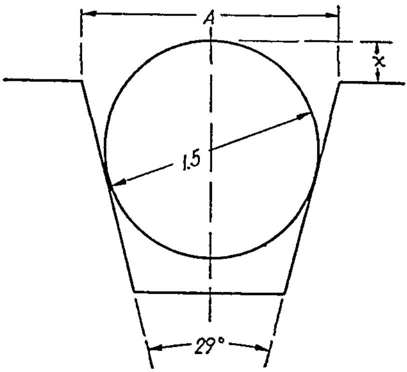


$A = 3$
Ans. $\beta = 70° \; 30' \; 16''$

9. Determine the angle β.

Variable		
No.	Sym.	Value
1	*A*	2.25
2	*A*	2.375
3	*A*	2.5
4	*A*	2.625
5	*A*	2.75
6	*A*	2.875

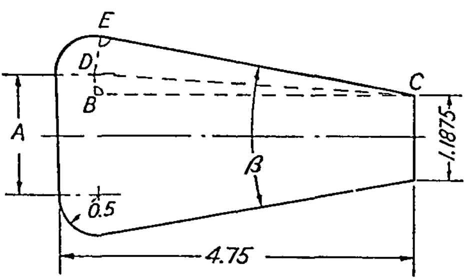
Worm or Acme Thread



$A = 1.645$
Ans. $x = .56506$

10. Determine the distance x.

Variable		
No.	Sym.	Value
1	*A*	1.695
2	*A*	1.745
3	*A*	1.804
4	*A*	1.855
5	*A*	1.913
6	*A*	1.927



$A = 2.25$
Ans. $\beta = 27° \; 39' \; 32''$

11. Determine the angle β. *(Solution on next page.)*

Variable		
No.	Sym.	Value
1	*A*	1.5
2	*A*	1.625
3	*A*	1.75
4	*A*	1.875
5	*A*	2.0
6	*A*	2.125

Solution for preceding problem:

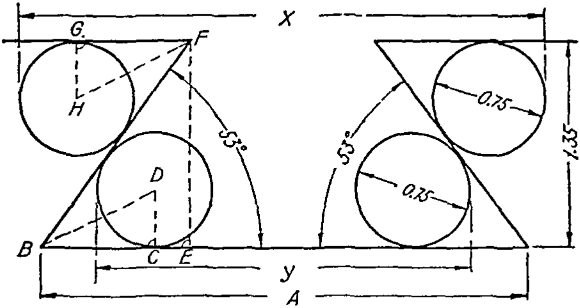
$BD = (A - 1.1875) \div 2.$

In $\triangle BCD$, solve for $\angle BDC$ and CD.

In $\triangle DCE$, solve for $\angle DCE$.

$\beta = 2(\angle BCD + \angle DCE).$

Dovetail



$A = 3.11$

Ans. $\begin{cases} x = 3.3298 \\ y = 2.3557 \end{cases}$

Variable

1. $A = 3.18$	**2.** $A = 3.26$	**3.** $A = 3.31$
4. $A = 3.39$	**5.** $A = 3.46$	**6.** $A = 3.58$

12. Determine the distance x.

13. Determine the distance y.

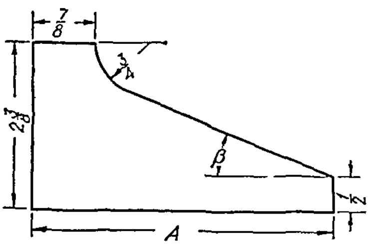
Solution:

In $\triangle BEF$, solve for BE.

$\angle DBC = 53° \div 2.$

In $\triangle DBC$, solve for BC.

$GF = BC.$ Why?



Variable

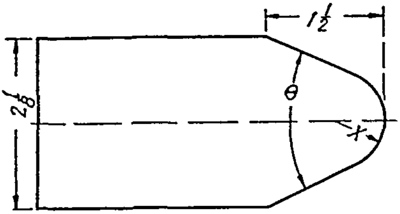
No.	Sym.	Value
1	A	4.25
2	A	4.375
3	A	4.5
4	A	4.625
5	A	4.75
6	A	4.875

$A = 5$

Ans. $\beta = 17° \; 51' \; 18''$

14. Determine the angle β.

Die Punch

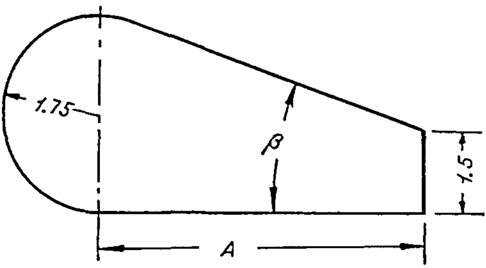


Variable		
No.	Sym.	Value
1	θ	46°
2	θ	48°
3	θ	50°
4	θ	52°
5	θ	54°
6	θ	56°

$\theta = 58°$

Ans. $x = .39221$

15. Determine the radius x.



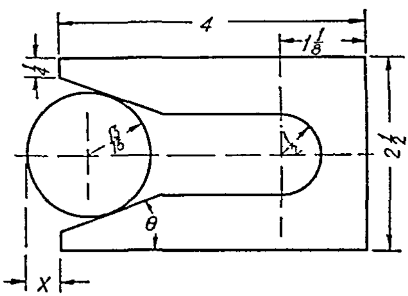
Variable		
No.	Sym.	Value
1	A	5.5
2	A	5.2
3	A	5.3
4	A	5.4
5	A	5.7
6	A	5.6

$A = 5.8$

Ans. $\beta = 20° \; 0' \; 44''$

16. Determine the angle β.

Gage



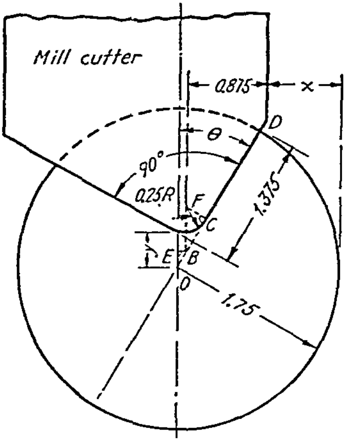
Variable		
No.	Sym.	Value
1	θ	16°
2	θ	17°
3	θ	18°
4	θ	19°
5	θ	20°
6	θ	21°

$\theta = 22°$

Ans. $x = .50636$

17. Determine the distance x.

Angular Milling Machine Cutter



Variable

No.	Sym.	Value
1	θ	30°
2	θ	32°
3	θ	34°
4	θ	36°
5	θ	38°
6	θ	40°

$$\theta = 42^\circ$$

$$Ans. \begin{cases} x = .64258 \\ y = .38175 \end{cases}$$

18. Determine the distance x.

19. Determine the distance y.

Solution:

$CD = 1.375 - CF.$

In $\triangle BCF$, solve for BC and BF.

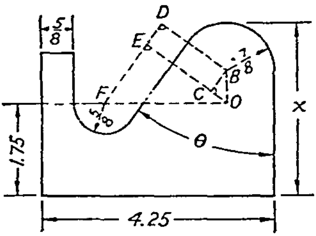
$OB = 1.75 - BC - CD.$

In $\triangle OBE$, solve for BE and EO

$x = 1.75 - .875 - BE.$

$y = EO + BF - .25.$

Gage



Variable

No.	Sym.	Value
1	θ	32°
2	θ	34°
3	θ	36°
4	θ	38°
5	θ	40°
6	θ	42°

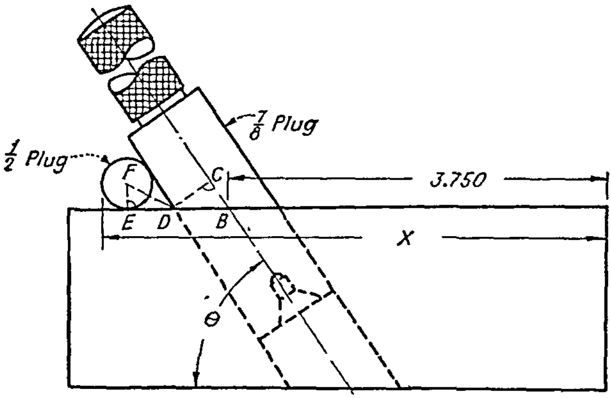
$$\theta = 44^\circ$$

$$Ans.\ x = 2.6661$$

20. Determine the distance x.

Dotted lines show diagrammatic hint.

Checking Angular Holes



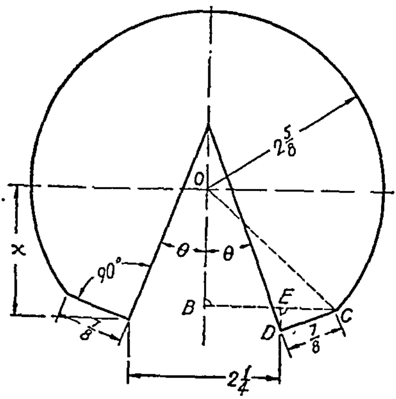
$\theta = 62°$
Ans. $x = 4.9115$

VARIABLE

1. $A = 50°$ 2. $A = 52°$ 3. $A = 54°$
4. $A = 56°$ 5. $A = 58°$ 6. $A = 60°$

21. Determine the distance x.
Solution:

$CD = .875 \div 2.$
In $\triangle DBC$, solve for BD.
$\angle FDE = \theta \div 2.$
In $\triangle EFD$, solve for DE.



VARIABLE

No.	Sym.	Value
1	θ	14°
2	θ	16°
3	θ	18°
4	θ	20°
5	θ	22°
6	θ	24°

$\theta = 26°$
Ans. $x = 2.1827$

22. Determine the distance x. (*Solution on next page.*)

Solution for preceding problem:

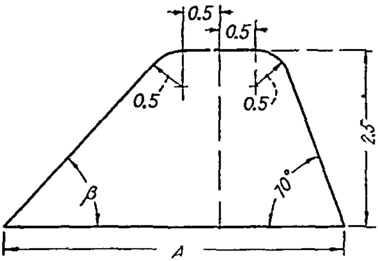
$\angle ECD = \theta$. Why?

In $\triangle DEC$, solve for CE and DE.

$BE = 2.25 \div 2$. $BC = BE + CE$.

In $\triangle BCO$, solve for $\angle BOC$ and OB.

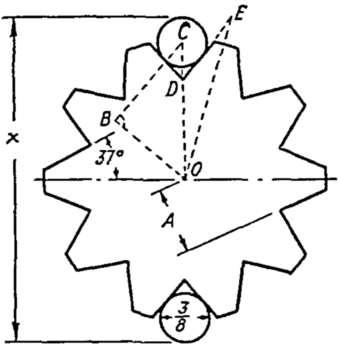
$OB + DE = x$.



VARIABLE No.	Sym.	Value
1	A	4.0
2	A	4.125
3	A	4.25
4	A	4.375
5	A	4.5
6	A	4.625

$A = 4.75$
Ans. $\beta = 47° 46' 37''$

23. Determine the angle β.



VARIABLE No.	Sym.	Value
1	A	.774
2	A	.792
3	A	.812
4	A	.855
5	A	.875
6	A	.892

$A = .751$
Ans. $x = 2.6664$

24. Determine the distance x.

Solution:

$\angle DEO = 37°$. $\angle DOE = 180° \div 10$. Why?

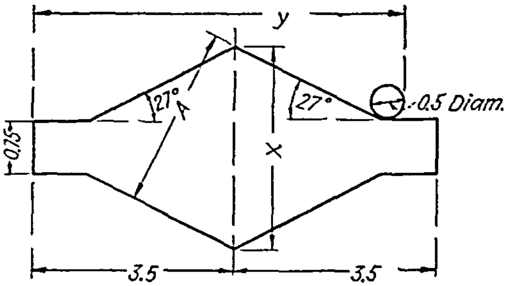
$\angle CDE = \angle DEO + \angle DOE$. Why?

$\angle BCO = \angle CDE$. Why?

$BO = A + .1875$.

In $\triangle BOC$, solve for CO.

Die Section



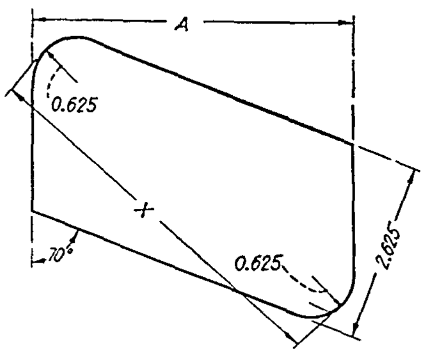
Variable		
No.	Sym.	Value
1	A	2.05
2	A	2.12
3	A	2.25
4	A	2.37
5	A	2.48
6	A	2.55

$A = 2.08$

Ans. $\begin{cases} x = 2.3343 \\ y = 5.3646 \end{cases}$

25. Determine the value of x.
26. Determine the value of y.

End View of Gib



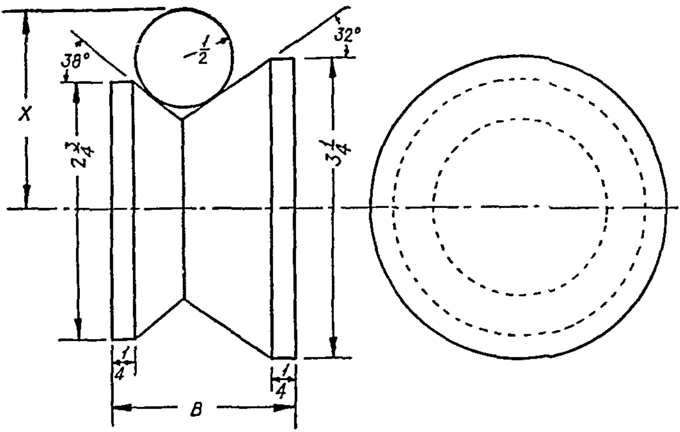
Variable		
No.	Sym.	Value
1	A	4.51
2	A	4.562
3	A	4.63
4	A	4.684
5	A	4.75
6	A	4.816

$A = 5.125$

Ans. $x = 6.0742$

27. Determine the distance x.

Form Roller



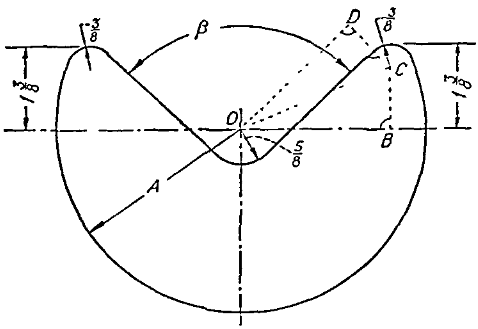
$B = 2.25$
Ans. $X = 2.0159$

VARIABLE

1. $B = 1.5$
2. $B = 1.625$
3. $B = 1.75$
4. $B = 1.875$
5. $B = 2.0$
6. $B = 2.125$

28. Determine the distance x.

Gage



VARIABLE

No.	Sym.	Value
1	A	2 875
2	A	3 0
3	A	3 125
4	A	3 25
5	A	3 357
6	A	3 5

$A = 3.625$
Ans. $\beta = 108° \; 19' \; 18''$

29. Determine the angle β.

Solution:

$OC = A - .375$. $CB = 1.375 - .375$.

In $\triangle OBC$, solve for $\angle COB$.

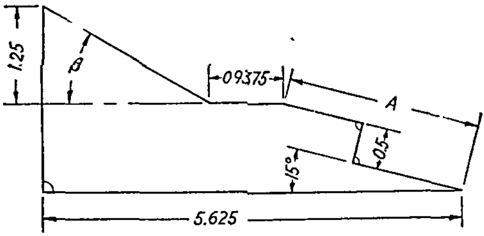
$CD = .625 + .375$. Why? *(Continued on next page.)*

Solution continued:

In $\triangle ODC$, solve for $\angle DOC$.

$\beta = 2(90° - \angle COB - \angle DOC)$.

Gage



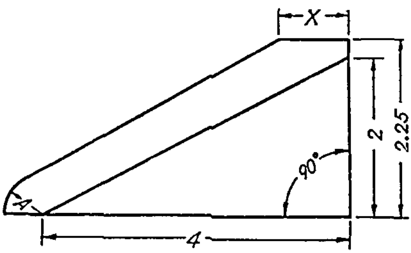
	Variable	
No.	Sym.	Value
1	A	1.875
2	A	2.0
3	A	2.25
4	A	2.375
5	A	2.5
6	A	2.625

$A = 2.75$

Ans. $\beta = 30° \, 3' \, 10''$

30. Determine the angle β.

Fixture Section

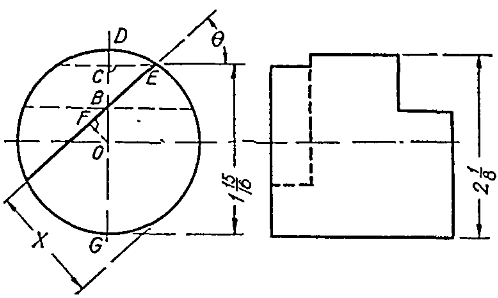


	Variable	
No.	Sym.	Value
1	A	.5
2	A	.625
3	A	.75
4	A	.875
5	A	1.0
6	A	1.125

$A \approx 1.25$

Ans. $x \approx 2.2951$

31. Determine the distance x.



	Variable	
No.	Sym.	Value
1	θ	42°
2	θ	44°
3	θ	46°
4	θ	48°
5	θ	50°
6	θ	52°

$\theta = 54°$

Ans. $x = 1.0891$

32. Determine the distance x. (*Solution on next page.*)

Solution for proceding problem:

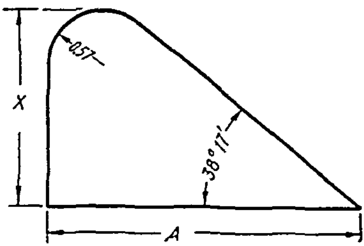
$CG = 1.9375$. $CD = 2.125 - 1.9375$.

In circle GED, solve for CE by geometry by P-39 and P-45.

In $\triangle BCE$, solve for BC. $OC = 1.9375 - (2.125 \div 2)$.

$OB = OC - BC$.

In $\triangle OFB$, solve for OF.



VARIABLE

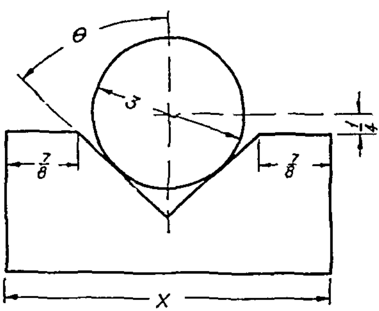
No.	Sym.	Value
1	A	3.11
2	A	3.32
3	A	3.73
4	A	3.84
5	A	3.55
6	A	3.96

$A = 3.65$

Ans. $x = 2.2748$

33. Determine the distance x.

V-Block



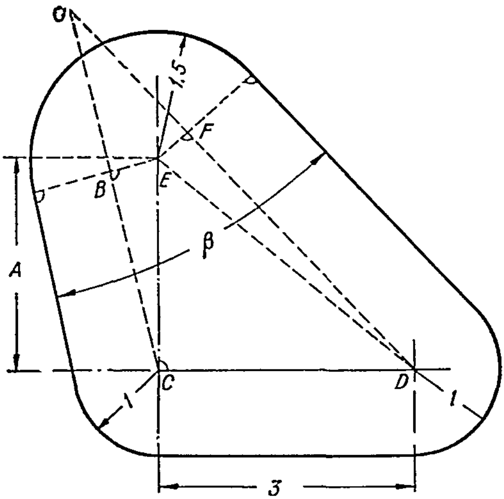
VARIABLE

No.	Sym.	Value
1	θ	38°
2	θ	40°
3	θ	42°
4	θ	44°
5	θ	46°
6	θ	48°

$\theta = 50°$

Ans. $x = 5.8212$

34. Determine the distance x.



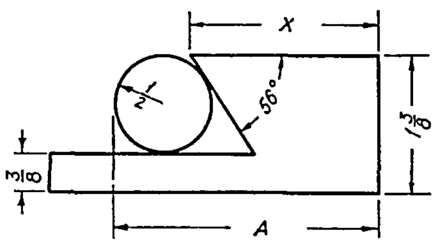
VARIABLE		
No.	Sym.	Value
1	A	2.0
2	A	2.125
3	A	2.25
4	A	2.375
5	A	2.5
6	A	2.625

$A = 2.75$
Ans. $\beta = 29° \, 57' \, 30''$

35. Determine the angle β.
Solution:

$BE = 1.5 - 1$. $EF = 1.5 - 1$. $\angle COD = \beta$. Why?
In $\triangle BCE$, solve for $\angle BCE$. In $\triangle ECD$, solve for $\angle CDE$ and DE.
$\angle BCD = \angle BCE + 90°$. In $\triangle EDF$, solve for $\angle EDF$.
$\angle CDF = \angle CDE + \angle EDF$.
$\beta = \angle COD = 180° - \angle BCD - \angle CDF$.

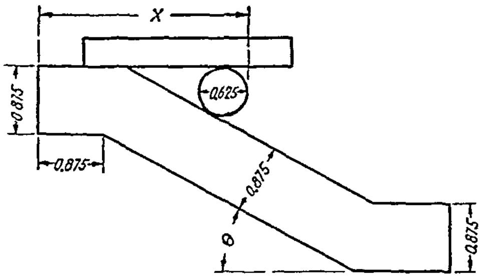
Dovetail Section



VARIABLE		
No.	Sym.	Value
1	A	2.125
2	A	2.25
3	A	2.375
4	A	2.5
5	A	2.625
6	A	2.75

$A = 2.875$
Ans. $x = 2.1091$

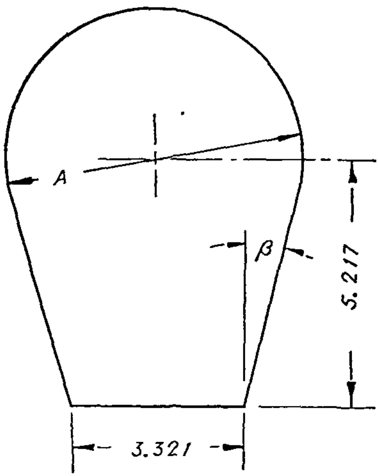
36. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	θ	23°
2	θ	25°
3	θ	27°
4	θ	29°
5	θ	31°
6	θ	33°

$\theta = 35°$
Ans. $x = 2.4545$

37. Determine the distance x.

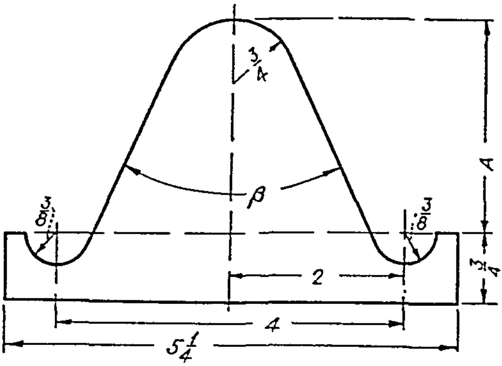


VARIABLE		
No.	Sym.	Value
1	A	5.51
2	A	5.67
3	A	5.82
4	A	5.95
5	A	6.18
6	A	6.32

$A = 6.27$
Ans. $\beta = 17° \ 16' \ 39''$

38. Determine the angle β.

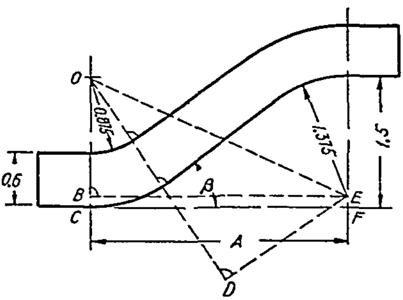
Fixture Section



Variable		
No.	Sym.	Value
1	A	2.125
2	A	2.25
3	A	2.375
4	A	2.5
5	A	2.625
6	A	2.75

$A = 2.875$
Ans. $\beta = 41° \ 10' \ 40''$

39. Determine the angle β.



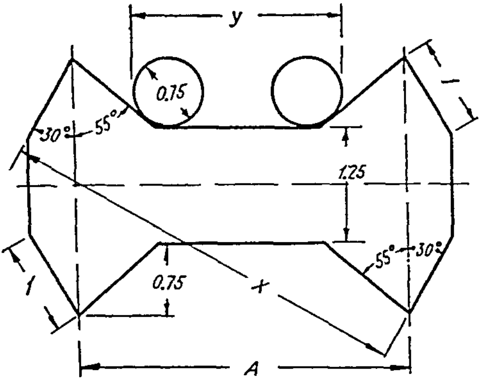
Variable		
No.	Sym.	Value
1	A	3.0
2	A	3.4
3	A	2.7
4	A	2.8
5	A	2.9
6	A	3.2

$A = 3.6$
Ans. $\beta = 27° \ 17'$

40. Determine the angle β.
Solution:

$EF = BC = 1.5 - 1.375$. $OB = OC - BC$.
In $\triangle OBE$, solve for $\angle BOE$ and OE.
$OD = .875 + .6 + 1.375$. Why?
In $\triangle ODE$, solve for $\angle DOE$.
$\beta = \angle BOE - \angle DOE$. Why?

Gage



Variable		
No.	Sym.	Value
1	A	3.125
2	A	3.25
3	A	3.375
4	A	3.5
5	A	3.625
6	A	3.75

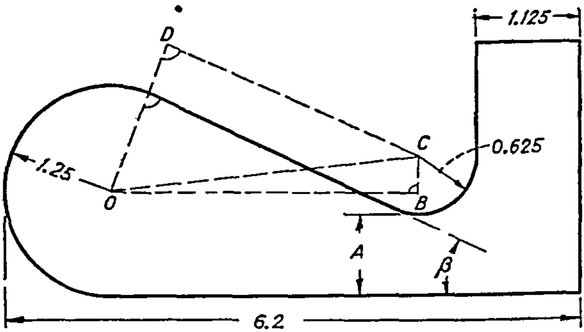
$A = 3.875$

Ans. $\begin{cases} x = 4.7306 \\ y = 2.2464 \end{cases}$

41. Determine the distance x.

42. Determine the distance y.

Gage



$A = .781$

Ans. $\beta = 33° 1' 40''$

Variable

1. $A = .91$	**2.** $A = .72$	**3.** $A = .83$
4. $A = .94$	**5.** $A = .65$	**6.** $A = .76$

43. Determine the angle β.

Solution:

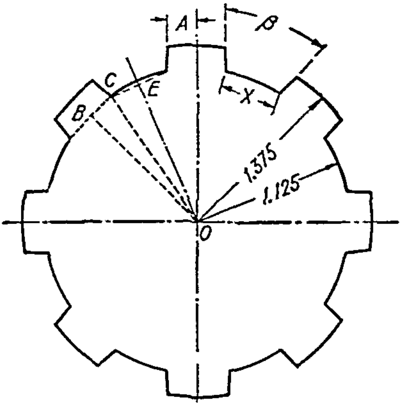
$BC = A + .625 - 1.25.$

In $\triangle OBC$, solve for $\angle BOC$ and CO.

In $\triangle ODC$, solve for $\angle DOC$.

$\beta = 90° - \angle BOC - \angle DOC$. Why?

Spline Shaft



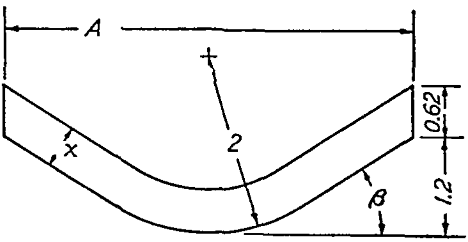
Variable		
No.	Sym.	Value
1	A	.375
2	A	.348
3	A	.321
4	A	.294
5	A	.267
6	A	.240

$$Ans. \begin{cases} A = .279 \\ x = .21864 \\ \beta = 45° \end{cases}$$

44. Determine the distance x.
45. Determine the angle β.
Solution:

$$\angle BOE = \frac{180°}{\text{number of splines}}.$$

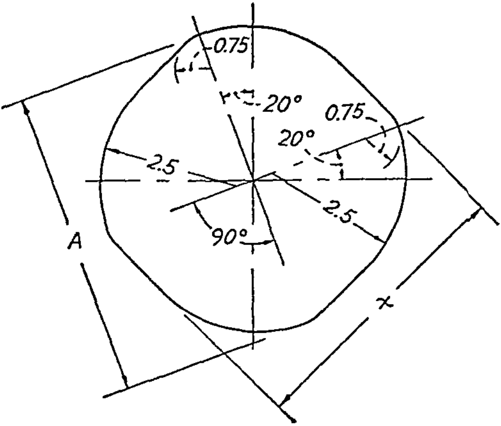
In $\triangle BCO$, solve for $\angle BOC$. $\angle COE = \angle BOE - \angle BOC$.
In $\triangle COE$, solve for CE. $x = 2(CE)$



Variable		
No.	Sym.	Value
1	A	4.375
2	A	4.500
3	A	4.625
4	A	4.750
5	A	4.875
6	A	5.000

$$Ans. \begin{cases} A = 5.125 \\ \beta = 30° \, 49' \, 19'' \\ x = .53243 \end{cases}$$

46. Determine the angle β.
47. Determine the distance x.



$A = 5.5$
Ans. $x = 5.1923$

VARIABLE

1. $A = 5.625$	2. $A = 5.750$	3. $A = 5.875$
4. $A = 6.000$	5. $A = 6.125$	6. $A = 6.625$

48. Determine the distance x.

OBLIQUE TRIANGLES

Many problems may be reduced directly to right triangles (as those of the previous group), but others must first be reduced to oblique triangles (D-29).

These oblique triangles may, in turn, be reduced to right triangles. If the three altitudes (D-30) of an oblique triangle are drawn, six right triangles are formed, as shown in Figs. 133 and 134.

The six right triangles formed in both the acute and the obtuse triangles are *ADC*, *BDC*, *AFB*, *AFC*, *BEC*, and *BEA*.

$+ .625 - 1.25.$
OBC, solve for $\angle BOC$.
ODC, solve for $\angle DOC$.
$90° - \angle BOC - \angle DOC$. Wh

Whenever it is desired to solve for any part of an oblique triangle by reducing it to right triangles, only two of the six right triangles are necessary. The two to be used depend upon what is given and required. The procedure for selecting the proper two right triangles is as follows:

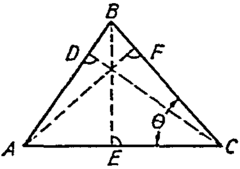


Fig. 133.

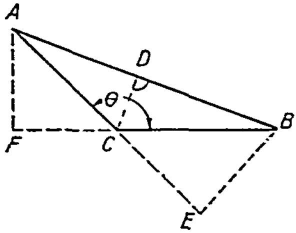


Fig. 134.

Given: AB, AC, and $\angle\theta$.

To solve for $\angle ABC$ and side BC.

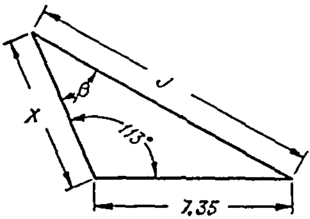
Since the $\angle\theta$ is given, the altitude from C cannot be used since it would divide the known angle into two unknown parts, thus giving two rt. $\triangle BDC$ and CDA, the first having only the right angle known and the second having the right angle and only one other part known. Hence, these right triangles cannot be used to find any unknown parts (see statement on page 181).

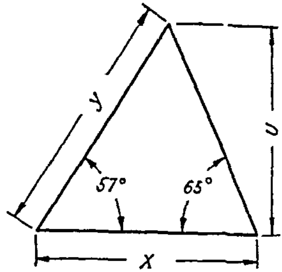
Drawing the altitudes from B would give two rt. $\triangle BEC$ and BEA each of which contains a right angle and only one other part known. The altitude AF forms two rt. $\triangle AFB$ and AFC, the first of which cannot be used (insufficient parts). The $\triangle AFC$, however, is determined because it has three known parts, AC, rt. $\angle AFC$ and $\angle ACF$ (which is θ or $180° - \theta$), and any unknown part of it may be obtained. Thus the side AF may be computed. Then the desired angle ABC may be obtained since in the rt. $\triangle AFB$, the parts AB, AF and the rt. $\angle AFB$ are given. In obtaining BC, note that in the acute $\triangle ABC$, $BC = BF + FC$ and in the obtuse $\triangle ABC$, $BC = BF - FC$. BF may be obtained from rt. $\triangle BFA$ which now contains the three known parts, AB, rt. $\angle BFA$ and $\angle ABF$. FC may be obtained from the rt. $\triangle AFC$ which now contains

the three known parts, AF, rt. $\angle AFC$, and $\angle\theta$ (or $180° - \theta$). Thus BC can be obtained.

PROBLEMS

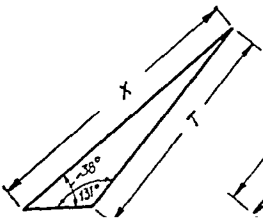
Simple Oblique Triangles

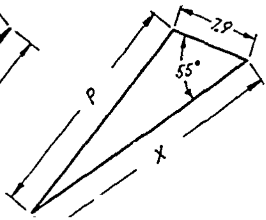




1. Determine the angle β.
2. Determine the distance x.
3. Determine the distance x.
4. Determine the distance y.

reduced



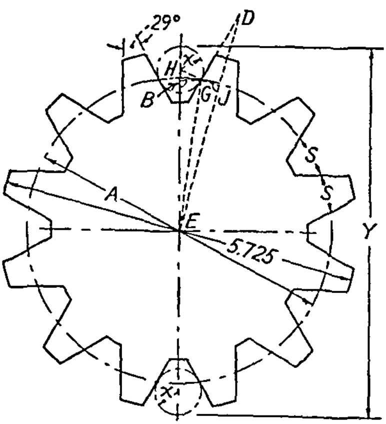


VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	J	10.2	10.4	10.6	10.8	11.1	11.3
2	J	10.2	10.4	10.6	10.8	11.1	11.3
3	U	4.65	4.93	5.42	5.68	5.89	6.24
4	U	4.65	4.93	5.42	5.68	5.89	6.24
5	T	8.21	8.43	8.65	8.78	9.12	9.37
6	P	9.12	9.25	9.37	9.48	9.62	9.75
7	S	12.2	12.8	13.4	13.9	14.5	15.2
8	F	3.21	3.32	3.53	3.64	3.75	3.96

PRACTICAL PROBLEMS

Involving Oblique Triangles



VARIABLE

No.	Sym.	Value
1	A	4.656
2	A	4.704
3	A	4.752
4	A	4.800
5	A	4.848
6	A	4.896

$A = 4.944$

Ans. $\begin{cases} x = .37071 \\ y = 6.0081 \end{cases}$

N = number of teeth in broach

1. Determine the distance y.

Solution:

$\angle BED = 180° \div N$.

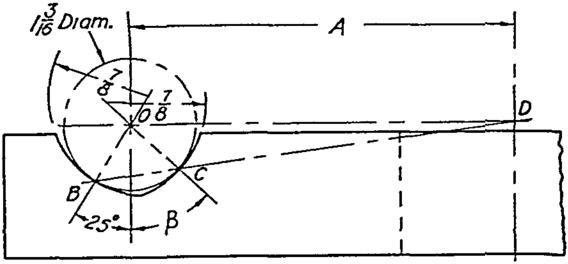
$\angle BEG = \angle GEJ$. Why?

In $\triangle BEG$, solve for BG and BE.

In $\triangle BGH$, solve for GH and BH.

$x = HG$. $y = BE + BH$.

Ball-bearing Racer



$A = 3.8125$
Ans. $\beta = 40° \; 5' \; 16''$

Variable

1. $A = 3.0625$	2. $A = 3.1875$	3. $A = 3.3125$
4. $A = 3.4375$	5. $A = 3.5625$	6. $A = 3.6875$

2. Determine the angle β.

Solution:

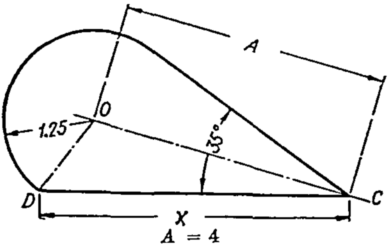
B and C are the points of tangency of the ball.

$OB = OC = 1.1875 \div 2. \quad \angle BOD = 90° + 25°$

In $\triangle BOD$, solve for $\angle OBD$.

$\angle BOC = 180° - 2\angle OBD.$ Why?

$\beta = \angle BOC - 25°.$

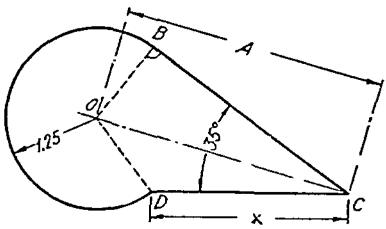


Variable

No.	Sym.	Value
1	A	3.25
2	A	3.375
3	A	3.5
4	A	3.625
5	A	3.75
6	A	3.875

$A = 4$
Ans. $x = 4.3062$

3. Determine the distance x.



Variable

No.	Sym.	Value
1	A	3.25
2	A	3.375
3	A	3.5
4	A	3.625
5	A	3.75
6	A	3.875

$A = 4$
Ans. $x = 3.3521$

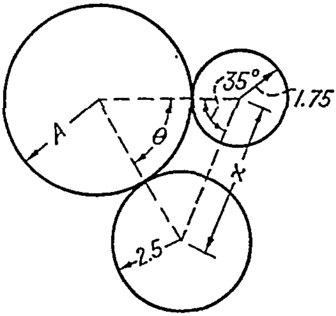
4. Determine the distance x. (*Solution on next page.*)

Solution for preceding problem:

$OB = 1.25$. In $\triangle OBC$, solve for $\angle BCO$.

$\angle OCD = 35° - \angle BCO$.

In $\triangle OCD$, solve for DC.



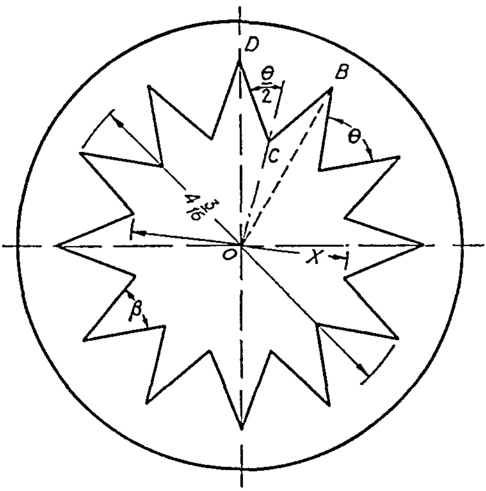
Variable		
No.	Sym.	Value
1	A	2.767
2	A	2.914
3	A	3.061
4	A	3.208
5	A	3.355
6	A	3.502

$$A = 2.62$$

Ans. $\begin{cases} \theta = 115° \, 41' \, 21'' \\ x = 8.0441 \end{cases}$

5. Determine the angle θ.

6. Determine the distance x.



Variable		
No.	Sym.	Value
1	θ	69°
2	θ	71°
3	θ	73°
4	θ	75°
5	θ	77°
6	θ	79°

$$\theta = 81°$$

Ans. $x = 2.7758$

7. Determine the angle β.

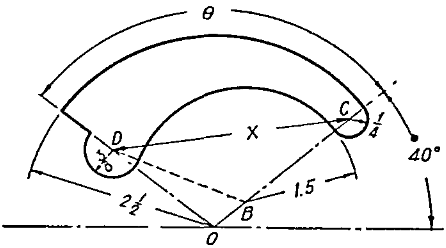
8. Determine the distance x.

Solution:

$\angle DOB = 360°$ divided by the number of points. Why?

$\angle DOC = \angle DOB \div 2$. Why?

The balance of the problem is left to the student.



Variable		
No.	Sym.	Value
1	θ	104°
2	θ	106°
3	θ	108°
4	θ	110°
5	θ	112°
6	θ	114°

$\theta = 116°$

Ans. $x = 3.2839$

9. Determine the distance x.

Solution:

$OB = 2.5 - .5 - 1.5$. $BD = 1.5 + .375$.

In $\triangle DOB$, solve for DO.

In $\triangle DOC$, solve for CD.

PROJECTION FORMULAS

One of the special types of oblique triangle problems is that in which the lengths of the three sides are given, and one or more of the angles are required.

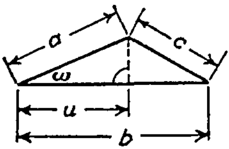


Fig. 135.

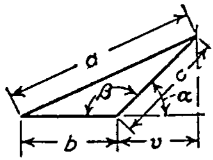


Fig. 136.

The projection formulas of Proposition 32 give the values or u and v as follows:

$$u = \frac{a^2 + b^2 - c^2}{2b}$$

and

$$v = \frac{a^2 - b^2 - c^2}{2b}.$$

Hence

$$\cos \omega = \frac{u}{a} = \frac{a^2 + b^2 - c^2}{2ba}$$

and

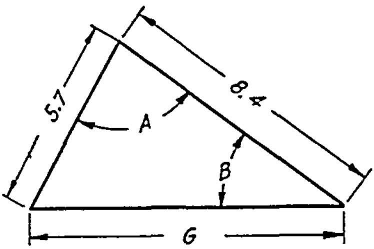
$$\cos \alpha = \frac{v}{c} = \frac{a^2 - b^2 - c^2}{2bc}$$

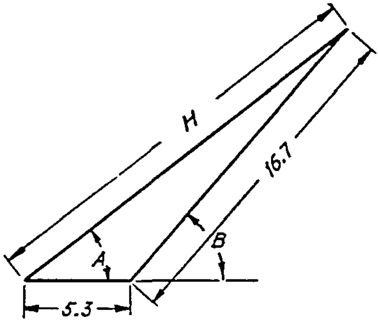
and

$$\beta = 180° - \alpha.$$

Note: The projection formulas can be used only when three sides are given.

PROBLEMS

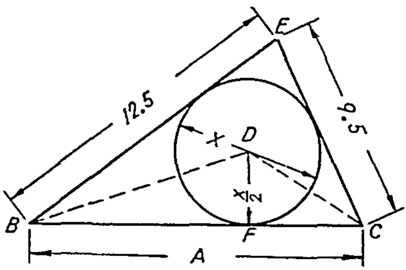




1. Determine the angle A.
2. Determine the angle B.
3. Determine the angle A.
4. Determine the angle B.

VARIABLES

Prob.	Sym.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
1	G	9.5	9.7	9.9	10.1	10.3	10.5
2	G	9.5	9.7	9.9	10.1	10.3	10.5
3	H	19.6	19.9	20.1	20.4	20.6	20.9
4	H	19.6	19.9	20.1	20.4	20.6	20.9



$A = 14.9$

Ans. $x = 6.4020$

VARIABLE

No.	Sym.	Value
1	A	15.3
2	A	16.4
3	A	17.5
4	A	18.4
5	A	19.9
6	A	20.5

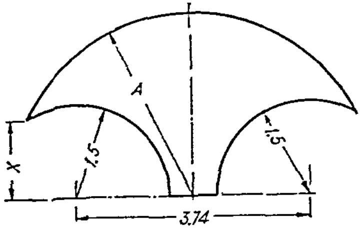
5. Determine the diameter x. (*Solution on next page.*)

Solution for preceding problem:

In $\triangle BCE$, solve for $\angle EBC$ and $\angle ECB$.

$\angle DBC = \angle EBC \div 2$. $\angle DCB = \angle ECB \div 2$.

In $\triangle BCD$, solve for DF.



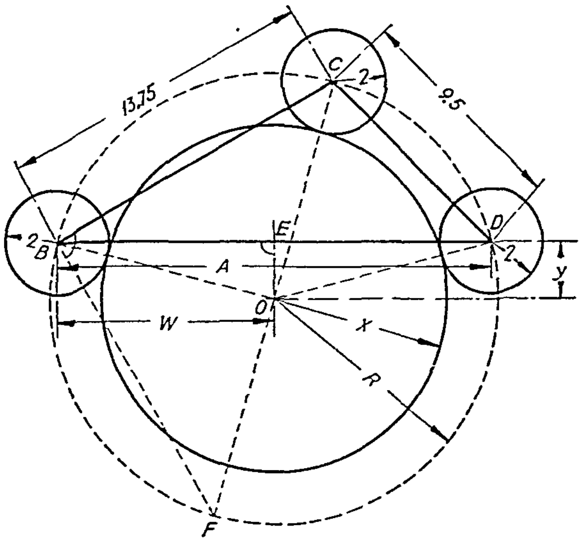
Variable		
No.	Sym.	Value
1	A	2.931
2	A	2.975
3	A	3.124
4	A	2.687
5	A	2.734
6	A	2.891

$A = 2.568$

Ans. $x = 1.4828$

6. Determine the distance x.

Locating the Center of a Gear Which Is in Mesh with Three Definitely Placed Gears Having Equal Numbers of Teeth.



$A = 20$

Ans. $\begin{cases} x = 9.2751 \\ y = 5.2082 \end{cases}$

Variable

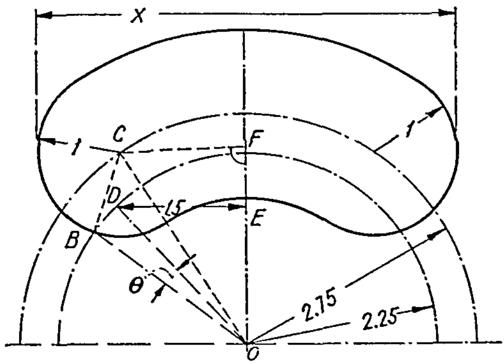
1. $A = 18.5$ 2. $A = 18.75$ 3. $A = 19$
4. $A = 19.25$ 5. $A = 19.5$ 6. $A = 19.75$

7. Determine the radius x.
8. Determine the distance y.

(*Solution on next page.*)

Solution for preceding problem:

In $\triangle BCD$, solve for $\angle CDB$.
$\angle BFC = \angle CDB$. Why?
In $\triangle BFC$, solve for CF. $R = CF \div 2$.
$ED = A \div 2$. In $\triangle OED$, solve for EO.

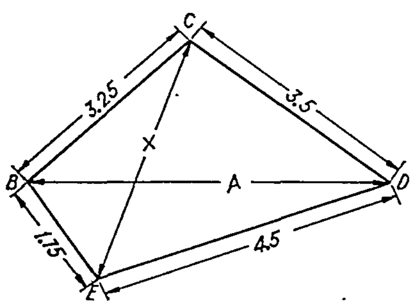


VARIABLE		
No.	Sym.	Value
1	θ	2°
2	θ	4°
3	θ	6°
4	θ	8°
5	θ	10°
6	θ	12°

$\theta = 14°$
Ans. $x = 5.2140$

9. Determine the distance x.
Solution:

In $\triangle BOC$, solve for $\angle BOC$.
$\angle DOC = \angle BOC - \theta$.
In $\triangle DOE$, solve for $\angle DOE$.
$\angle COF = \angle DOE - \angle DOC$.
In $\triangle COF$, solve for CF.



VARIABLE		
No.	Sym.	Value
1	A	4.625
2	A	4.75
3	A	4.875
4	A	5.0
5	A	5.125
6	A	5.25

$A = 5.5$
Ans. $x = 3.5331$

10. Determine the distance x.
Solution:

In $\triangle BDC$, solve for $\angle DBC$.
In $\triangle BDE$, solve for $\angle DBE$.
(*Solution continued on next page.*)

Solution continued:

$\angle CBE = \angle DBC + \angle DBE.$

In $\triangle CBE$, solve for x.

COTANGENT FORMULAS

Another of the special types of oblique triangle problems is that in which a side and the two adjacent angles are given.

A special formula which will be referred to as the "cotangent formula" will be used in solving problems of this type and will now be developed.

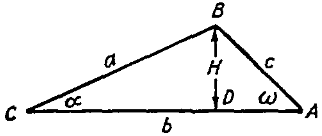


FIG. 137.

Given: Side b and the adjacent $\measuredangle$ α and ω of the $\triangle ABC$. Draw a similar $\triangle A'B'C'$ whose altitude is unity.

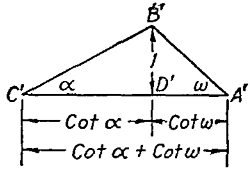


FIG. 138.

Since $B'D' = 1$, $A'D' = \cot\ \omega$ and $C'D' = \cot\ \alpha$ and $A'C' = \cot \omega + \cot \alpha$. Since the triangles in Figs. 137 and 138 are similar,

$$\frac{H}{1} = \frac{b}{\cot \alpha + \cot \omega} \qquad \text{(D-34).}$$

or

$$H = \frac{b}{\cot \alpha + \cot \omega}.$$

This relation may be stated in the form of a rule as follows:

When a side and two adjacent angles are given, the altitude to that side is equal to the length of the side divided by the sum of the cotangents of the two adjacent angles.

If AB (Fig. 137) is required, it can be obtained by multiplying the altitude by csc ω. Similarly BC can be obtained.

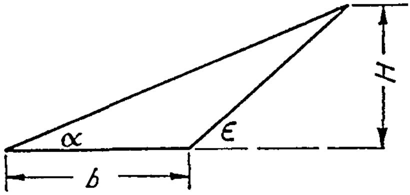


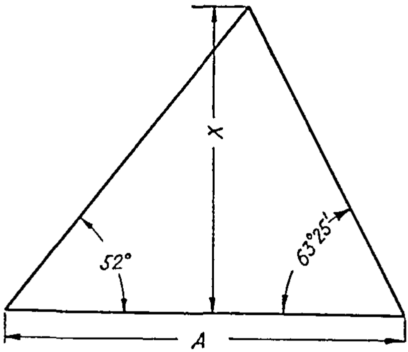
FIG. 139.

In the case of an obtuse triangle, the cotangent formula becomes

$$H = \frac{b}{\cot \alpha - \cot \varepsilon}.$$

The proof of this is left to the student.

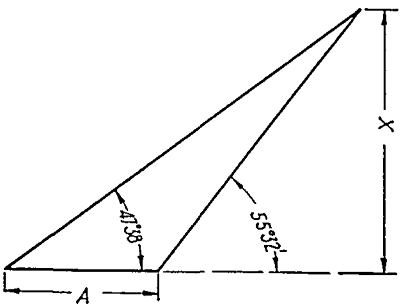
PROBLEMS



VARIABLE		
No.	Sym.	Value
1	A	4.68
2	A	5.09
3	A	5.85
4	A	6.43
5	A	6.94
6	A	7.65

$A = 7.76$
Ans. $x = 6.0549$

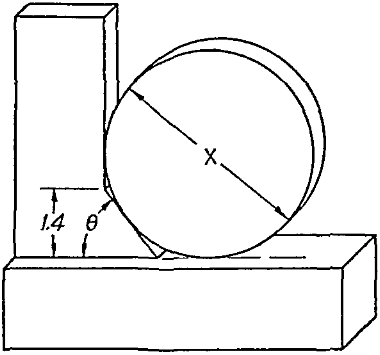
1. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	8.43
2	A	8.64
3	A	8.87
4	A	8.93
5	A	9.22
6	A	9.76

$A = 9.87$
Ans. $x = 43.742$

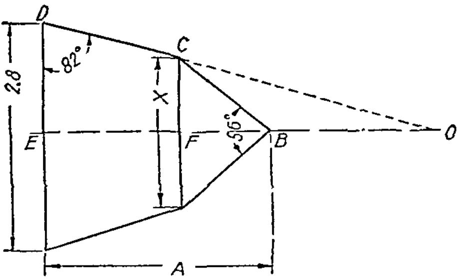
2. Determine the distance x.



Variable		
No.	Sym.	Value
1	θ	50°
2	θ	48°
3	θ	46°
4	θ	44°
5	θ	42°
6	θ	40°

$\theta = 38°$
Ans. $x = 5.4660$

3. Determine the diameter x.



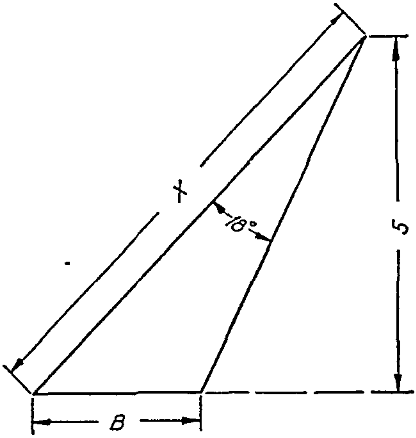
Variable		
No.	Sym.	Value
1	A	3.6
2	A	3.9
3	A	4.2
4	A	4.8
5	A	5.3
6	A	5.9

$A = 5.6$
Ans. $x = 1.6664$

4. Determine the distance x.

Solution:

$DE = 2.8 \div 2$. In $\triangle EOD$, solve for EO.
$BO = EO - A$. $\angle EOD = 90° - 82°$. $\angle CBE = 56° \div 2$.
In $\triangle BOC$, solve for CF.

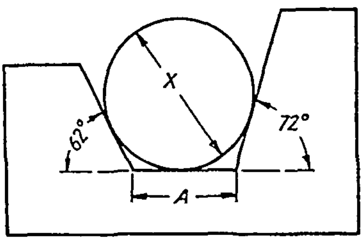


No.	Sym.	Value
1	B	2.21
2	B	2.42
3	B	2.73
4	B	3.24
5	B	3.45
6	B	3.76

(VARIABLE)

$B = 3.35$
Ans. $x = 8.6783$

5. Determine the distance x.

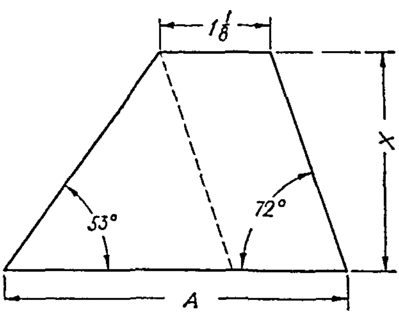


No.	Sym.	Value
1	A	1.3
2	A	1.7
3	A	1.9
4	A	2.2
5	A	2.4
6	A	2.8

(VARIABLE)

$A = 2.9$
Ans. $x = 4.3694$

6. Determine the diameter x.

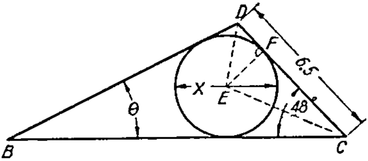


No.	Sym.	Value
1	A	3.25
2	A	3.375
3	A	3.5
4	A	3.625
5	A	3.75
6	A	3.875

(VARIABLE)

$A = 4$
Ans. $x = 2.6659$

7. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	θ	28°
2	θ	30°
3	θ	32°
4	θ	34°
5	θ	36°
6	θ	38°

$\theta = 40°$
Ans. $x = 4.0477$

8. Determine the diameter x.

Solution:

$\angle EDC = (180° - \theta - 48°) \div 2.$ Why?

$\angle ECD = 48° \div 2.$ Why?

Solve for EF by the cotangent formula.

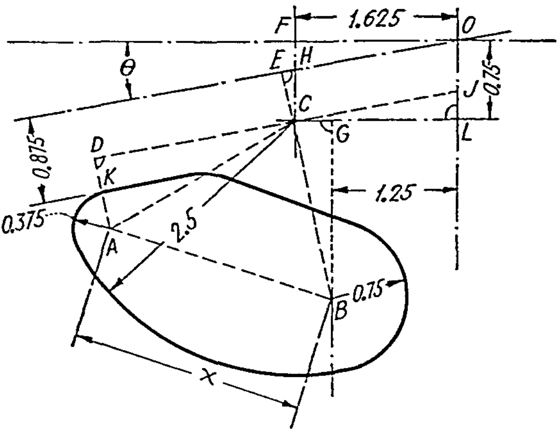
GENERAL METHOD OF PROCEDURE IN SOLVING TRIGONOMETRIC PROBLEMS

Problems 1 to 199 are all practical problems taken from tool rooms, die rooms, or drawing rooms. They are arranged in approximate order of complexity. In solving these problems there are certain methods of procedure with which the student should become thoroughly familiar. First, a drawing should be made which shows all the given dimensions and the required distance or angle. To determine an unknown distance or angle, a triangle (right or oblique) should be searched for, which contains the desired part and has sufficient other parts known to enable the student to determine the required side or angle. If no such triangle exists, auxiliary lines should be drawn to form one. In general, these auxiliary lines should consist of given lines produced or new lines drawn parallel or perpendicular to given dimensions, and usually these lines will be drawn through vertices or through centers of circles already drawn or tangent to given circles. Frequently, the auxiliary lines are simply lines connecting given vertices and centers of given circles, etc.

Often no triangle can be drawn which will have enough given parts to lead directly to a solution of the required side or angle. In that case it will be necessary to draw a second triangle which will include one of the sides or angles of the first triangle (or a line or an angle equal to a side or angle of the first tri-

angle), and which will contain enough given parts to allow a solution. A third, and even a fourth, triangle may be necessary before a triangle is finally reached which contains sufficient known parts. The method is thus to start with the side or angle in question and to continue forming related triangles until one is found which can be solved. Then work in the reverse order through these same triangles to obtain finally the required side or angle.

Illustrative Problem:



VARIABLE		
No.	Sym.	Value
1	θ	7°
2	θ	8°
3	θ	9°
4	θ	10°
5	θ	11°
6	θ	12°

FIG. 140.

Determine the distance x.

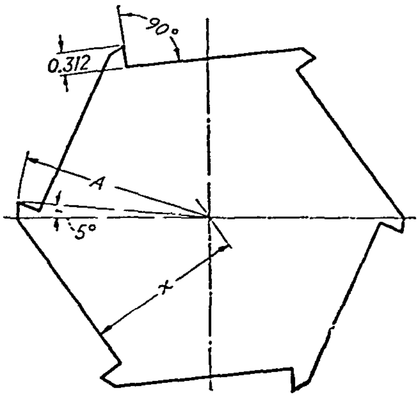
Draw the known auxiliary lines CA and CB to form a triangle containing the required distance x. In this $\triangle ABC$, if $\angle ACB$ were known, x could be determined. Thus $\angle ACB$ must be determined. Draw line $DJ \parallel EO$ through C. If $\measuredangle JCB$ and DCA were determined, $\angle ACB$ would equal 180° minus their sum. $\angle JCG = \angle\theta$. In the rt. $\triangle GCB$, $GC = 1.625 - 1.25$ and CB is known, so $\angle GCB$ may be determined. Thus $\angle JCB$ is determined. To obtain $\angle DCA$, draw $AD \perp DC$. If AD were known, $\angle DCA$ would be known. If KD were known, AD would be known. Draw $CE \parallel KD$. If CE were known, KD would be known. If CH were known, CE would be known. If FH were known, CH would be known. But FH of the right $\triangle FHO$ can be computed since $\angle\theta$ is given. Starting from this triangle, work in the reverse order to obtain the $\angle ACB$ and the distance x.

There are certain types of problems that can be best solved by special methods. Thus, in many problems involving two lines tangent to a given circle, it is often necessary to determine the location of the point of intersection of the two tangents with respect to some set of perpendicular lines as axes. (Sometimes only the distance from one axis is necessary.) The distances of this point of intersection from the two axes can then be used in determining the unknown distance or angle (see Problems 29, 33, 89, 99, 119, etc.).

Solutions will be given for many of the following problems. This will help the student to acquire the ability to analyze a given problem and to draw the proper auxiliary lines necessary *for the solution.*

Too much space would be required to describe each of the actual problems involved in the following figures, but tool makers, die makers, and draftsmen will recognize them as problems similar to those that they have been confronted with in their work, and it is hoped that the student will solve all of these problems in order to obtain the practice and experience necessary to enable him to solve other problems which he will meet with in his own work.

PRACTICAL PROBLEMS TAKEN FROM DIE ROOMS, TOOL ROOMS, AND DRAFTING ROOMS

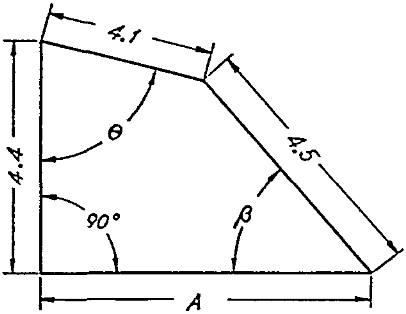


Variable		
No.	Sym.	Value
1	A	2.25
2	A	2 375
3	A	2.5
4	A	2.625
5	A	2.75
6	A	2.875

$A = 3$

Ans. $x = 2.4881$

1. Determine the distance x.

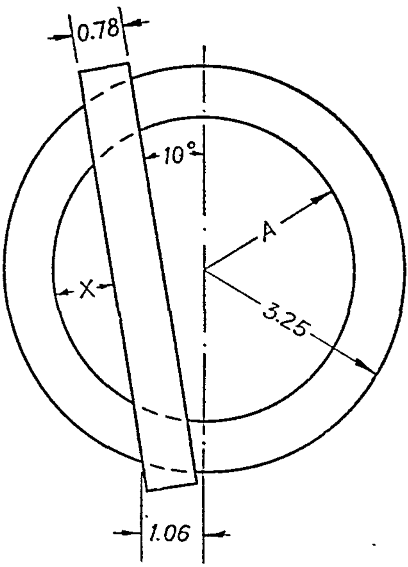


Variable		
No.	Sym.	Value
1	A	5.51
2	A	5.64
3	A	5.77
4	A	5.90
5	A	6.03
6	A	6.16

$A = 6.29$

Ans. $\begin{cases} \theta = 83° \; 12' \; 23'' \\ \beta = 60° \; 27' \; 29'' \end{cases}$

2. Determine the angle θ.
3. Determine the angle β.



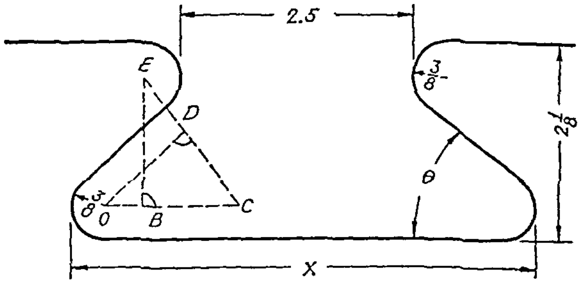
Variable		
No.	Sym.	Value
1	A	1.751
2	A	1.842
3	A	1.933
4	A	2.024
5	A	2.115
6	A	2.206

$A = 2.29$

Ans. $x = .7197$

4. Determine the distance x.

Dovetail



$\theta = 48°$
Ans. $X = 4.4576$

Variable

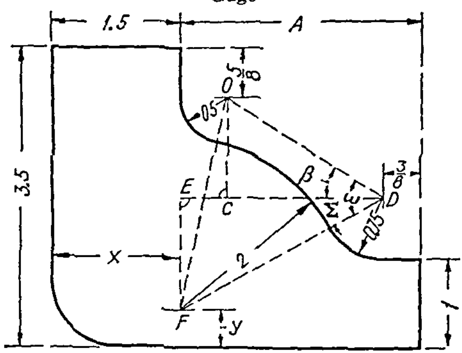
1. $\theta = 35°$	2. $\theta = 38°$	3. $\theta = 40°$
4. $\theta = 42°$	5. $\theta = 44°$	6. $\theta = 46°$

5. Determine the distance x.

Solution:

$BE = 2.125 - .375 - .375.$
$\angle BEC = \theta.$ Why?
In $\triangle EBC$, solve for BC and EC.
$CD = EC - .375 - 375.$
$\angle DOC = \theta.$ In $\triangle ODC$, solve for OC.
$OB = OC - BC.$

Gage



Variable

No.	Sym.	Value
1	A	2 5
2	A	2 625
3	A	2 75
4	A	2 875
5	A	3
6	A	3 125

$A = 3.25$
Ans. $\begin{cases} x = 1.9934 \\ y = .37502 \end{cases}$

6. Determine the distance x.
7. Determine the distance y.

Solution for preceding problem:

$OC = 3.5 - 1 - .75 - .625.$

$CD = A - .5 - .375.$

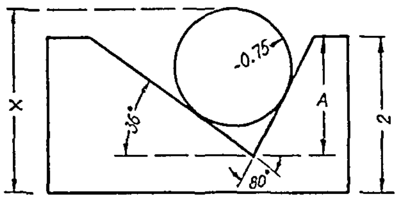
In $\triangle OCD$, solve for β and OD.

In $\triangle OFD$, solve for ω.

$\Sigma = \omega - \beta.$

In $\triangle EDF$, solve for DE and EF.

V-Block

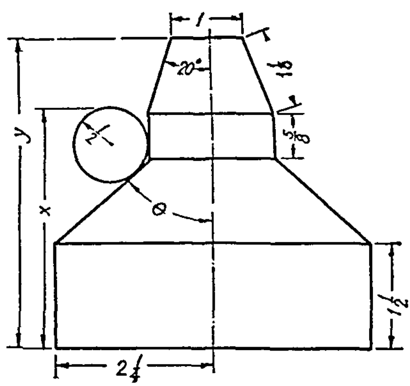


VARIABLE		
No.	Sym.	Value
1	A	.875
2	A	1.0
3	A	1.125
4	A	1.25
5	A	1.375
6	A	1.5

$A = 1.625$

Ans. $x = 2.2571$

8. Determine the distance x.

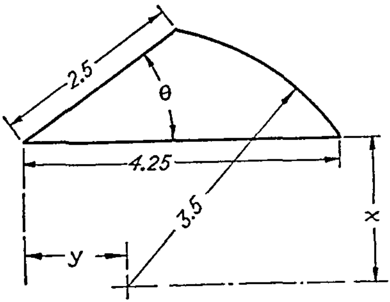


VARIABLE		
No.	Sym.	Value
1	θ	36°
2	θ	38°
3	θ	40°
4	θ	42°
5	θ	44°
6	θ	46°

$\theta = 48°$

Ans. $\begin{cases} x = 3.4518 \\ y = 4.4113 \end{cases}$

9. Determine the distance x.
10. Determine the distance y.



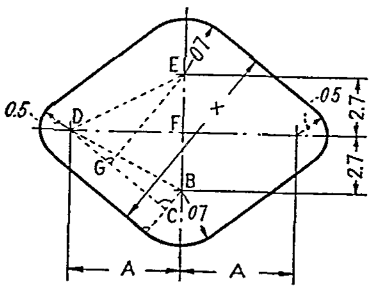
VARIABLE		
No.	Sym.	Value
1	θ	36°
2	θ	38°
3	θ	40°
4	θ	42°
5	θ	44°
6	θ	46°

$\theta = 48°$

Ans. $\begin{cases} x = 1.601 \\ y = 1.1376 \end{cases}$

11. Determine the distance x.

12. Determine the distance y.



VARIABLE		
No.	Sym.	Value
1	A	3.07
2	A	3.16
3	A	3.23
4	A	3.29
5	A	3.35
6	A	3.47

$A = 3.42$

Ans. $x = 5.4802$

13. Determine the distance x.

Solution:

In $\triangle EDF$, solve for $\angle EDF$ and DE.

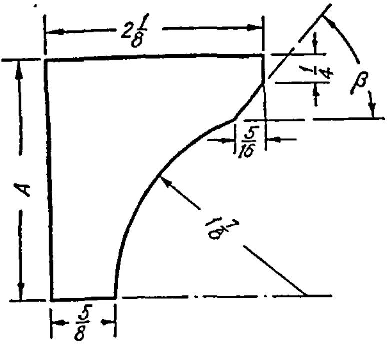
$\angle EDF = \angle FDB$. $DE = BD$. $BC = .7 - .5$.

In $\triangle BDC$, solve for $\angle BDC$.

$\angle EDC = \angle EDF + \angle FDB + \angle BDC$.

In $\triangle EDG$, solve for EG.

$x = EG + .7 + .5$.



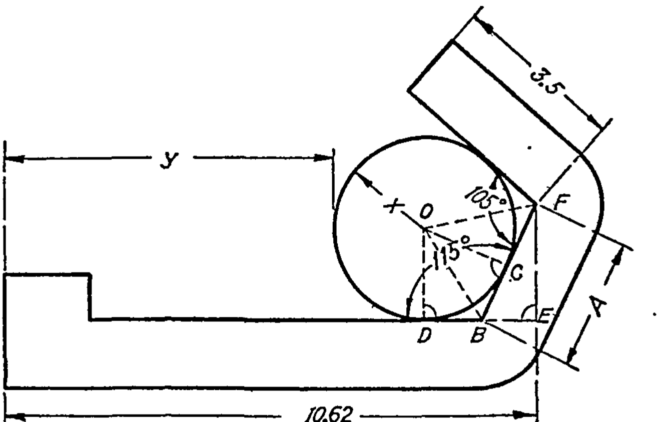
VARIABLE

No.	Sym.	Value
1	A	2.365
2	A	2.375
3	A	2.385
4	A	2.395
5	A	2.405
6	A	2.415

$A = 2.425$

Ans. $\beta = 54° 1' 48''$

14. Determine the angle β.



$A = 2.731$

Ans. $\begin{cases} x = 1.9446 \\ y = 6.2829 \end{cases}$

VARIABLE

1. $A = 2.1641$	**2.** $A = 2.3752$	**3.** $A = 2.4113$
4. $A = 2.564$	**5.** $A = 2.675$	**6.** $A = 2.726$

15. Determine the radius x.

16. Determine the distance y.

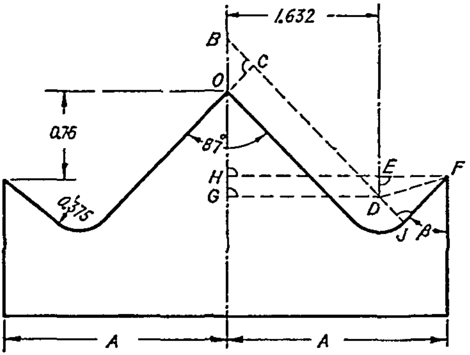
Solution:

In $\triangle OBF$, use cotangent formula, solve for OC.

$OC = OD = x$.

In $\triangle BFE$, solve for BE.

In $\triangle OBD$, solve for BD.



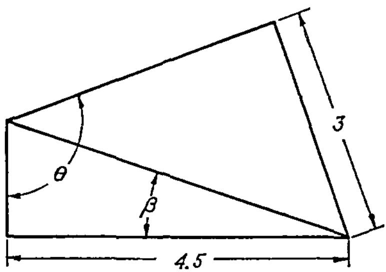
VARIABLE		
No.	Sym.	Value
1	A	2.72
2	A	2.93
3	A	3.24
4	A	3.35
5	A	3.56
6	A	3.67

$A = 2.51$
Ans. $\beta = 41°\ 33'\ 30''$

17. Determine the angle β.

Solution:

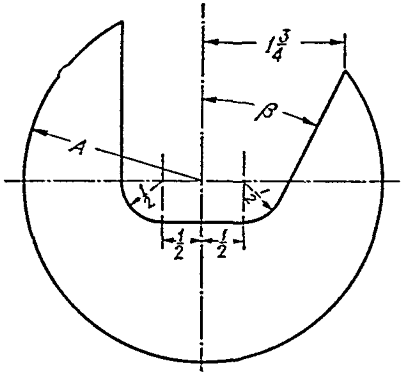
$OC = .375.$
In $\triangle OBC$, solve for OB.
In $\triangle BGD$, solve for GB.
$OH = .75.\quad GH = GB - OB - OH = ED.$
$EF = A - HE.$
In $\triangle DEF$, solve for $\angle EFD$ and DF.
$DJ = .375.$
In $\triangle DJF$, solve for $\angle DFJ$.
$\beta = 90° - \angle EFD - \angle DFJ.$



VARIABLE		
No.	Sym.	Value
1	θ	110°
2	θ	112°
3	θ	114°
4	θ	116°
5	θ	118°
6	θ	120°

$\theta = 122°$
Ans. $\beta = 9°\ 9'\ 36''$

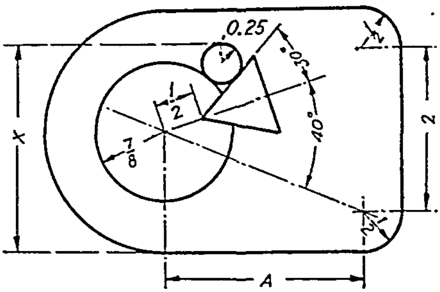
18. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	1.8125
2	A	1.9375
3	A	2.0625
4	A	2.1875
5	A	2.3125
6	A	2.4375

$A = 2.5625$
Ans. $\beta = 20°\ 53'57''$

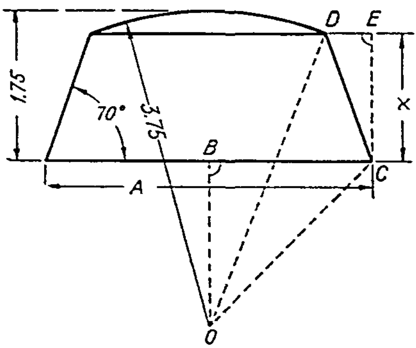
19. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	1.75
2	A	2.0
3	A	2.25
4	A	2.5
5	A	2.75
6	A	3.0

$A = 3.25$
Ans. $x = 2.6472$

20. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	3.5
2	A	3.625
3	A	3.75
4	A	3.875
5	A	4.0
6	A	4.125

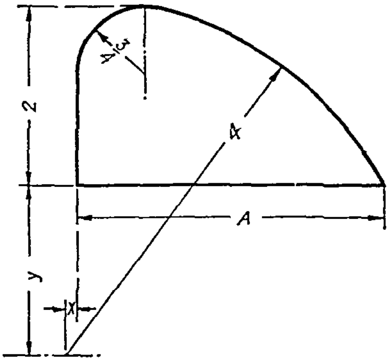
$A = 4.25$
Ans. $x = 1.3812$

21. Determine the distance x.

(Solution on next page.)

Solution for preceding problem:

$BC = A \div 2.$
$BO = 3.75 - 1.75.$
In $\triangle OBC$, solve for $\angle BCO$ and OC.
$\angle DCO = 70° + \angle BCO.$ $\quad OD = 3.75.$
In $\triangle ODC$, solve for CD.
$\angle DCE = 90° - 70°.$
In $\triangle DEC$, solve for CE. $\quad CE = x.$

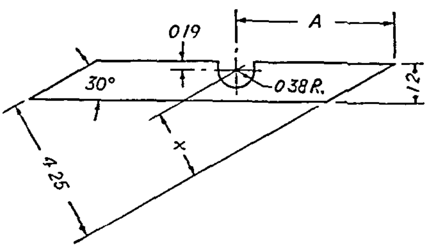


Variable		
No.	Sym.	Value
1	A	2.5
2	A	2.625
3	A	2.75
4	A	2.875
5	A	3 0
6	A	3.125

$A = 3.25$

Ans. $\begin{cases} x = .31096 \\ y = 1.8219 \end{cases}$

22. Determine the distance x.
23. Determine the distance y.

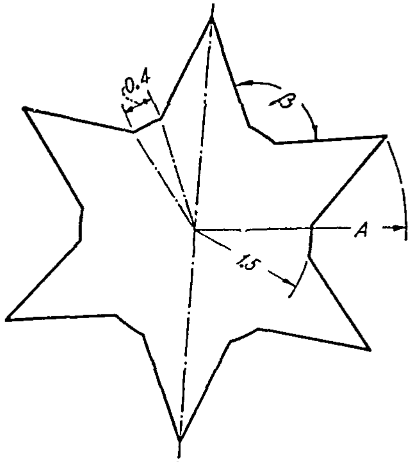


Variable		
No.	Sym.	Value
1	A	3.750
2	A	3.875
3	A	4.000
4	A	4.125
5	A	4.250
6	A	4.375

$A = 4.500$
Ans. $x = 2.0852$

24. Determine the distance x.

Nozzle Spray

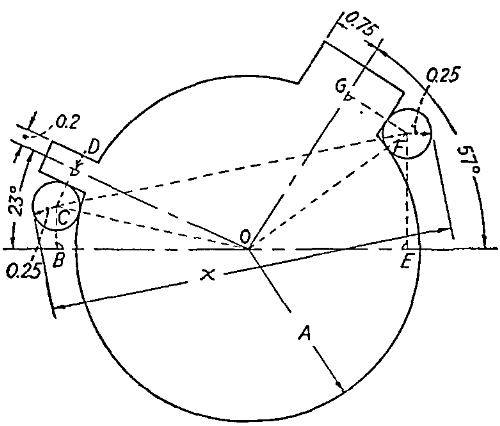


Variable		
No.	Sym.	Value
1	A	2.0
2	A	2.125
3	A	2.25
4	A	2.375
5	A	2.5
6	A	2.625

$A = 2.75$

Ans. $\beta = 105° \; 26' \; 18''$ where 0.4 equals length of arc.

25. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	1.187
2	A	1.225
3	A	1.295
4	A	1.342
5	A	1.390
6	A	1.438

$A = 1.125$

Ans. $x = 3.2245$

26. Determine the distance x.

Solution:

$CD = .45. \quad FG = 1.$

$OC = OF = A + .25.$

In $\triangle OCD$, solve for $\angle COD$.

$\angle COB = 23° - \angle COD$.

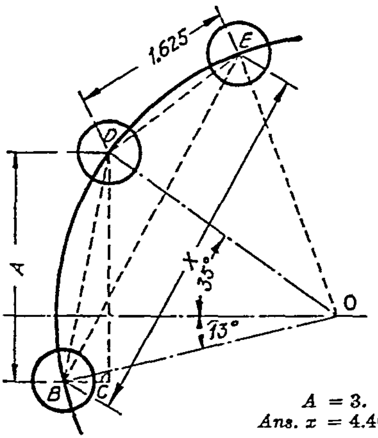
In $\triangle OFG$, solve for $\angle GOF$.

(*Continued on next page.*)

Solution continued:

$\angle FOE = 57° - \angle GOF.$
$\angle COF = 180° - \angle COB - \angle FOE.$
In $\triangle COF$, solve for CF.

Checking the Position of Holes



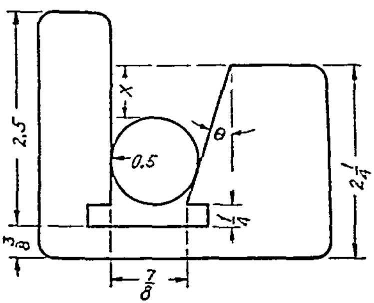
VARIABLE		
No.	Sym.	Value
1	A	2 25
2	A	2.375
3	A	2.5
4	A	2.625
5	A	2.75
6	A	2.875

$A = 3.$
Ans. $x = 4.4698$

27. Determine the distance x.
Solution:

$\angle DBO = (180° - 13° - 35°) \div 2.$
$\angle DBC = \angle DBO + 13°.$
In $\triangle BCD$, solve for BD.
In $\triangle DBO$, solve for OD.
In $\triangle DOE$, solve for $\angle DOE$.
$\angle BOE = \angle DOE + 13° + 35°.$
In $\triangle BOE$, solve for BE.

Gage

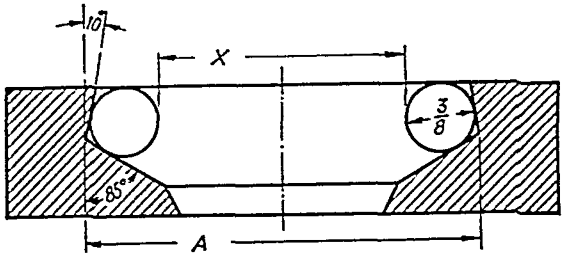


VARIABLE		
No.	Sym.	Value
1	θ	14°
2	θ	15°
3	θ	16°
4	θ	17°
5	θ	18°
6	θ	19°

$\theta = 20°$
Ans. $x = .69341$

28. Determine the distance x.

Checking Angular Rings

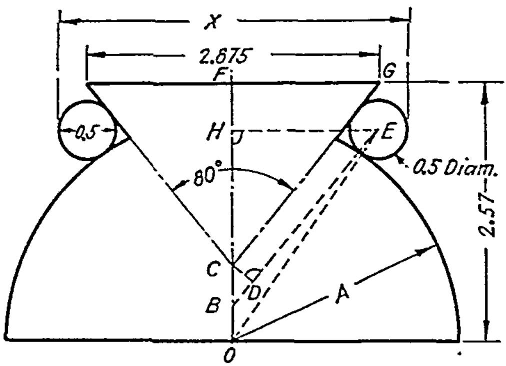


$A = 9.5$
Ans. $x = 8.6846$

VARIABLE

1. $A = 9.8$	**2.** $A = 10.2$	**3.** $A = 10.7$
4. $A = 11.2$	**5.** $A = 11.8$	**6.** $A = 12.4$

29. Determine the distance x.



VARIABLE

No.	Sym.	Value
1	A	1.81
2	A	1.87
3	A	1.95
4	A	2.12
5	A	2.41
6	A	2.62

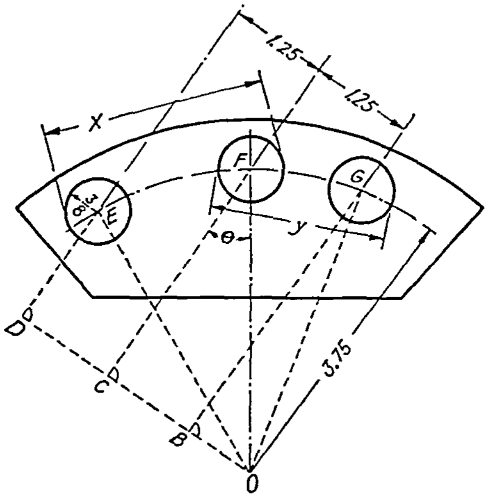
$A = 1.75$
Ans. $x = 2.581$

30. Determine the distance x.

Solution:

$FG = 2.875 \div 2$. $\angle FCG = 80° \div 2$.
In $\triangle CFG$, solve for CF. $CD = .25$.
$\angle CBD = \angle FCG$. In $\triangle CBD$, solve for CB.
$OB = 2.57 - CF - CB$. $OE = A + .25$.
In $\triangle OBE$, solve for $\angle BOE$.
In $\triangle OEH$, solve for EH.

Die Section



Variable		
No.	Sym.	Value
1	θ	22°
2	θ	24°
3	θ	26°
4	θ	28°
5	θ	30°
6	θ	32°

$\theta = 34°$

Ans. $\begin{cases} x = 2.6399 \\ y = 2.1132 \end{cases}$

31. Determine the distance x.

32. Determine the distance y.

Solution for x:

$OE = OF = OG = 3.75.$

$\angle COF = 90° - \theta.$

In $\triangle COF$, solve for CO.

$OD = CO + 1.25.$

In $\triangle DOE$, solve for $\angle DOE$.

$\angle EOF = \angle COF - \angle DOE.$

In sector EOF, solve for chord EF.

Solution for y:

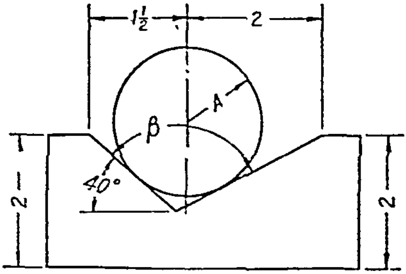
$OB = OC - 1.25.$

In $\triangle BOG$, solve for $\angle BOG$.

$\angle FOG = \angle BOG - \angle COF.$

In sector FOG, solve for chord FG.

V-Block

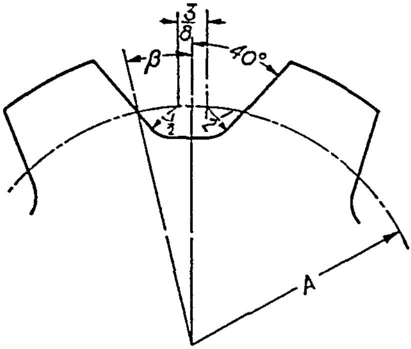


Variable		
No.	Sym.	Value
1	A	1.0625
2	A	1.125
3	A	1.1875
4	A	1.25
5	A	1.3125
6	A	1.375

$A = 1.4375$
Ans. $\beta = 113° 47' 50''$

33. Determine the angle β.

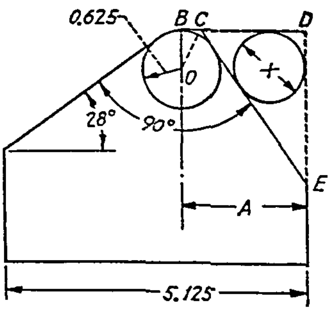
Chain Gear Teeth



Variable		
No.	Sym.	Value
1	A	2.5
2	A	2.625
3	A	2.75
4	A	2.875
5	A	3.
6	A	3.125

$A = 3.25$
Ans. $\beta = 13° 39' 30''$

34. Determine the angle β.



Variable		
No.	Sym.	Value
1	A	2.2
2	A	2.3
3	A	2.4
4	A	2.5
5	A	2.6
6	A	2.7

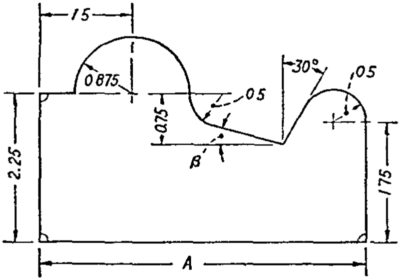
$A = 2.1$
Ans. $x = 1.2946$

35. Determine the distance x.

(*Solution on next page.*)

Solution for preceding problem:

$OB = .625$. $\angle BCO = (90° + 28°) \div 2$. Why?
In $\triangle OBC$, solve for BC. $CD = A - BC$.
$\angle CED = 28°$. Why?
In $\triangle CDE$, solve for CE and DE.
By geometry solve for x.

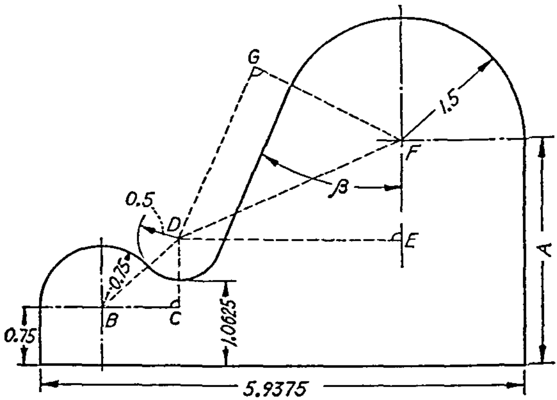


$A = 6.50$
Ans. $\beta = 4° 53' 8''$

VARIABLE

1. $A = 5.00$	**2.** $A = 5.25$	**3.** $A = 5.50$
4. $A = 5.75$	**5.** $A = 6.00$	**6.** $A = 6.25$

36. Determine the angle β.



$A = 2.9375$
Ans. $\beta = 22° 34' 17''$

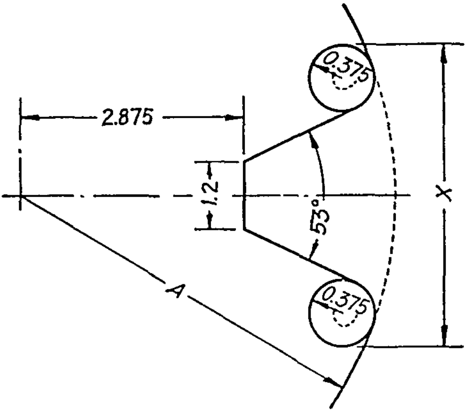
VARIABLE

1. $A = 2.25$	**2.** $A = 2.375$	**3.** $A = 2.5$
4. $A = 2.625$	**5.** $A = 2.75$	**6.** $A = 2.875$

37. Determine the angle β. *(Solution on next page.)*

Solution for preceding problem:

$CD = 1.0625 - .75 + .5$. Why? $BD = .75 + .5$.
In $\triangle BCD$, solve for BC. $DE = 5.9375 - .75 - BC - 1.5$.
$EF = A - 1.0625 - .5$. In $\triangle DEF$, solve for $\angle EDF$ and DF.
$FG = 1.5 + .5$. Why?
In $\triangle DFG$, solve for $\angle GDF$.
$\beta = 90° - \angle EDF - \angle GDF$.

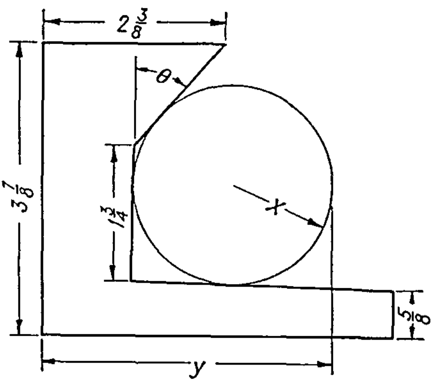


Variable		
No.	Sym.	Value
1	A	5.031
2	A	5.122
3	A	5.243
4	A	5.324
5	A	5.415
6	A	5.496

$A = 5.561$
Ans. $x = 4.7022$

38. Determine the distance x.

Gage

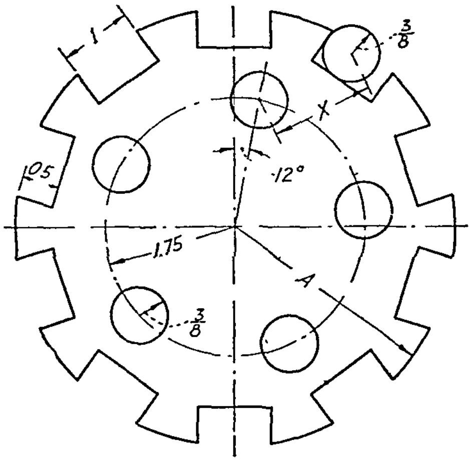


Variable		
No.	Sym.	Value
1	θ	40°
2	θ	42°
3	θ	44°
4	θ	46°
5	θ	48°
6	θ	50°

$\theta = 52°$

Ans. $\begin{cases} x = 1.1763 \\ y = 2.8077 \end{cases}$

39. Determine the radius x.
40. Determine the distance y.

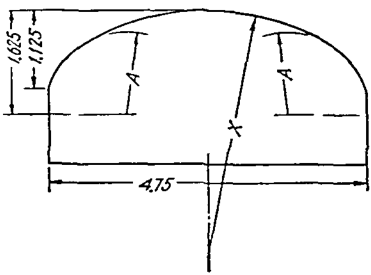


VARIABLE

No.	Sym.	Value
1	A	2.75
2	A	2.875
3	A	3.0
4	A	3 125
5	A	3.25
6	A	3.375

$A = 3.5$
Ans. $x = 1.8376$

41. Determine the distance x.

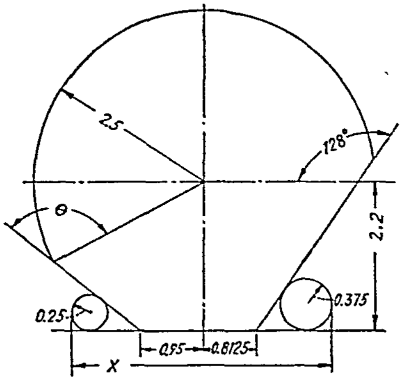


VARIABLE

No.	Sym.	Value
1	A	1.125
2	A	1.160
3	A	1.195
4	A	1.230
5	A	1.265
6	A	1.300

$A = 1.335$
Ans. $x = 3.709$

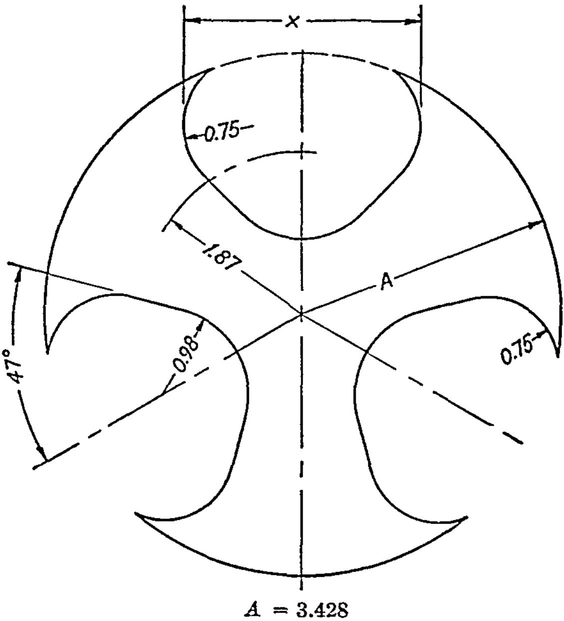
42. Determine the distance x.



Variable		
No.	Sym.	Value
1	θ	112°
2	θ	114°
3	θ	116°
4	θ	118°
5	θ	120°
6	θ	122°

$\theta = 110°$
Ans. $x = 3.9561$

43. Determine the distance x.

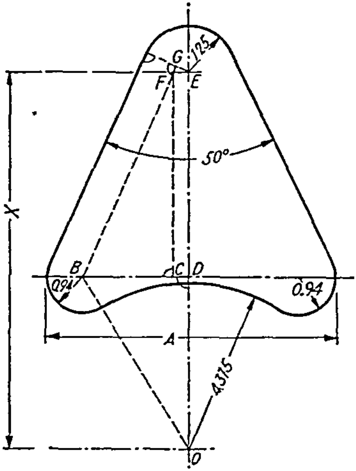


$A = 3.428$
Ans. $x = 3.4945$

Variable

1. $A = 2.750$	2. $A = 2.863$	3. $A = 2.976$
4. $A = 3.089$	5. $A = 3.202$	6. $A = 3.315$

44. Determine the distance x.



No.	Sym.	Value
1	A	5.86
2	A	6.10
3	A	6.34
4	A	6.58
5	A	6.82
6	A	7.16

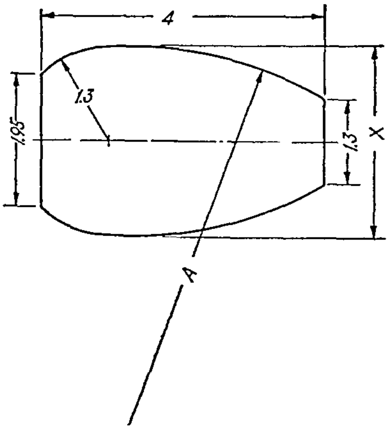
VARIABLE (columns above)

$A = 5.62$
Ans. $x = 8.2518$

45. Determine the distance x.

Solution:

$GE = 1.25 - .94$. Why? $\angle GEF = 50° \div 2$. Why?
In $\triangle FEG$, solve for FE. $BD = (A - .94 - .94) \div 2$.
$BC = BD - CD$. $CD = FE$. In $\triangle BCF$, solve for FC.
In $\triangle OBD$, solve for DO.
$x = DO + FC$.

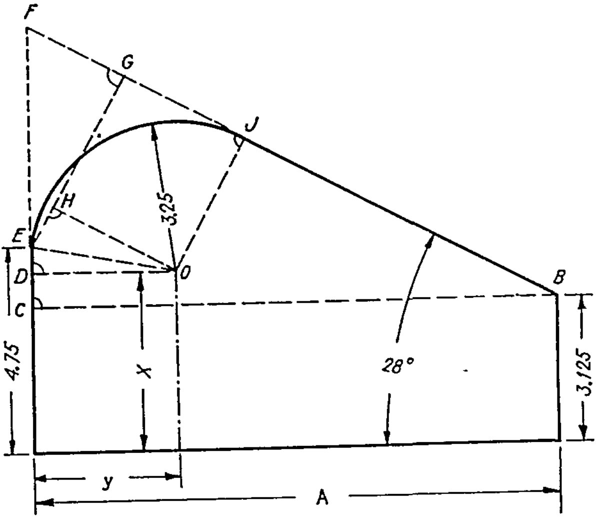


VARIABLE

No.	Sym.	Value
1	A	3 59
2	A	3 71
3	A	3 83
4	A	3 95
5	A	4.07
6	A	4.19

$A = 3.47$
Ans. $x = 2.9733$

6. Determine the distance x.



$$Ans. \begin{cases} A = 11.677 \\ x = 3.9748 \\ y = 3.1562 \end{cases}$$

VARIABLE

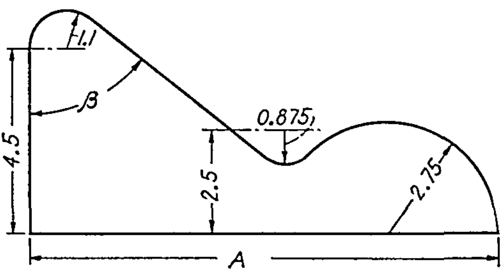
1. A = 10.351	2. A = 10.572	3. A = 10.793
4. A = 11.014	5. A = 11.235	6. A = 11.456

47. Determine the distance x.

48. Determine the distance y.

Solution:

$\angle CBF = \angle FEG = 28°$. $OJ = 3.25$.
In $\triangle BCF$, solve for CF. $EF = CF + 3.125 - 4.75$.
In $\triangle EFG$, solve for EG. $EH = EG - OJ$.
$EO = 3.25$. In $\triangle EOH$, solve for $\angle EOH$.
$\angle DOH = \angle FEG = 28°$. Why? $\angle EOD = 28° - \angle EOH$.
In $\triangle EOD$, solve for ED and DO.

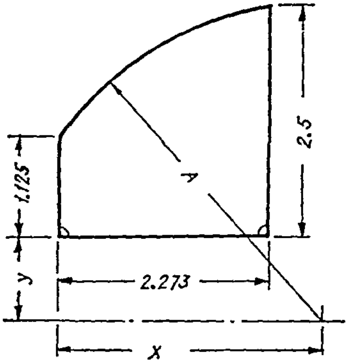


VARIABLE

No.	Sym.	Value
1	A	13.25
2	A	12.96
3	A	12.67
4	A	12.38
5	A	12.09
6	A	11.80

$A = 11.51$
Ans. $\beta = 46° 57' 24''$

49. Determine the angle β.



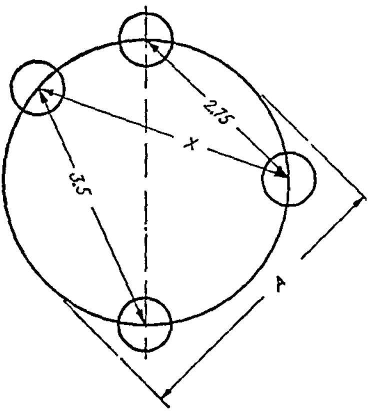
Variable		
No.	Sym.	Value
1	A	3.75
2	A	3.85
3	A	4.08
4	A	4.25
5	A	4.42
6	A	4.59

$A = 3.52$

Ans. $\begin{cases} x = 2.8237 \\ y = .97661 \end{cases}$

50. Determine the distance x.

51. Determine the distance y.

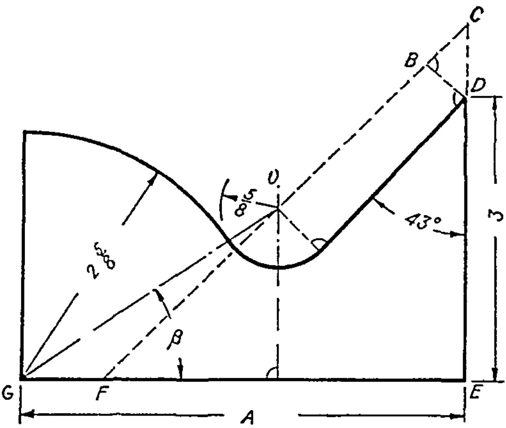


Variable		
No.	Sym.	Value
1	A	3 75
2	A	3 875
3	A	4.00
4	A	4.125
5	A	4.25
6	A	4 375

$A = 4.5$

Ans. $x = 4.3775$

52. Determine the distance x.



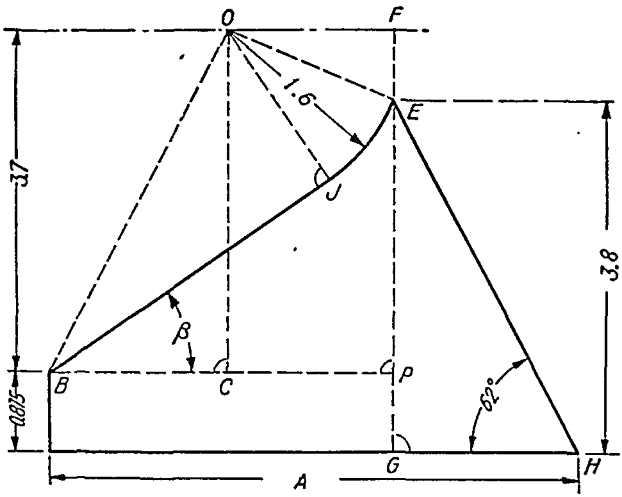
Variable		
No.	Sym.	Value
1	A	4.75
2	A	4.875
3	A	5.00
4	A	5.125
5	A	5.25
6	A	5.375

$A = 5.5$
Ans. $\beta = 22° 23' 48''$

53. Determine the angle β.

Solution:

$\angle BCD = 43°$. In $\triangle BCD$, solve for CD.
In $\triangle CEF$, solve for EF.
$GF = A - EF$. $GO = 2.625 + .625$.
$\angle CFE = 90° - 43°$.
In $\triangle GOF$, solve for β.



$A = 6.57$
Ans. $\beta = 30° 22' 7''$

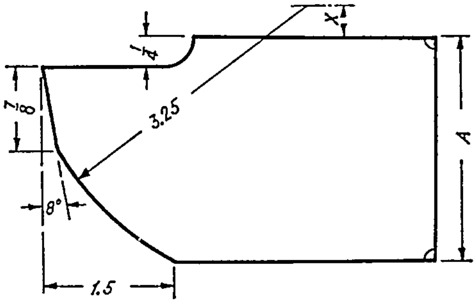
Variable

1. $A = 5.31$ **2.** $A = 5.52$ **3.** $A = 5.73$
4. $A = 5.94$ **5.** $A = 6.15$ **6.** $A = 6.36$

54. Determine the angle β. (*Solution on next page.*)

Solution for preceding problem:

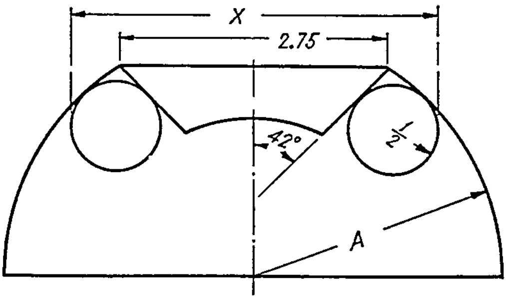
In $\triangle GHE$, solve for GH. $BP = A - GH$. $OE = OJ = 6.1$
$PE = 3.8 - .875$. $EF = .875 + 3.7 - 3.8$
In $\triangle EOF$, solve for OF. $BC = BP - OF$.
$OC = 3.7$. In $\triangle OBC$, solve for $\angle OBC$ and OB.
In $\triangle OBJ$, solve for $\angle OBJ$
$\beta = \angle OBC - \angle OBJ$.



Variable		
No.	Sym.	Value
1	A	2.0
2	A	2.125
3	A	2.25
4	A	2.375
5	A	2.5
6	A	2.625

$A = 2.75$
Ans. $x = .04760$

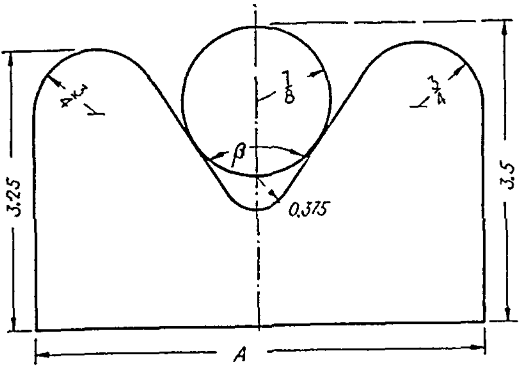
55. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	2 75
2	A	2.875
3	A	3.0
4	A	3.125
5	A	3.25
6	A	2.375

$A = 3.5$
Ans. $x = 3.9856$

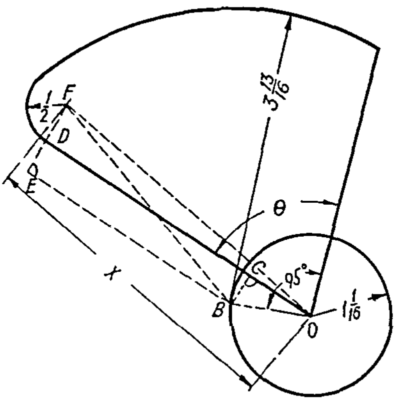
56. Determine the distance x.



Variable		
No.	Sym.	Value
1	A	4.881
2	A	4.912
3	A	4.963
4	A	4.992
5	A	5.035
6	A	5.116

$A = 5.146$
Ans. $\beta = 62° \; 16' \; 2''$

57. Determine the angle β.



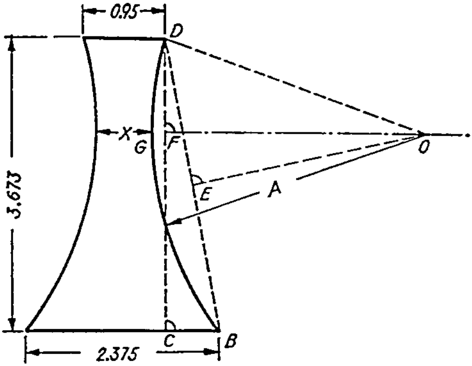
Variable		
No.	Sym.	Value
1	θ	66°
2	θ	68°
3	θ	70°
4	θ	72°
5	θ	74°
6	θ	76°

$\theta = 78°$
Ans. $x = 4.2574$

58. Determine the distance x.

Solution:

$\angle BOC = 95° - \theta$. $BC = ED$. In $\triangle BOC$, solve for BC and OC.
$EF = ED + .5$. $BF = 3.8125 - .5$.
In $\triangle EBF$, solve for $\angle EBF$ and EB. $EB = DC$.
$OD = DC + OC$. $DF = .5$.
In $\triangle FDO$, solve for $\angle DOF$ and OF.



Variable		
No.	Sym.	Value
1	A	4.22
2	A	4.43
3	A	4.64
4	A	4.75
5	A	4.96
6	A	5.17

$A = 4.11$
Ans. $x = .6278$

59. Determine the distance x.

Solution:

$BC = 2.375 \div 2 - .95 \div 2.$

In $\triangle BCD$, solve for $\angle BDC$ and BD.

$DE = BD \div 2. \quad DO = A.$

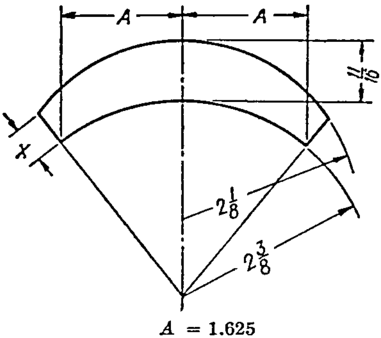
In $\triangle DEO$, solve for $\angle DOE$.

$\angle EOF = \angle BDC.$ Why?

$\angle FOD = \angle DOE - \angle EOF.$

In $\triangle DOF$, solve for OF.

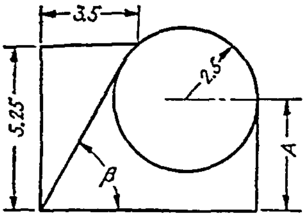
$x = .95 - 2(A - OF).$ Why?



Variable		
No.	Sym.	Value
1	A	.875
2	A	1.000
3	A	1.125
4	A	1.25
5	A	1.375
6	A	1.5

$A = 1.625$
Ans. $x = .33463$

60. Determine the distance x.



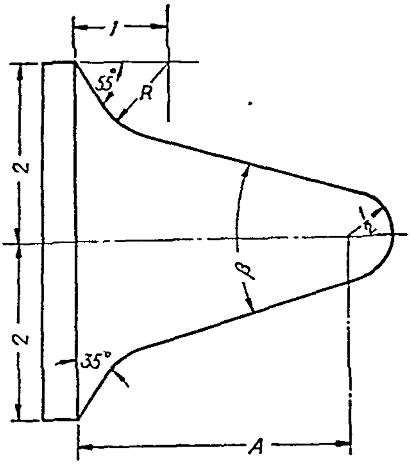
Variable		
No.	Sym.	Value
1	A	2.9
2	A	3.0
3	A	3.1
4	A	3.2
5	A	3.3
6	A	3.4

$A = 2.8$

Ans. $\beta = 65° 49' 33''$

61. Determine the angle β.

Die Punch

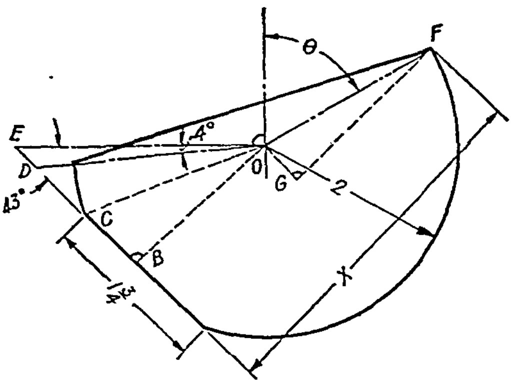


Variable		
No.	Sym.	Value
1	A	3.
2	A	3.125
3	A	3.25
4	A	3.375
5	A	3.5
6	A	3.625

$A = 3.75$

Ans. $\beta = 26° 24' 10''$

62. Determine the angle β.



Variable		
No.	Sym.	Value
1	θ	54°
2	θ	56°
3	θ	58°
4	θ	60°
5	θ	62°
6	θ	64°

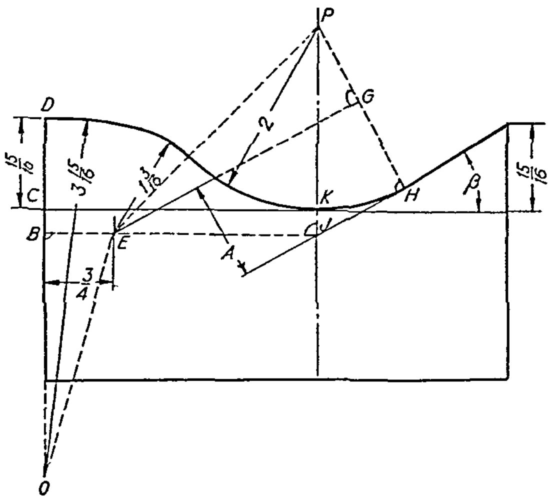
$\theta = 66°$

Ans. $x = 3.6394$

63. Determine the distance x. (*Solution on next page.*)

Solution for preceding problem:

$BC = 1.75 \div 2$. Why? $OC = OF = 2$.
In $\triangle BCO$, solve for OB.
$\angle DOB = 90° - 43° - 4°$. Why?
$\angle FOG = 270° - \angle DOB - 4° - 90° - \theta$. Why?
In $\triangle OFG$, solve for FG.
$x = OB + FG$.



$A = 1.5$
Ans. $\beta = 38° 35' 14''$

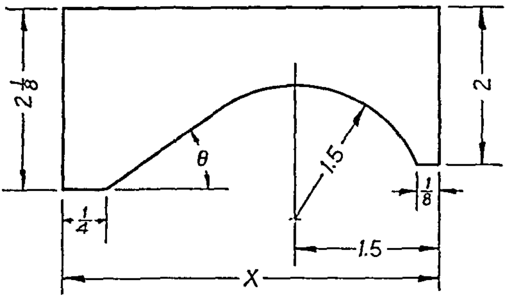
VARIABLE

1. $A = .75$	2. $A = .875$	3. $A = 1.00$
4. $A = 1.125$	5. $A = 1.25$	6. $A = 1.375$

64. Determine the angle β.

Solution:

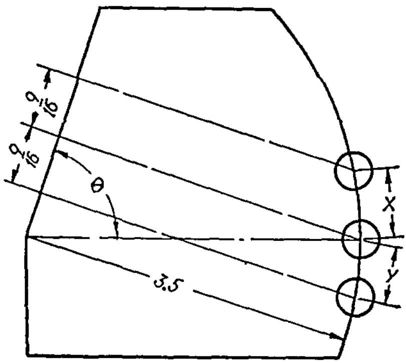
$OE = 3.9375 - 1.1875$. In $\triangle BOE$, solve for OB.
$BC = JK = 3.9375 - .9375 - OB$.
$EP = 1.1875 + 2$. $PG = PH - A$.
In $\triangle EPG$, solve for $\angle EPG$.
In $\triangle EPJ$, solve for $\angle EPJ$.
$\angle KPH = \angle EPG - \angle EPJ$.
$\beta = \angle KPH$. Why?



Variable		
No.	Sym.	Value
1	θ	30°
2	θ	32°
3	θ	34°
4	θ	36°
5	θ	38°
6	θ	40°

$\theta = 42°$
Ans. $x = 3.4648$

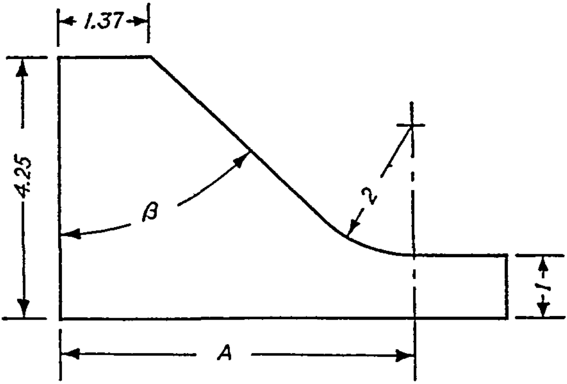
65. Determine the distance x.



Variable		
No.	Sym.	Value
1	θ	80°
2	θ	77°
3	θ	74°
4	θ	71°
5	θ	68°
6	θ	65°

$\theta = 62°$
Ans. $\begin{cases} x = .67473 \\ y = .61096 \end{cases}$

66. Determine the distance x.
67. Determine the distance y.

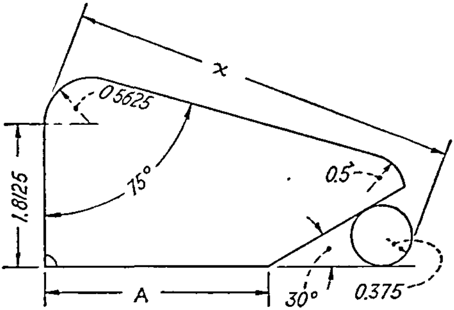


$A = 6.250$
Ans. $\beta = 52° 14' 28''$

VARIABLE

1. $A = 5.500$	2. $A = 5.625$	3. $A = 5.750$
4. $A = 5.875$	5. $A = 6.000$	6. $A = 6.125$

68. Determine the angle β.



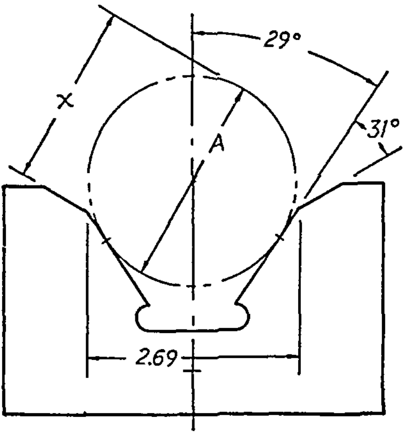
$A = 3.5625$
Ans. $x = 5.5657$

VARIABLE

1. $A = 2.8125$	2. $A = 2.9375$	3. $A = 3.0625$
4. $A = 3.1875$	5. $A = 3.3125$	6. $A = 3.4375$

69. Determine the distance x.

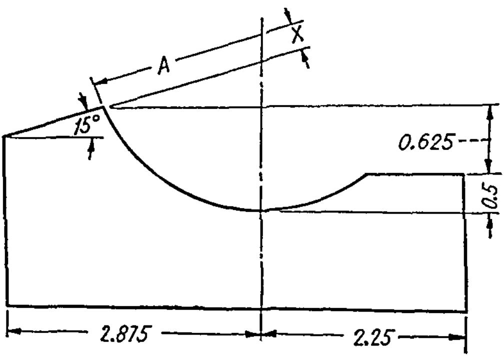
Gage



No.	Sym.	Value
1	A	2.375
2	A	2.5
3	A	2.625
4	A	2.75
5	A	2.875
6	A	3.0

$A = 3.125$
Ans. $x = 2.9248$

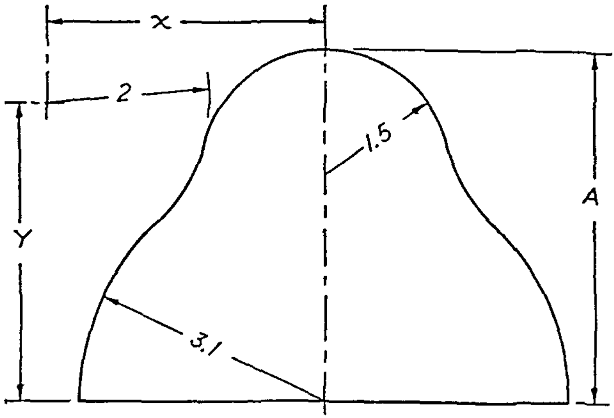
70. Determine the distance x.



No.	Sym.	Value
1	A	2.2
2	A	2.4
3	A	2.6
4	A	2.8
5	A	3.0
6	A	3.2

$A = 2.$
Ans. $x = .37970$

71. Determine the distance x.



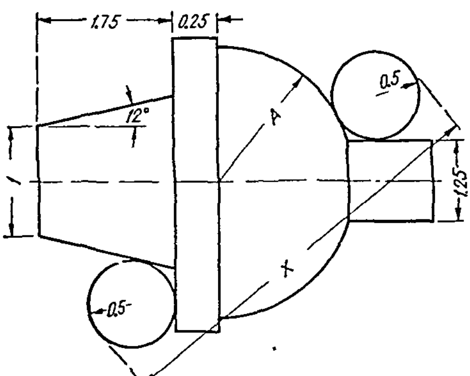
$A = 4\ 250$

Ans. $\begin{cases} x = 3.3136 \\ y = 3.8768 \end{cases}$

VARIABLE

1. $A = 3.500$	2. $A = 3.625$	3. $A = 3.750$
4. $A = 3.875$	5. $A = 4.000$	6. $A = 4.125$

72. Determine the distance x.
73. Determine the distance y.

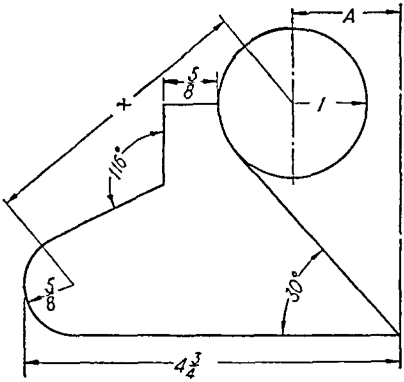


VARIABLE

No.	Sym.	Value
1	A	1 5
2	A	1 625
3	A	1 75
4	A	1 875
5	A	2.0
6	A	2.125

$A = 2.25$
Ans. $x = 5\ 0487$

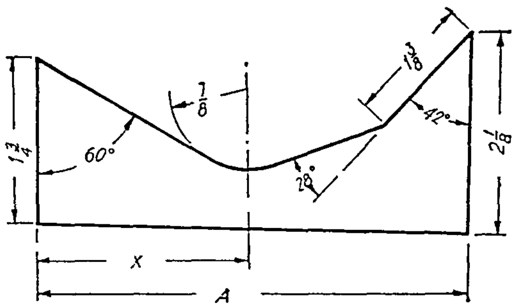
74. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	1 375
2	A	1 5
3	A	1 625
4	A	1.75
5	A	1.875
6	A	2.0

$A = 2.125$
Ans. $x = 2.6618$

75. Determine the distance x.

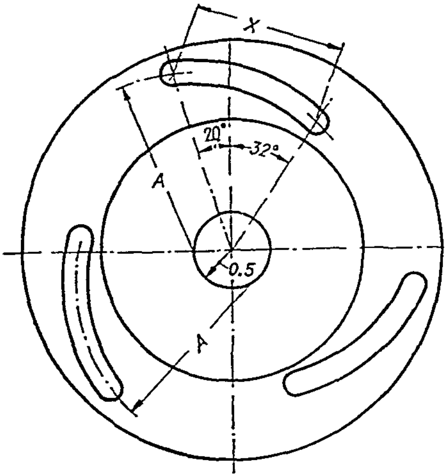


VARIABLE		
No.	Sym.	Value
1	A	4.25
2	A	4.375
3	A	4.5
4	A	4.625
5	A	4.75
6	A	4.875

$A = 5$
Ans. $x = 2.3488$

76. Determine the distance x.

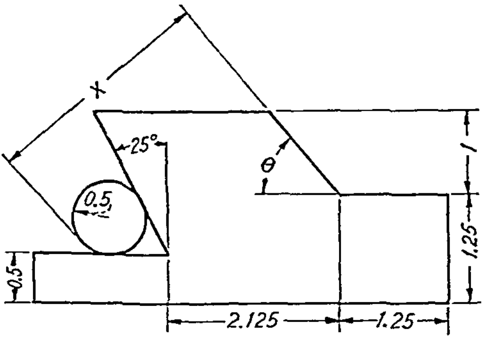
Lock Nut



Variable		
No.	Sym.	Value
1	A	2.25
2	A	2 285
3	A	2 32
4	A	2.355
5	A	2.39
6	A	2.425

$A = 2.46$
Ans. $x = 2.1148$

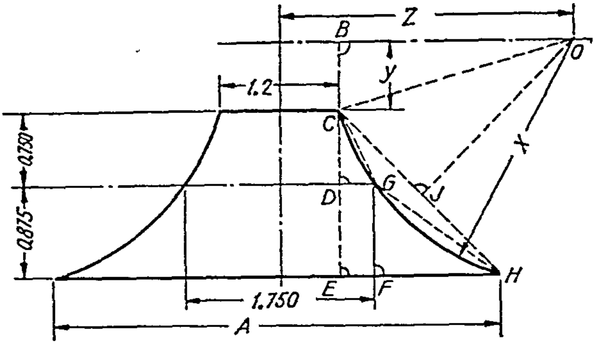
77. Determine the distance x.



Variable		
No.	Sym.	Value
1	θ	46°
2	θ	48°
3	θ	50°
4	θ	52°
5	θ	54°
6	θ	56°

$\theta = 58°$
Ans. $x = 3.1001$

78. Determine the distance x.



VARIABLE

No.	Sym.	Value
1	A	4.32
2	A	4.53
3	A	4.74
4	A	4.85
5	A	4.96
6	A	5.07

$$A = 4.11$$

$$Ans. \begin{cases} x = 1.9862 \\ y = .2948 \\ z = 2.563 \end{cases}$$

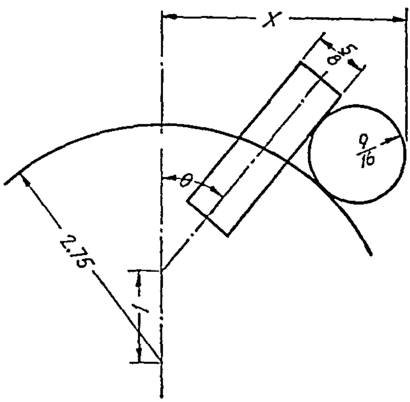
79. Determine the distance x.

80. Determine the distance y.

81. Determine the distance z.

Solution:

$DG = 1.75 \div 2 - 1.2 \div 2$. In $\triangle DCG$, solve for $\angle DCG$ and CG.
In $\triangle FGH$, solve for GH. In $\triangle CEH$, solve for CH.
In $\triangle CGH$, solve for $\angle GCH$ and x.
$CJ = CH \div 2$. $CO = x$.
In $\triangle CJO$, solve for $\angle JCO$.
$\angle BCO = 180° - \angle DCG - \angle GCH - \angle JCO$.
In $\triangle BCO$, solve for BC and BO.



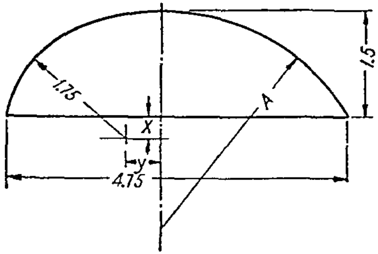
VARIABLE

No.	Sym.	Value
1	θ	43°
2	θ	44°
3	θ	45°
4	θ	46°
5	θ	47°
6	θ	48°

$$\theta = 42°$$

$$Ans.\ x = 2.9275$$

82. Determine the distance x.



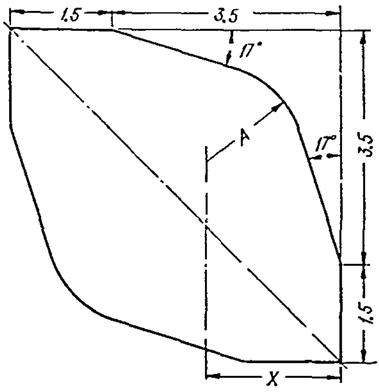
Variable		
No.	Sym.	Value
1	A	2.75
2	A	2.875
3	A	3.0
4	A	3.125
5	A	3.25
6	A	3.375

$$A = 3.5$$

$$Ans. \begin{cases} x = .25597 \\ y = .14670 \end{cases}$$

83. Determine the distance x.

84. Determine the distance y.

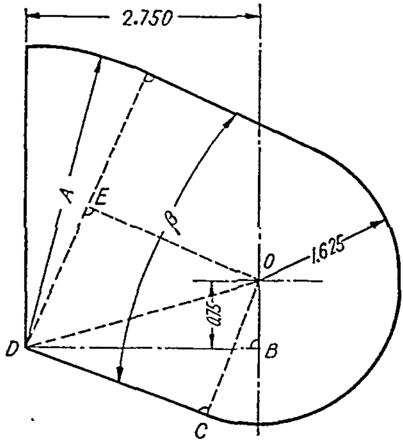


Variable		
No.	Sym.	Value
1	A	1.252
2	A	1.259
3	A	1.325
4	A	1.386
5	A	1.442
6	A	1.453

$$A = 1.516$$

$$Ans.\ x = 2.0336$$

85. Determine the distance x.



VARIABLE

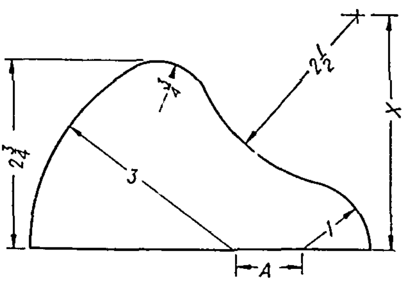
No.	Sym.	Value
1	A	3.42
2	A	3.53
3	A	3.64
4	A	3.75
5	A	3.86
6	A	3.97

$A = 3.31$
Ans. $\beta = 1° 28' 54''$

86. Determine the angle β.

Solution:

In $\triangle DBO$, solve for DO.
$DE = A - 1.625$. Why?
In $\triangle DOE$, solve for $\angle DOE$. $CO = 1.625$.
In $\triangle DOC$, solve for $\angle DOC$.
$\beta = \angle DOE + \angle DOC - 90°$. Why?

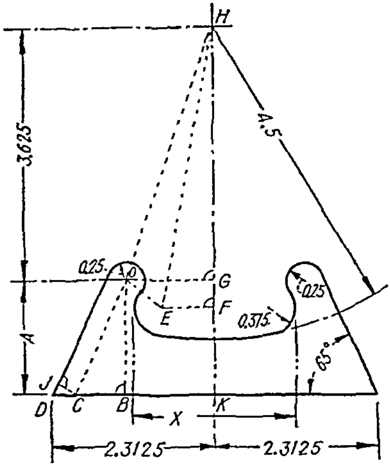


VARIABLE

No.	Sym.	Value
1	A	.5
2	A	.625
3	A	.75
4	A	.875
5	A	1.00
6	A	1.125

$A = 1.25$
Ans. $x = 3.4424$

87. Determine the distance x.



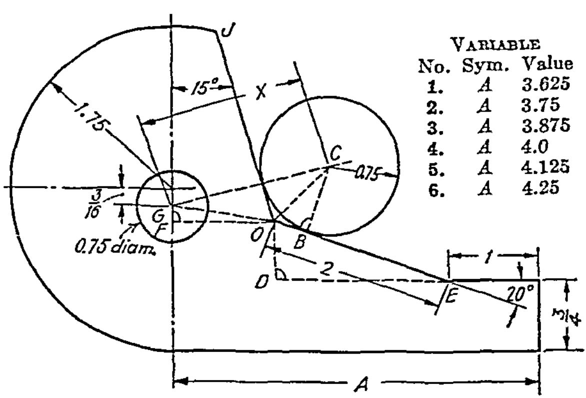
Variable No.	Sym.	Value
1	A	1.5625
2	A	1.715
3	A	1.829
4	A	2.106
5	A	2.207
6	A	2.312

$A = 1.5$
Ans. $x = 2.4778$

88. Determine the distance x.

Solution:

$\angle JDC = \angle OCB = 65°$. $CJ = .25$. $BO = A - .25$.
In $\triangle JDC$, solve for DC. In $\triangle OCB$, solve for CB.
$DK - DC - CB = BK = OG$. $HG = 3.625$.
In $\triangle HOG$, solve for $\angle OHG$ and OH. $OE = .25 + .375$.
$EH = 4.5 - .375$. In $\triangle OHE$, solve for $\angle OHE$.
$\angle EHF = \angle OHG - \angle OHE$. In $\triangle HEF$, solve for EF.
$x = 2(EF + .375)$.



Variable No.	Sym.	Value
1.	A	3.625
2.	A	3.75
3.	A	3.875
4.	A	4.0
5.	A	4.125
6.	A	4.25

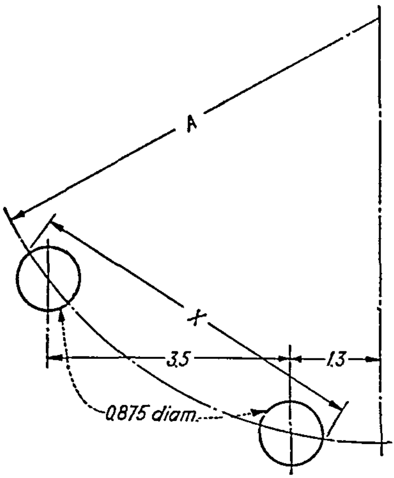
$A = 4.375$
Ans. $x = 2.1687$

89. Determine the distance x. (*Solution on next page.*)

Solution for preceding problem:

$\angle OED = 20°$. In $\triangle OED$, solve for DE and DO.
$FO = A - DE - 1$. $FG = 1.75 - .1875 - DO - .75$.
In $\triangle GFO$, solve for $\angle GOF$ and GO.
$\angle GOJ = 90° - 15° - \angle GOF$.
$\angle JOE = 90° + 15° + 20°$. Why?
$\angle JOC = \angle COB = \angle JOE \div 2$. In $\triangle COB$, solve for CO.
$\angle GOC = \angle GOJ + \angle JOC$.
In $\triangle GOC$, solve for CG.

Checking Position of Holes

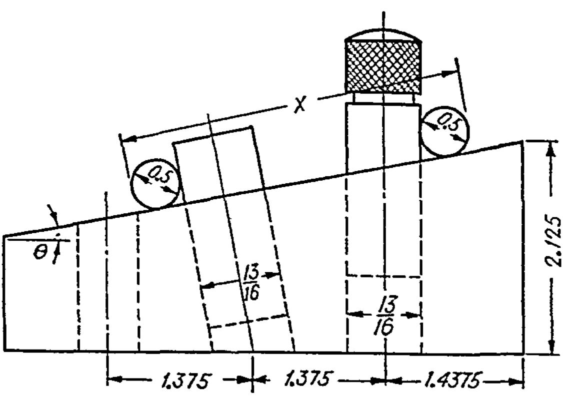


VARIABLE		
No.	Sym.	Value
1	A	7.2
2	A	8.3
3	A	9.4
4	A	10.5
5	A	11.6
6	A	11.9

$A = 6.1$
Ans. $x = 5.0066$

90. Determine the distance x.

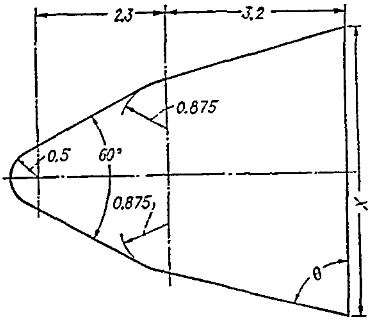
Checking Angular Holes



VARIABLE		
No.	Sym.	Value
1.	θ	8°
2.	θ	10°
3.	θ	12°
4.	θ	14°
5.	θ	16°
6.	θ	18°

$\theta = 20°$
Ans. $x = 3.7854$

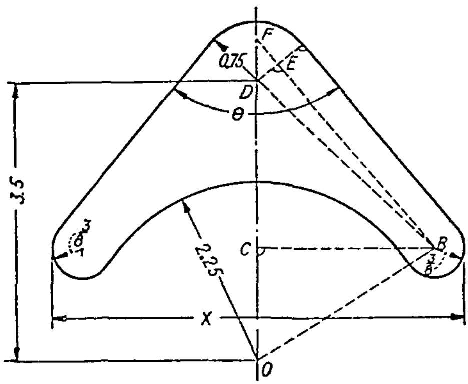
91. Determine the distance x.



Variable		
No.	Sym.	Value
1	θ	73°
2	θ	69°
3	θ	71°
4	θ	70°
5	θ	74°
6	θ	75°

$\theta = 72°$
Ans. $x = 5.7093$

92. Determine the distance x.



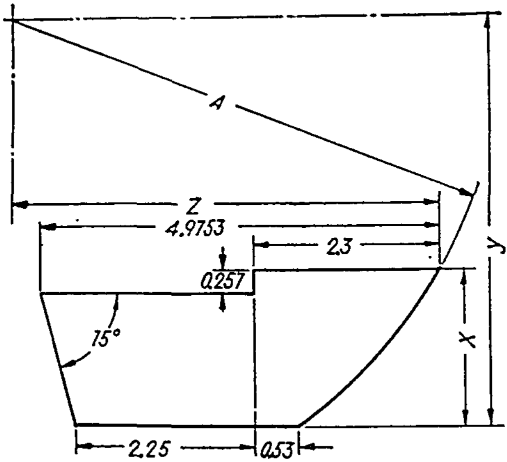
Variable		
No.	Sym.	Value
1	θ	68°
2	θ	70°
3	θ	72°
4	θ	74°
5	θ	76°
6	θ	78°

$\theta = 80°$
Ans. $x = 4.8189$ or 4.7237

93. Determine the distance x.

Solution:

$\angle DFE = \theta \div 2$. $DE = .75 - .375$.
In $\triangle DFE$, solve for DF. $FO = DO + DF$.
$BO = 2.25 + .375$. In $\triangle FOB$, solve for $\angle FOB$.
In $\triangle OCB$, solve for BC.

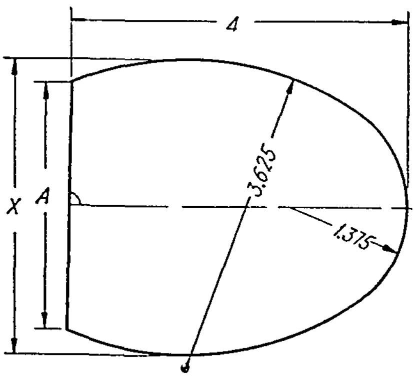


Variable		
No.	Sym.	Value
1	A	6.18
2	A	6.25
3	A	6.37
4	A	6.45
5	A	6.56
6	A	6.67

$$A = 6.12$$

$$Ans. \begin{cases} x = 1.8442 \\ y = 5.0665 \\ z = 5.2029 \end{cases}$$

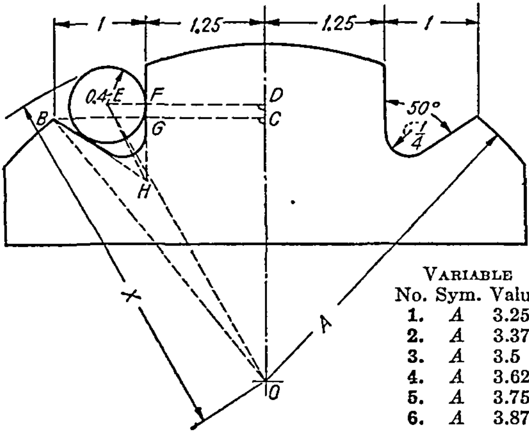
94. Determine the distance x.
95. Determine the distance y.
96. Determine the distance z.



Variable		
No.	Sym.	Value
1	A	3.0
2	A	2.25
3	A	2.375
4	A	2.5
5	A	2.625
6	A	2.75

$$A = 2.875$$
$$Ans.\ x = 3.4526$$

97. Determine the distance x.



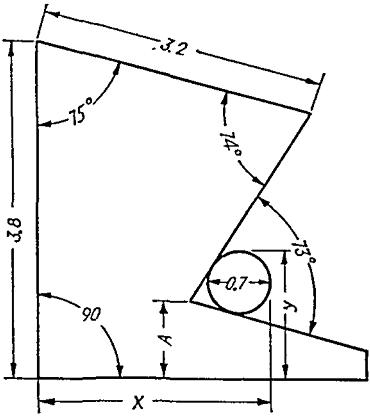
VARIABLE		
No.	Sym.	Value
1.	A	3.25
2.	A	3.375
3.	A	3.5
4.	A	3.625
5.	A	3.75
6.	A	3.875

$A = 4$
Ans. $x = 4.1126$

98. Determine the distance x.

Solution:

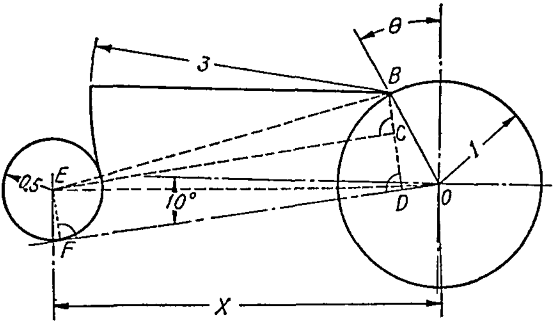
$BC = 1 + 1.25$, $BO = A$. In $\triangle BOC$, solve for CO.
$\angle BHG = 50°$. In $\triangle BHG$, solve for GH.
$\angle EHF = 50° \div 2$. In $\triangle EHF$, solve for FH.
$FG = FH - GH = CD$. $DO = CO + CD$.
$DE = 1.25 + .4$.
In $\triangle EDO$, solve for EO.



VARIABLE		
No.	Sym.	Value
1	A	1 2
2	A	.93
3	A	.64
4	A	.85
5	A	.76
6	A	1.09

$A = .71$
Ans. $\begin{cases} x = 2.6255 \\ y = 1.2851 \end{cases}$

99. Determine the distance x.
100. Determine the distance y.

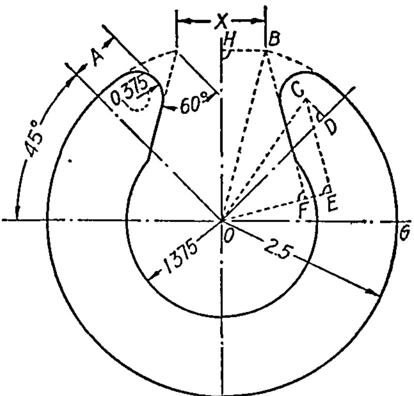


Variable		
No.	Sym.	Value
1	θ	16°
2	θ	18°
3	θ	20°
4	θ	22°
5	θ	24°
6	θ	26°

$\theta = 28°$
Ans. $x = 3.8128$

101. Determine the distance x.
Solution:

$\angle DBO = \theta - 10°$. Why?
In $\triangle DOB$, solve for BD. $BC = BD - CD$.
$EF = .5 = CD$. $BE = 3 + .5$.
In $\triangle BEC$, solve for $\angle EBC$.
$\angle EBO = \angle EBC + \angle DBO$.
In $\triangle EBO$, solve for EO.

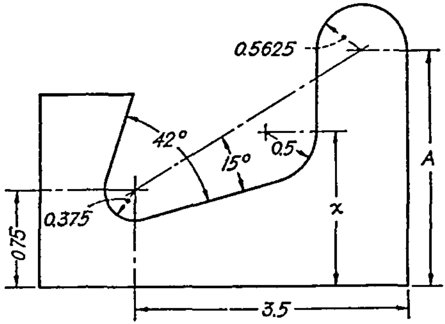


Variable		
No.	Sym.	Value
1	A	.625
2	A	.6875
3	A	.75
4	A	.8125
5	A	.875
6	A	.9375

$A = 1.0000$
Ans. $X = .90055$

102. Determine the distance x.
Solution:

In $\triangle COD$, solve for $\angle COD$. $\angle DOE = 90° - 60°$. Why?
In $\triangle COE$, solve for OE. $OF = OE - .375$. Why?
In $\triangle BOF$, solve for $\angle BOF$. $\angle EOG = 15°$. Why?
$\angle HOB = 90° - \angle BOF - \angle EOG$.
In $\triangle HOB$, solve for BH.

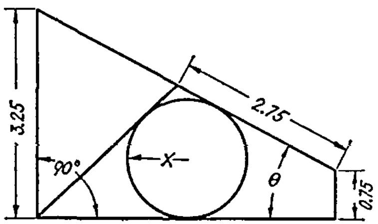


VARIABLE

No.	Sym.	Value
1	A	2 5
2	A	2 625
3	A	2 75
4	A	2 875
5	A	3.0
6	A	3.125

$A = 3.25$
Ans. $x = 1.7786$

103. Determine the distance x.

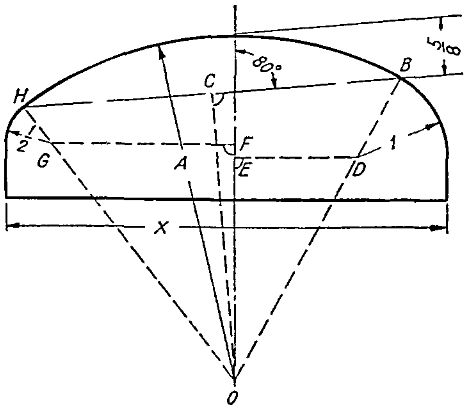


VARIABLE

No.	Sym.	Value
1	θ	25°
2	θ	26°
3	θ	27°
4	θ	28°
5	θ	29°
6	θ	30°

$\theta = 24°$
Ans. $x = .87910$

104. Determine the radius x.



VARIABLE

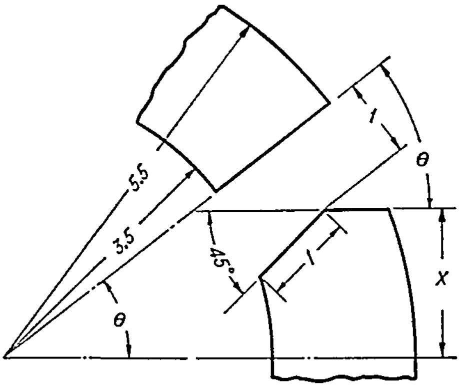
No.	Sym.	Value
1	A	3 5
2	A	3 625
3	A	3 75
4	A	3 875
5	A	4 000
6	A	1 125

$A = 1.25$
Ans. $x = 2.3962$

105. Determine the distance x. (*Solution on next page.*)

Solution for preceding problem:

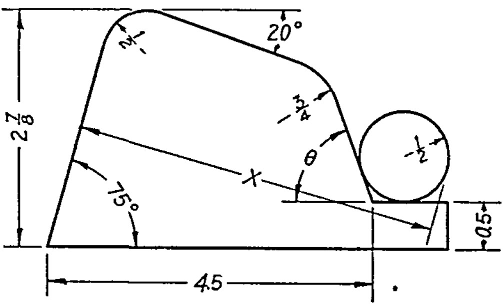
$CO = A - .625$. In $\triangle COB$, solve for $\angle COB$.
$\angle COF = 10°$. Why? $\angle FOB = \angle COB - 10°$.
$DO = A - 1$. In $\triangle EOD$, solve for DE.
$\angle HOC = \angle COB$. $\angle GOF = \angle HOC + 10°$.
In $\triangle GOF$, solve for FG.



Variable No.	Sym.	Value
1	θ	36°
2	θ	37°
3	θ	38°
4	θ	39°
5	θ	40°
6	θ	41°

$\theta = 42°$
Ans. $x = 2.1586$

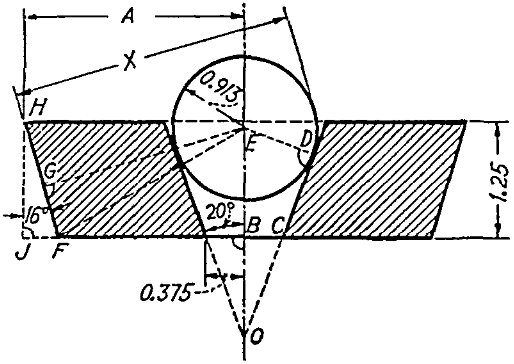
106. Determine the distance x.



Variable No.	Sym.	Value
1	θ	68°
2	θ	70°
3	θ	72°
4	θ	74°
5	θ	76°
6	θ	78°

$\theta = 80°$
Ans. $x = 4.9930$

107. Determine the distance x.



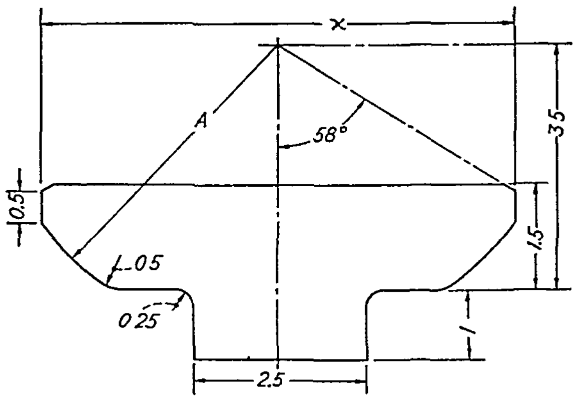
Variable		
No.	Sym.	Value
1	A	2.67
2	A	2 78
3	A	2.91
4	A	3.12
5	A	3.25
6	A	3.39

A = 2.55
Ans. x = 3.4714

108. Determine the distance x.

Solution:

BC = .375. In $\triangle BCO$, solve for BO.
DE = .913. In $\triangle EDO$, solve for EO.
$BE = EO - BO$. In $\triangle HJF$, solve for FJ.
$BF = A - FJ$. In $\triangle FBE$, solve for $\angle EFB$ and EF.
$\angle GFE = 180° - \angle HFJ - \angle EFB$.
In $\triangle GFE$, solve for EG.

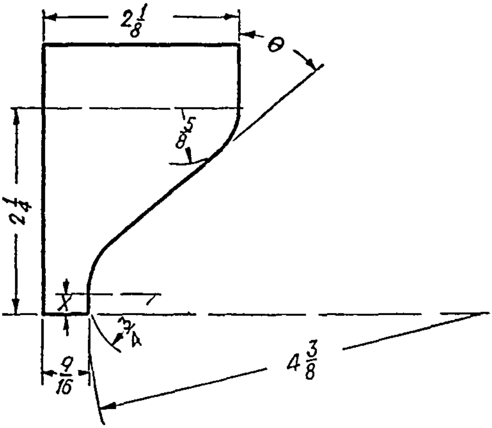


A = 4.375
Ans. x = 6.9360

Variable

1. A = 3.625	**2.** A = 3.750	**3.** A = 3.875
4. A = 4.000	**5.** A = 4.125	**6.** A = 4.250

109. Determine the distance x.

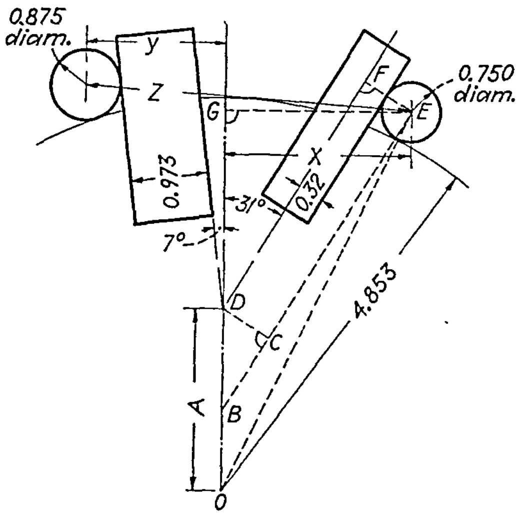


VARIABLE		
No.	Sym.	Value
1	θ	47°
2	θ	49°
3	θ	51°
4	θ	53°
5	θ	55°
6	θ	57°

$\theta = 59°$

Ans. $x = .55930$

110. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	2.12
2	A	2.35
3	A	2.48
4	A	2.59
5	A	2.67
6	A	2.78

$A = 2.09$

Ans. $\begin{cases} x = 2.3585 \\ y = 1.7763 \\ z = 4.1472 \end{cases}$

111. Determine the distance x.
112. Determine the distance y.
113. Determine the distance z.

Solution:

$EF = CD = .32 + .375.$

$\angle CBD = 31°.$

(*Solution continued on next page.*)

Solution continued:

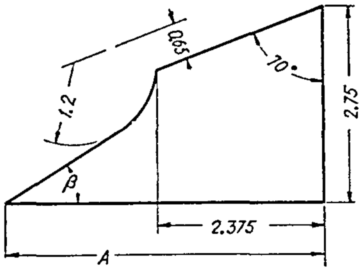
In $\triangle CBD$, solve for BD.

$BO = A - BD$. $EO = 4.853 + .375$. In $\triangle BOE$, solve for $\angle BOE$.

In $\triangle OEG$, solve for EG.

The solution for y is similar to that of x.

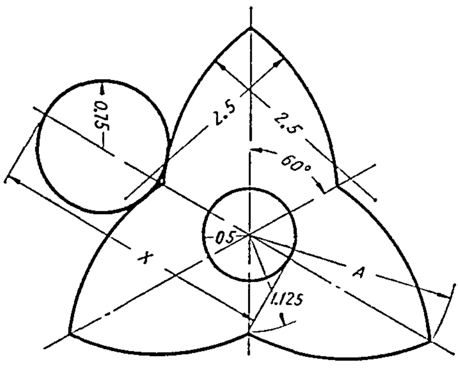
The solution for z is left to the student.



VARIABLE		
No.	Sym.	Value
1	A	4 25
2	A	4 375
3	A	4 5
4	A	4 625
5	A	4 75
6	A	4 875

$A = 5$
Ans. $\beta = 28° \, 24' \, 50''$

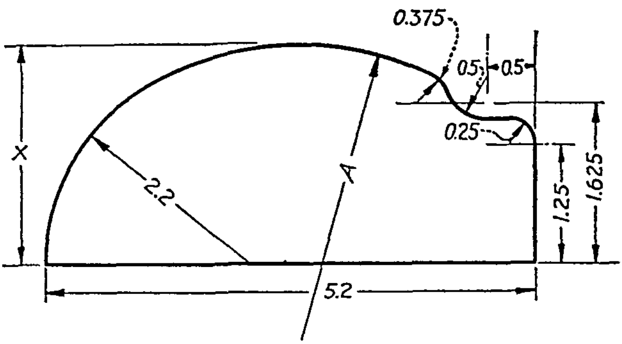
114. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	A	2 25
2	A	2 375
3	A	2 5
4	A	2 625
5	A	2.75
6	A	2 875

$A = 2.125$
Ans. $x = 3.1603$

115. Determine the distance x.



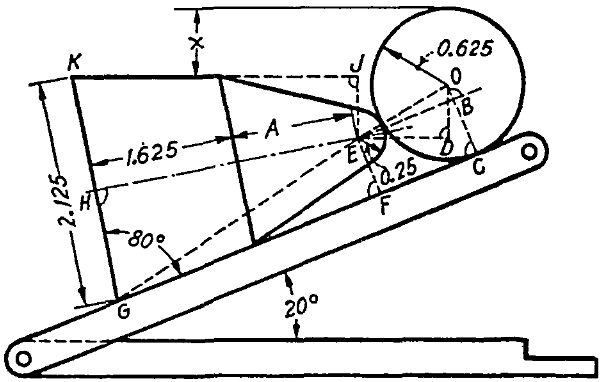
$A = 3.8$
Ans. $x = 2.2546$

VARIABLE

1. $A = 3.9$	**2.** $A = 2.9$	**3.** $A = 3.1$
4. $A = 3.3$	**5.** $A = 3.5$	**6.** $A = 3.7$

116. Determine the distance x

Checking by Means of a Sine Bar



$A = 1.06$
Ans. $x = .38593$

VARIABLE

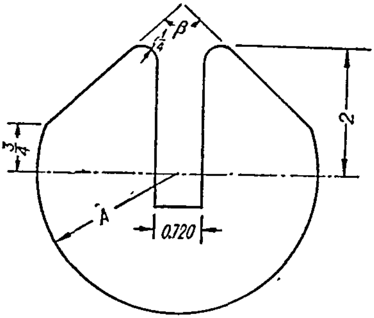
1. $A = 1.1$	**2.** $A = 1.16$	**3.** $A = 1.2$
4. $A = 1.26$	**5.** $A = 1.3$	**6.** $A = 1.34$

117. Determine the distance x.
Solution:

BE is parallel to GF.
DE is parallel to JK. Hence $\angle BED = 20°$. Why?
In $\triangle HEG$, solve for $\angle HGE$ and EG. $\angle EGF = 80° - \angle HGE$.
In $\triangle GEF$, solve for EF. $EF = BC$. $CO = .625$.
$BO = CO - BC$. In $\triangle OEB$, solve for $\angle OEB$.
$\angle OED = \angle OEB + \angle BED$. (*Continued on next page.*)

Solution continued:

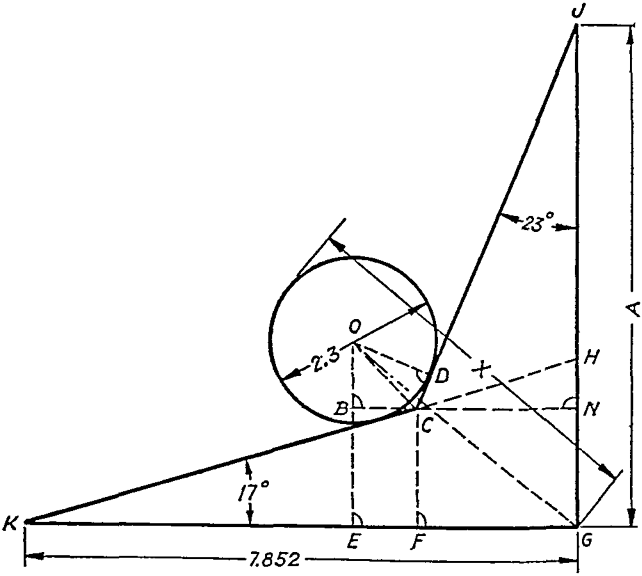
In $\triangle OED$, solve for DO. $EJ = EF$. Why?
$x = DO + .625 - EJ$.



VARIABLE		
No.	Sym.	Value
1	A	2.125
2	A	2.1875
3	A	2.25
4	A	2.3125
5	A	2.375
6	A	2.5

$A = 2.0625$
Ans. $\beta = 87°\ 53'\ 58''$

118. Determine the angle β.



$A = 5.51$
Ans. $x = 4.875$

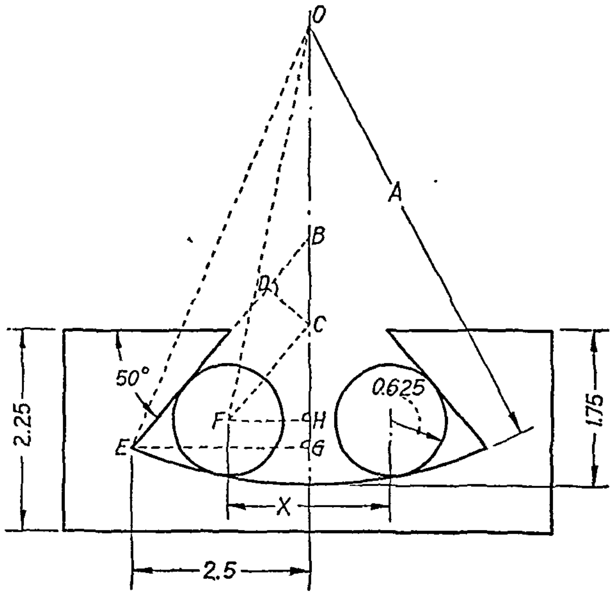
VARIABLE

1. $A = 5.72$	**2.** $A = 5.93$	**3.** $A = 6.14$
4. $A = 6.25$	**5.** $A = 6.35$	**6.** $A = 6.45$

119. Determine the distance x. *(Solution on next page.)*

Solution for preceding problem:

In $\triangle HKG$, solve for GH. $\angle KHG = 90° - 17°$. $JH = GJ - GH$.
In $\triangle JCH$, solve for CN. $CN = FG$. $FK = GK - FG$.
In $\triangle CKF$, solve for CF. $CF = BE$.
$\angle KCJ = 90° + 17° + 23°$. $\angle OCD = \angle OCK = \angle KCJ \div 2$.
$DO = 2.3 \div 2$. In $\triangle OCD$, solve for CO.
$\angle OCB = \angle OCK - 17°$. In $\triangle OCB$, solve for BO and BC.
$EG = BC + FG$. $EO = BE + BO$.
In $\triangle OEG$, solve for GO.



$A = 4.875$
Ans. $x = 3.0135$

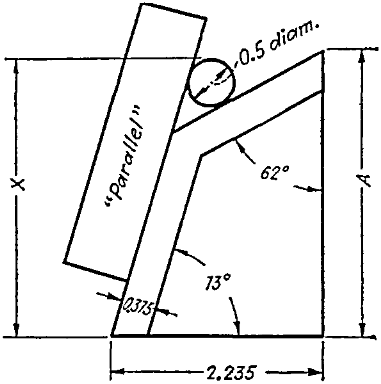
VARIABLE

1. $A = 5$	2. $A = 5.125$	3. $A = 5.25$
4. $A = 5.375$	5. $A = 5.5$	6. $A = 5.625$

120. Determine the distance x.

Solution:

In $\triangle OEG$, solve for GO. $\angle BEG = \angle BCD = 50°$. Why?
In $\triangle BEG$, solve for GB. $CD = .625$. In $\triangle DCB$, solve for BC.
$CO = GO - GB + BC$. $FO = A - .625$.
In $\triangle OFC$, solve for FH.

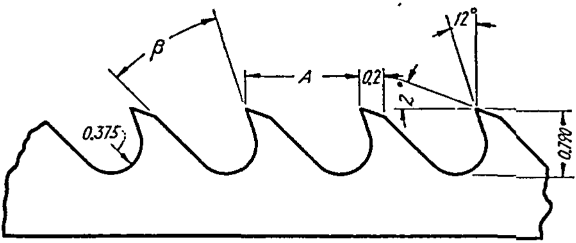


No.	Sym.	Value
1	A	3 25
2	A	3.37
3	A	3.43
4	A	3.52
5	A	3.65
6	A	3.78

$A = 3.12$
Ans. $x = 3.0604$

121. Determine the distance x.

Broach



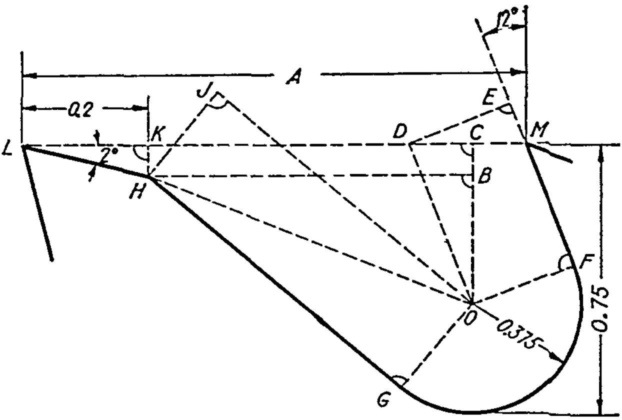
$A = 1.425$
Ans. $\beta = 34° 1' 3''$

VARIABLE

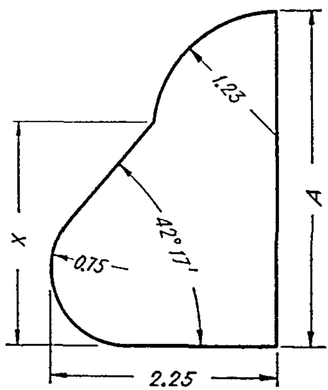
1. $A = 1.125$ 2. $A = 1.175$ 3. $A = 1.225$
4. $A = 1.275$ 5. $A = 1.325$ 6. $A = 1.375$

122. Determine the angle β. *(Solution on next page.)*

Solution for preceding problem:



$\angle DOC = \angle EDM = 12°$. $CO = .75 - .375$.
$DE = FO = .375$. In $\triangle DOC$, solve for CD.
In $\triangle DEM$, solve for DM. $CM = DM - CD$.
In $\triangle LKH$, solve for LK and HK.
$CK = BH = A - CM - LK$. $HK = CB$.
$BO = CO - CB$. In $\triangle HOB$, solve for $\angle HOB$ and HO.
$OG = HJ = .375$. In $\triangle HOJ$, solve for $\angle HOJ$.
$\beta = \angle JOD = \angle HOB - 12° - \angle HOJ$.



VARIABLE

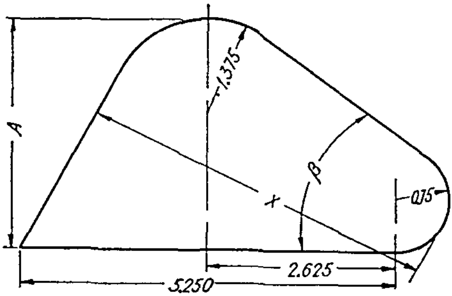
No.	Sym.	Value
1	A	3.12
2	A	3.21
3	A	3.35
4	A	3.42
5	A	3.84
6	A	3.91

$A = 3.05$
Ans. $x = 2.0248$

123. Determine the distance x.

Solution continued:

In $\triangle CBO$, solve for CO and BC.
In $\triangle NCO$, solve for NP, NC, and NO.
$x = NP$.
$\angle DCE = 90° - 75°$.
$\angle SCN = \angle NCO - \angle DCE$.
In $\triangle SCN$, solve for SC.
In $\triangle DCE$, solve for CE.
$y = x + SC - CE$.
In $\triangle FDG$, solve for DG and FG.
$HJ = A - FG - 1.125 - BC$.
In $\triangle KHJ$, solve for KJ.
$OK = 4 - DG - BO - KJ$.
In $\triangle OKL$, solve for OL.
$\angle NOR = \angle NOM + 20°$. Why?
In $\triangle NOR$, solve for OR.
$z = OL - OR - x$.



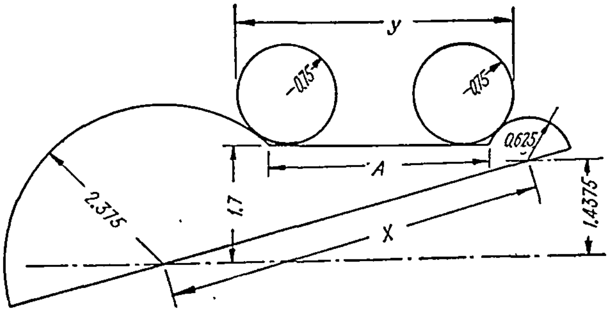
Variable		
No.	Sym.	Value
1	A	3 82
2	A	3 93
3	A	3 14
4	A	3 24
5	A	3 66
6	A	3.78

$A = 3.51$

Ans. $\begin{cases} x = 5.0924 \\ \beta = 39° 58' 24'' \end{cases}$

132. Determine the distance x.

133. Determine the angle β.



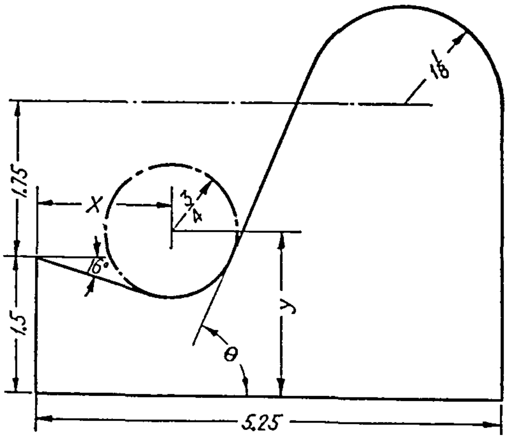
$A = 3.97$

Ans. $\begin{cases} x = 6.3603 \\ y = 4.8255 \end{cases}$

VARIABLE

1. $A = 3.11$	**2.** $A = 3.22$	**3.** $A = 3.43$
4. $A = 3.34$	**5.** $A = 3.55$	**6.** $A = 3.76$

134. Determine the distance x.
135. Determine the distance y.



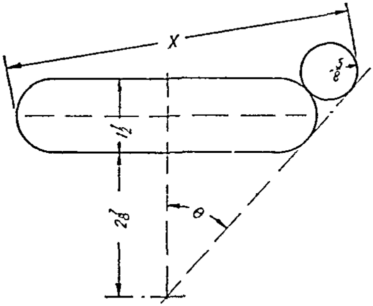
VARIABLE

No.	Sym.	Value
1	θ	62°
2	θ	64°
3	θ	66°
4	θ	68°
5	θ	70°
6	θ	72°

$\theta = 60°$

Ans. $\begin{cases} x = 1.2011 \\ y = 1.9357 \end{cases}$

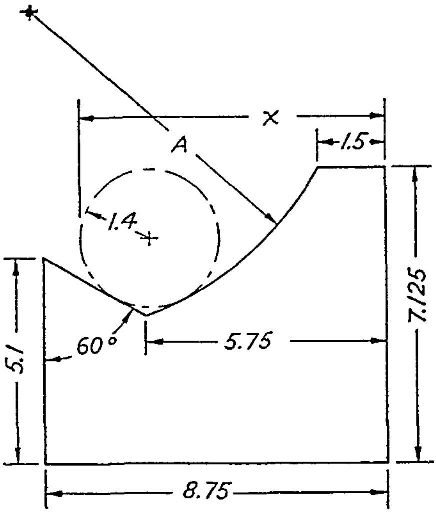
136. Determine the distance x.
137. Determine the distance y.



Variable		
No.	Sym.	Value
1	θ	36°
2	θ	38°
3	θ	40°
4	θ	42°
5	θ	44°
6	θ	46°

$\theta = 48°$
Ans. $x = 8.3352$

138. Determine the distance x.

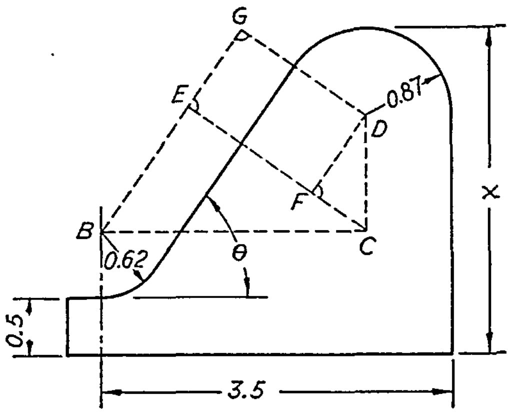


$A = 9.287$
Ans. $x = 7.0724$

Variable

1. $A = 8.501$	2. $A = 8.632$	3. $A = 8.763$
4. $A = 8.894$	5. $A = 9.025$	6. $A = 9.156$

139. Determine the distance x.



$\theta = 61°$
Ans. $x = 3.6613$

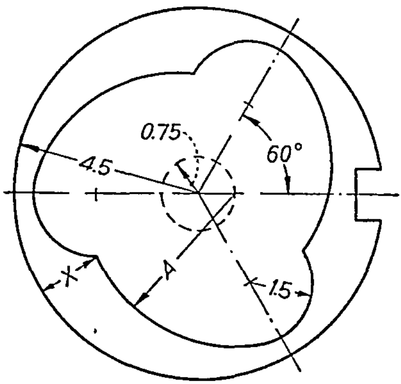
VARIABLE

1. $\theta = 55°$	**2.** $\theta = 56°$	**3.** $\theta = 57°$
4. $\theta = 58°$	**5.** $\theta = 59°$	**6.** $\theta = 60°$

140. Determine the distance x.

Solution:

$\angle EBC = \theta$. Why?
In $\triangle BCE$, solve for EC.
$EF = .62 + .87$. Why?
In $\triangle CFD$, solve for CD.

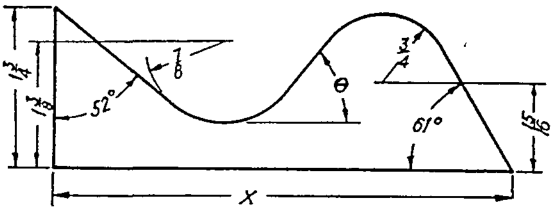


VARIABLE

No.	Sym.	Value
1	A	2.998
2	A	3.093
3	A	3.188
4	A	3.283
5	A	3.378
6	A	3.473

$A = 3.948$
Ans. $x = 1.2342$

141. Determine the distance x.

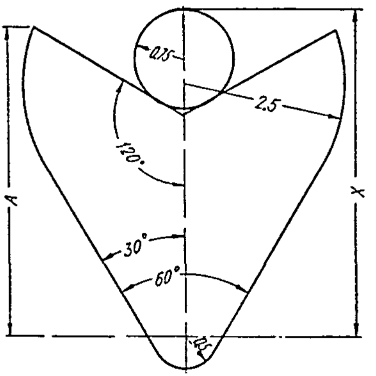


$\theta = 51°$
Ans. $x = 5.0148$

VARIABLE

1. $\theta = 39°$	2. $\theta = 41°$	3. $\theta = 43°$
4. $\theta = 45°$	5. $\theta = 47°$	6. $\theta = 49°$

142. Determine the distance x.

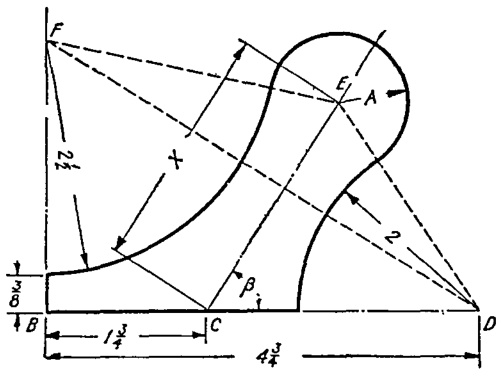


VARIABLE

No.	Sym.	Value
1	A	4.0
2	A	4.125
3	A	4.25
4	A	4.375
5	A	4.5
6	A	4.625

$A = 4.75$
Ans. $x = 4.9890$

143. Determine the distance x.



VARIABLE

No.	Sym.	Value
1	A	.625
2	A	.6875
3	A	.75
4	A	.8125
5	A	.875
6	A	.9375

$A = 1$

Ans. $\begin{cases} \beta = 57° \ 20' \ 49'' \\ x = 3.2375 \end{cases}$

144. Determine the angle β.

145. Determine the distance x.

Solution:

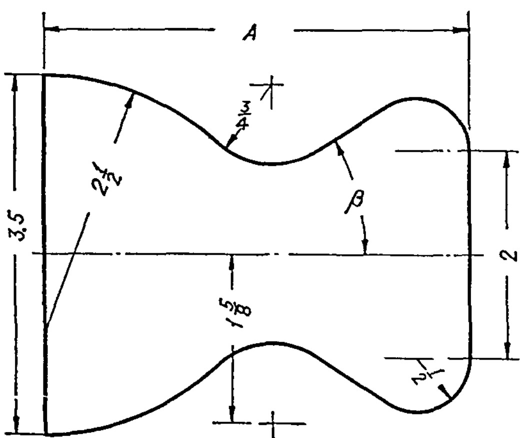
In $\triangle FBD$, solve for $\angle BDF$ and DF.

$EF = 2.5 + A$. $DE = 2 + A$

In $\triangle FDE$, solve for $\angle FDE$.

$\angle CDE = \angle BDF + \angle FDE$.

In $\triangle CDE$, solve for $\angle ECD$ and CE.



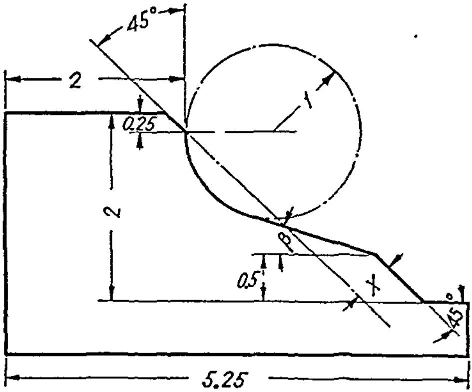
VARIABLE

No.	Sym.	Value
1	A	4.75
2	A	4.875
3	A	4.00
4	A	4.125
5	A	4.25
6	A	4.375

$A = 4.5$

Ans. $\beta = 13° \ 20' \ 32''$

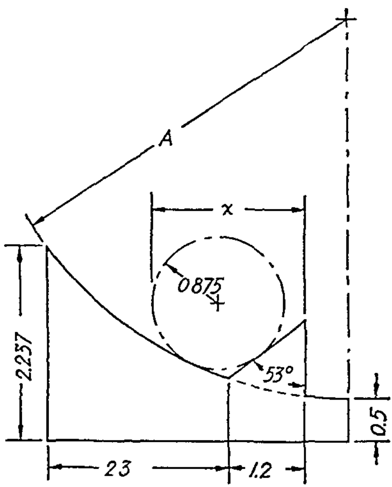
146. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	β	14°
2	β	16°
3	β	18°
4	β	20°
5	β	22°
6	β	24°

$\beta = 26°$
Ans. $x = .02243$

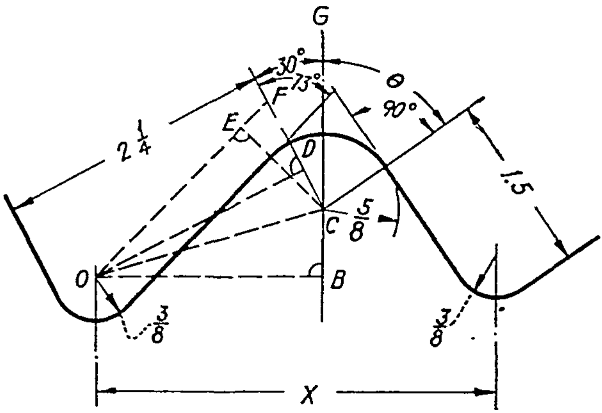
147. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	5.23
2	A	5.47
3	A	5.62
4	A	5.74
5	A	5.92
6	A	6.04

$A = 5.12$
Ans. $x = 2.2194$

148. Determine the distance x.



$\theta = 41°$
Ans. $x = 3.3651$

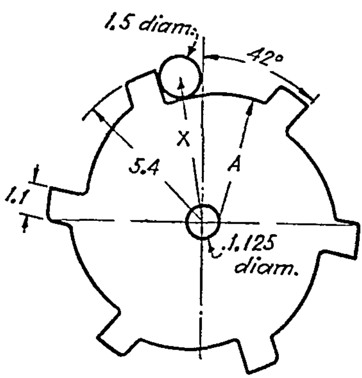
VARIABLE

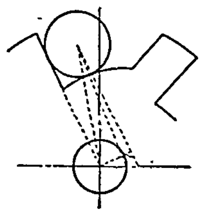
1. $\theta = 53°$	2. $\theta = 51°$	3. $\theta = 49°$
4. $\theta = 47°$	5. $\theta = 45°$	6. $\theta = 43°$

149. Determine the distance x.

Solution:

$\angle CFE = 73°$. Why? $OD = 1.875$
In $\triangle ODF$, solve for OF.
In $\triangle ECF$, solve for EF. $OE = OF - EF$.
$CE = .375 + .625$. In $\triangle OEC$, solve for $\angle EOC$ and OC.
$\angle ECG = 30° + 90° - 73°$. Why?
$\angle EOB = \angle ECG$. Why?
$\angle COB = \angle EOB - \angle EOC$.
In $\triangle OBC$, solve for OB.
The balance of the problem is left to the student.





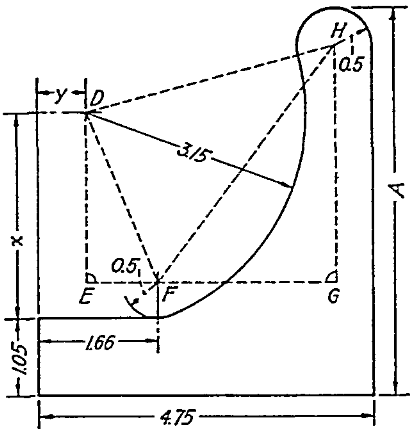
Diagrammatic Hint

VARIABLE

No.	Sym.	Value
1	A	3.75
2	A	3.818
3	A	3.886
4	A	3.954
5	A	4.022
6	A	4.09

$A = 4.43$
Ans. $x = 5.0576$

150. Determine the distance x.



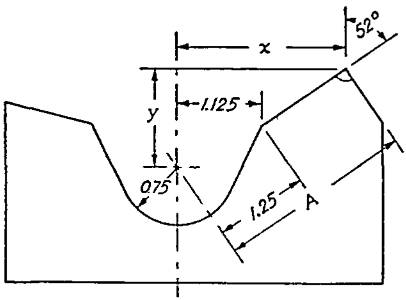
Variable		
No.	Sym.	Value
1	A	4.750
2	A	4.875
3	A	5.000
4	A	5.125
5	A	5.250
6	A	5.375

$A = 5.500$

Ans. $\begin{cases} x = 2.9817 \\ y = .73078 \end{cases}$

151. Determine the distance x.
152. Determine the distance y.
Solution:

In $\triangle FGH$, solve for $\angle HFG$ and FH.
In $\triangle DFH$, solve for $\angle DFH$.
$\angle EFD = 180° - \angle DFH - \angle HFG$.
In $\triangle EFD$, solve for ED and EF.

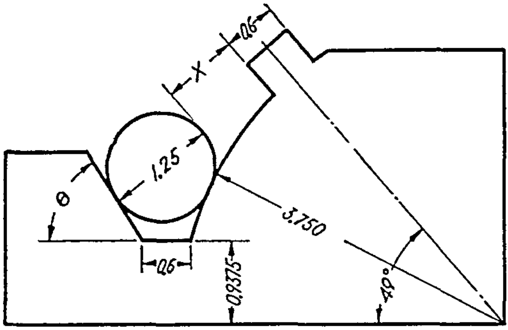


Variable		
No.	Sym.	Value
1	A	2 000
2	A	2.125
3	A	2.250
4	A	2.375
5	A	2.500
6	A	2.625

$A = 2.750$

Ans. $\begin{cases} x = 2.3068 \\ y = 1.5139 \end{cases}$

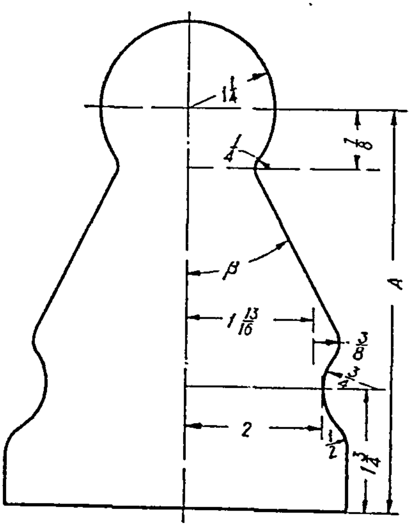
153. Determine the distance x.
154. Determine the distance y.



VARIABLE		
No.	Sym.	Value
1	θ	56°
2	θ	57°
3	θ	58°
4	θ	59°
5	θ	60°
6	θ	61°

$\theta = 55°$
Ans. $x = .9765$

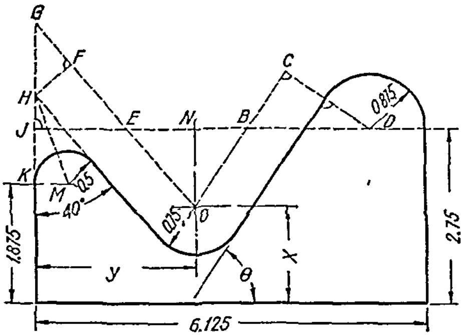
155. Determine the distance x.



VARIABLE		
No.	Sym.	Value
1	A	5.375
2	A	5.5
3	A	5.625
4	A	5.75
5	A	5.875
6	A	6.000

$A = 5.25$
Ans. $\beta = 33° \ 55' \ 33''$

156. Determine the angle β.



VARIABLE		
No.	Sym.	Value
1	θ	52°
2	θ	54°
3	θ	56°
4	θ	58°
5	θ	60°
6	θ	62°

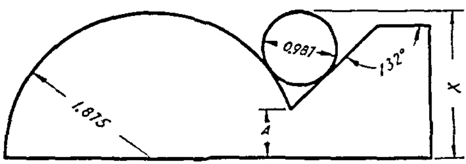
$\theta = 64°$

Ans. $\begin{cases} x = 1.2093 \\ y = 2.6904 \end{cases}$

157. Determine the distance x.
158. Determine the distance y.

Solution:

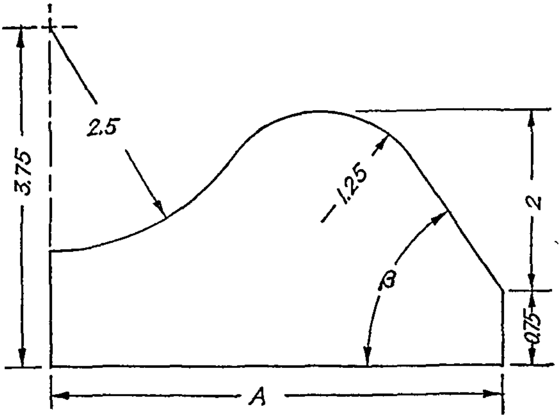
$CD = .875 + .75$. $\angle CBD = \theta$. In $\triangle CBD$, solve for BD.
$\angle KHM = 40° \div 2$. $KM = .5$. In $\triangle KHM$, solve for HK.
$HF = .75$. $\angle HGF = 40°$. In $\triangle HGF$, solve for GH.
$GJ = 1.875 + HK + GH - 2.75$.
In $\triangle JGE$, solve for EJ. $EB = 6.125 - .875 - BD - EJ$.
$\angle NEO = 90° - 40°$. $\angle NBO = \theta$. In $\triangle EON$, solve for NO and NE.
$NE + EJ = y$. $2.75 - NO = x$.



VARIABLE		
No.	Sym.	Value
1	A	.682
2	A	.783
3	A	.854
4	A	.985
5	A	1.086
6	A	.932

$A = .561$
Ans. $x = 1.9118$

159. Determine the distance x.

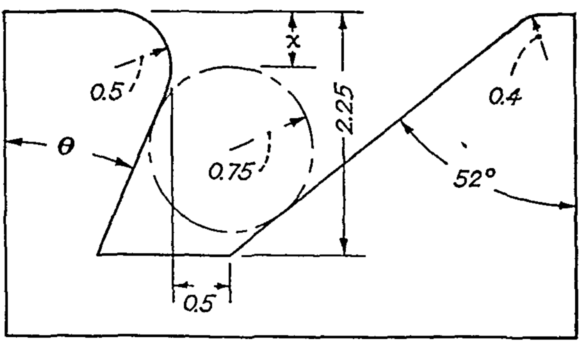


$A = 5.500$
Ans. $\beta = 45° \, 18' \, 51''$

VARIABLE

1. $A = 4.750$	2. $A = 4.875$	3. $A = 5.000$
4. $A = 5.125$	5. $A = 5.250$	6. $A = 5.375$

160. Determine the angle β.

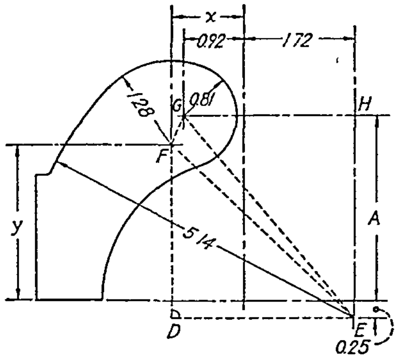


$\theta = 23°$
Ans. $x = .5259$

VARIABLE

1. $\theta = 17°$	2. $\theta = 18°$	3. $\theta = 19°$
4. $\theta = 20°$	5. $\theta = 21°$	6. $\theta = 22°$

161. Determine the distance x.



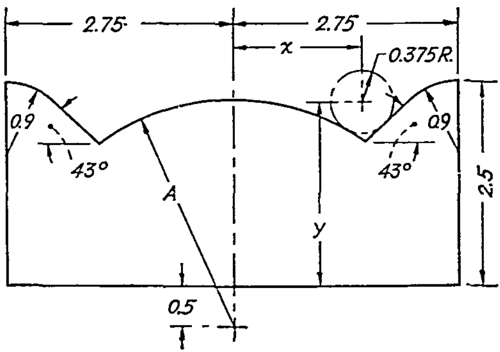
$$A = 3.172$$

Ans. $\begin{cases} x = .70126 \\ y = 2.7561 \end{cases}$

VARIABLE

1. $A = 2.593$	2. $A = 2.691$	3. $A = 2.787$
4. $A = 2.884$	5. $A = 2.981$	6. $A = 3.078$

162. Determine the distance x.
163. Determine the distance y.



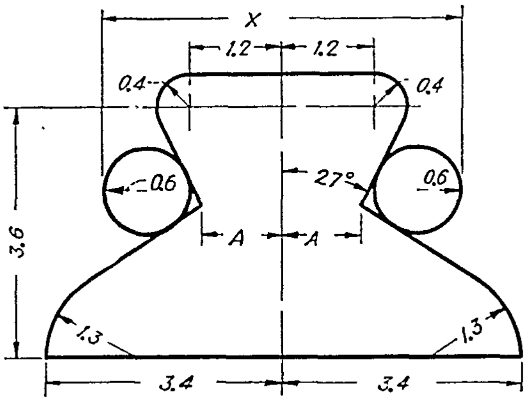
$$A = 2.858$$

Ans. $\begin{cases} x = 1.6253 \\ y = 2.2947 \end{cases}$

VARIABLE

1. $A = 2.537$	2. $A = 2.588$	3. $A = 2.642$
4. $A = 2.695$	5. $A = 2.750$	6. $A = 2.804$

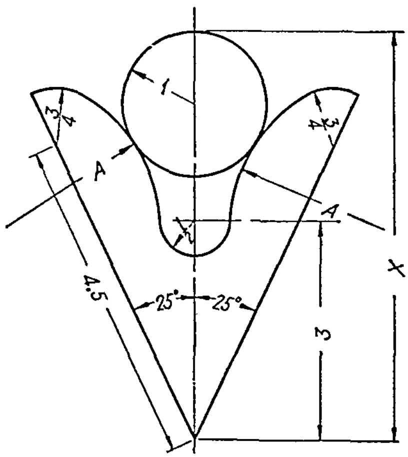
164. Determine the distance x.
165. Determine the distance y.



Variable No.	Sym.	Value
1	A	.56
2	A	.67
3	A	.62
4	A	.77
5	A	.82
6	A	.89

$A = .72$
Ans. $x = 4.3600$

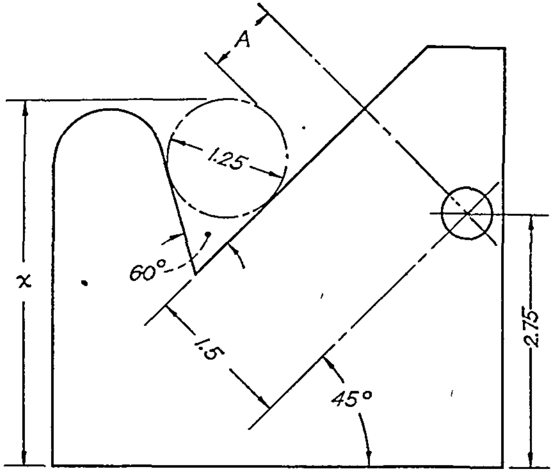
166. Determine the distance x.



Variable No.	Sym.	Value
1	A	2.375
2	A	2.5
3	A	2.625
4	A	2.75
5	A	2.875
6	A	3.0

$A = 3.125$
Ans. $x = 5.4521$

167. Determine the distance x.



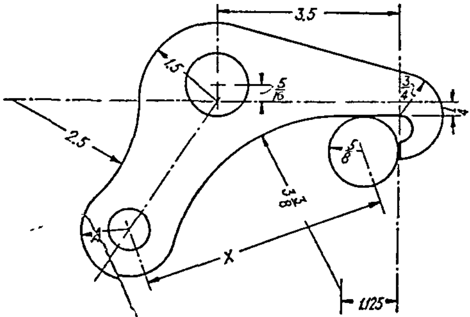
$A = .875$
Ans. $x = 3.8168$

VARIABLE

1. $A = .125$
2. $A = .250$
3. $A = .375$
4. $A = .500$
5. $A = .625$
6. $A = .750$

168. Determine the distance x.

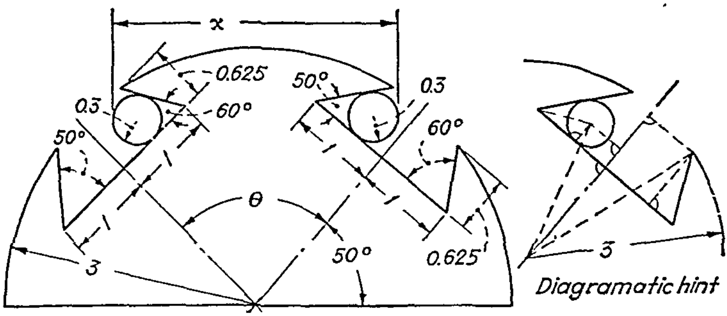
Trip Lever



VARIABLE		
No.	Sym.	Value
1	A	1.0
2	A	1 125
3	A	1 25
4	A	1 375
5	A	1.5
6	A	1.625

$A = 875$
Ans. $x = 4.8696$

169. Determine the distance x.



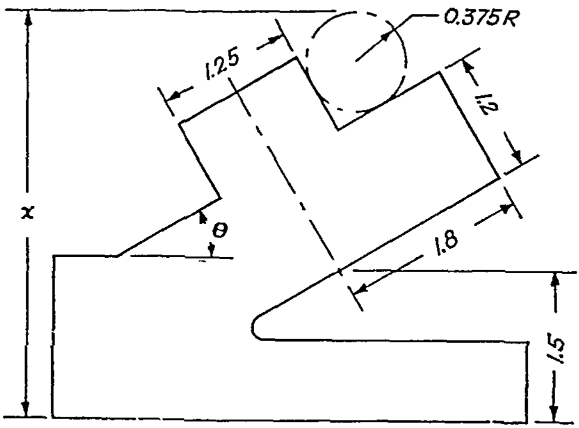
$\theta = 82°$

Ans. $x = 3.3889$

Variable

1. $\theta = 94°$	**2.** $\theta = 92°$	**3.** $\theta = 90°$
4. $\theta = 88°$	**5.** $\theta = 86°$	**6.** $\theta = 84°$

170. Determine the distance x.



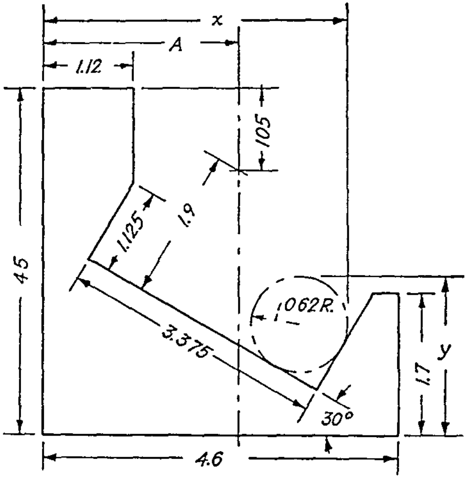
$\theta = 33°$

Ans. $x = 3.7405$

Variable

1. $\theta = 27°$	**2.** $\theta = 28°$	**3.** $\theta = 29°$
4. $\theta = 30°$	**5.** $\theta = 31°$	**6.** $\theta = 32°$

171. Determine the distance x.



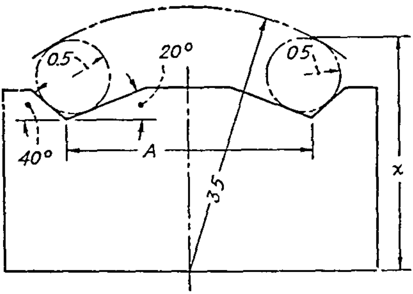
Variable		
No.	Sym.	Value
1	A	2.525
2	A	2.595
3	A	2.665
4	A	2.735
5	A	2.805
6	A	2.875

$A = 2.945$

Ans. $\begin{cases} x = 3.8734 \\ y = 2.4139 \end{cases}$

172. Determine the distance x.

173. Determine the distance y.



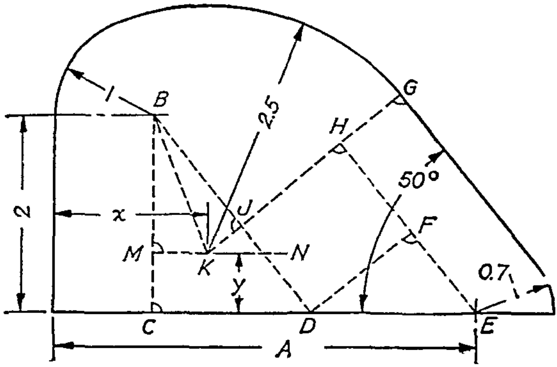
$A = 3.500$

Ans. $x = 3.0056$

Variable

1. $A = 2.750$
2. $A = 2.875$
3. $A = 3.000$
4. $A = 3.125$
5. $A = 3.250$
6. $A = 3.375$

174. Determine the distance x.



$A = 4.4$

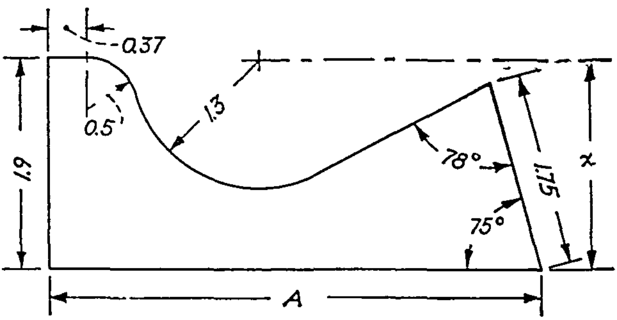
Ans. $\begin{cases} x = 1.5447 \\ y = .60240 \end{cases}$

VARIABLE

1. $A = 3.8$	2. $A = 3.9$	3. $A = 4.0$
4. $A = 4.1$	5. $A = 4.2$	6. $A = 4.3$

175. Determine the distance x.

176. Determine the distance y.



$A = 4.968$

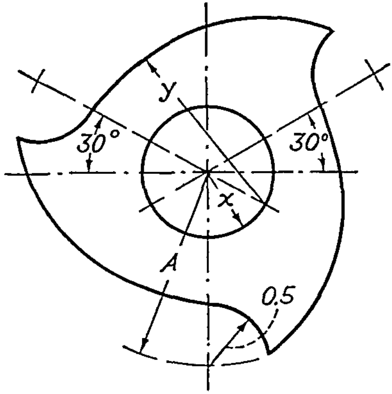
Ans. $x = 1.9170$

VARIABLE

1. $A = 4.560$	2. $A = 4.628$	3. $A = 4.696$
4. $A = 4.764$	5. $A = 4.832$	6. $A = 4.900$

177. Determine the distance x.

Cam



VARIABLE		
No.	Sym.	Value
1	A	3 2
2	A	2 3
3	A	2 4
4	A	2 5
5	A	3 6
6	A	3 9

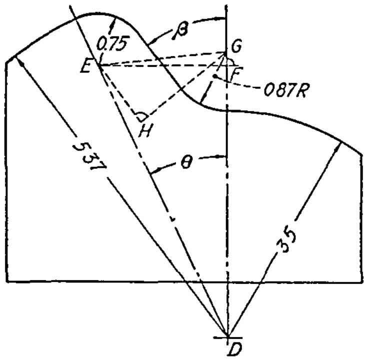
$A = 2.1$

Ans. $\begin{cases} x = .4222 \\ y = 2.0222 \end{cases}$

185. Determine the radius x.

186. Determine the radius y.

Cam



VARIABLE		
No.	Sym.	Value
1	θ	22°
2	θ	23°
3	θ	24°
4	θ	25°
5	θ	26°
6	θ	27°

$\theta = 28°$

Ans $\beta = 49° \, 52' \, 56''$

187. Determine the angle β.

Solution.

In $\triangle EDF$, solve for EF and DF.

$FG = 3\ 5 + .87 - DF$.

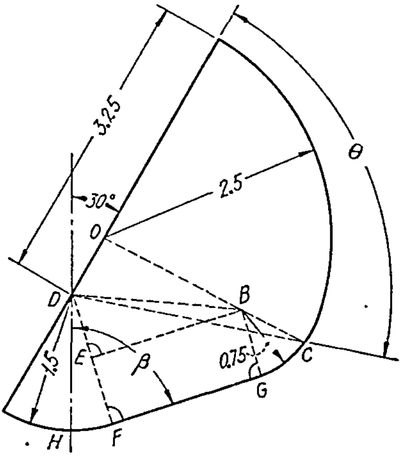
In $\triangle EFG$, solve for $\angle GEF$ and EG.

In $\triangle EGH$, solve for $\angle GEH$.

$\angle GEH - \angle GEF = \angle FEH$.

$\beta = 90° - \angle FEH$. Why?

Cam



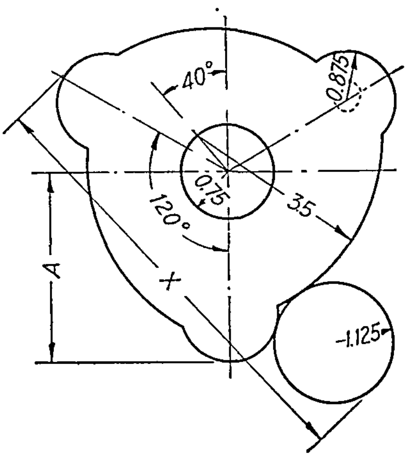
VARIABLE		
No.	Sym.	Value
1	θ	76°
2	θ	74°
3	θ	72°
4	θ	70°
5	θ	68°
6	θ	66°

$\theta = 64°$
Ans. $\beta = 66° 31' 2''$

188. Determine the angle β.

Solution:

$CO = 2.5$. $DO = 3.25 - 2.5$. $\angle ODC = \theta$.
In $\triangle ODC$, solve for $\angle OCD$ and CD. $BC = .75$.
In $\triangle BCD$, solve for $\angle BDC$ and BD. $DE = 1.5 - .75$.
In $\triangle DEB$, solve for $\angle BDE$. $\angle CDE = \angle BDE - \angle BDC$.
$\angle FDH = 180° - \theta - \angle CDE - 30°$.
$\beta = 90° - \angle FDH$. Why?

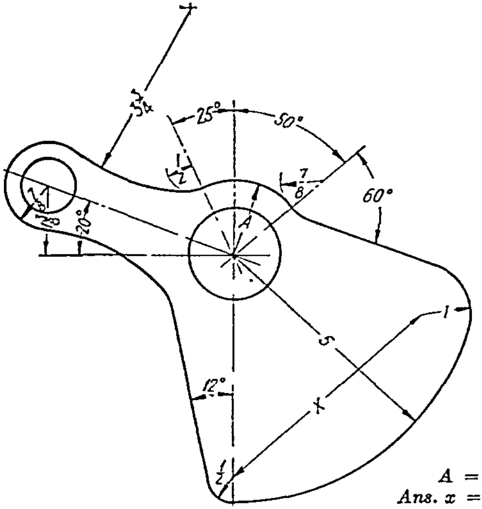


VARIABLE		
No.	Sym.	Value
1	A	3.72
2	A	3.43
3	A	3.54
4	A	3.65
5	A	3.76
6	A	3.87

$A = 3.61$
Ans. $x = 8.3887$

189. Determine the distance x.

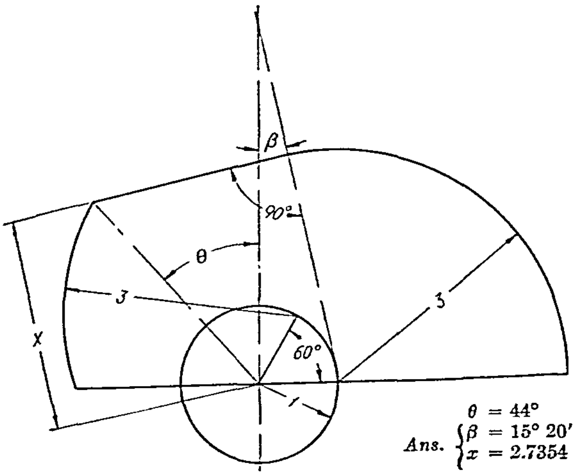
Cam



VARIABLE		
No.	Sym.	Value
1	A	1.125
2	A	1.1875
3	A	1.25
4	A	1.3125
5	A	1 375
6	A	1.4375

$A = 1.5$
Ans. $x = 5.0485$

190. Determine the distance x.



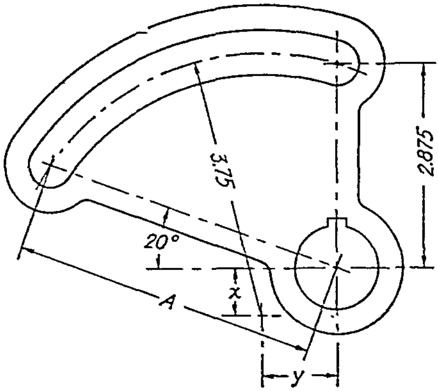
$\theta = 44°$

Ans. $\begin{cases} \beta = 15° \, 20' \, 35'' \\ x = 2.7354 \end{cases}$

VARIABLE

1. $\theta = 42°$	**2.** $\theta = 40°$	**3.** $\theta = 38°$
4. $\theta = 36°$	**5.** $\theta = 34°$	**6.** $\theta = 32°$

191. Determine the angle β. **192.** Determine the distance x.



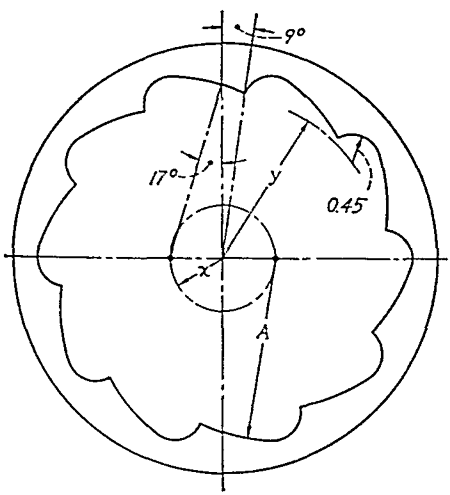
VARIABLE

No.	Sym.	Value
1	A	3.625
2	A	3.750
3	A	3.875
4	A	4.000
5	A	4.125
6	A	4.250

$A = 4.375$

Ans. $\begin{cases} x = .71532 \\ y = 1.0825 \end{cases}$

193. Determine the distance x.
194. Determine the distance y.



VARIABLE

No.	Sym.	Value
1	A	2.125
2	A	2.250
3	A	2.375
4	A	2.500
5	A	2.625
6	A	2.750

$A = 2.875$

Ans. $\begin{cases} x = .84056 \\ y = 2.4032 \end{cases}$

195. Determine the distance x.
196. Determine the distance y.

Figures 141 and 142 show a circular form cutter in relation to the work. It is required to determine the angle Σ and the depth F of the cutter in order to produce a given angle and a depth E of the work.

From Fig. 142, the following formulas may be derived:

For M given:

$$\sin \omega = \frac{B}{M}. \qquad E = R - S$$

$$\cot \psi = \frac{M}{E \sin \omega} - \cot \omega.$$

$$N = E \sin \omega \csc \psi.$$

$$F = M - N.$$

$$\tan \Sigma = \frac{F}{D}.$$

For N given:

$$\cos \phi = \frac{B}{N}.$$

$$\cot \omega' = \tan \phi + \frac{E'}{B}.$$

$$M' = B \csc \omega'.$$

$$F' = M' - N.$$

$$\tan \Sigma' = \frac{F'}{D'}.$$

D and D' are the distances along the axis between the maximum and minimum radii of the cutter represented by M and N and N and M', respectively. These distances correspond to the distances along the axis between the minimum and maximum radii of the work represented by S and R and R and S', respectively, as shown in Fig. 141.

Form 2. Rake on Cutting Face of Cutter

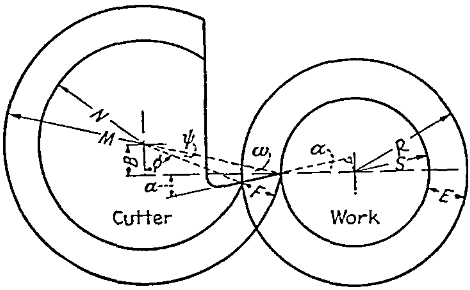


FIG. 143.

From Fig. 143, the following formulas may be derived:
For M given:

$$\sin \omega = \frac{B}{M} \qquad \sin \rho = \frac{S \sin \alpha}{R}$$

$$\cot \psi = \frac{M \csc (\alpha + \omega)}{R \cos \rho - S \cos \alpha} - \cot (\alpha + \omega)$$

$$N = (R \cos \rho - S \cos \alpha) \sin (\alpha + \omega) \csc \psi$$

$$F = M - N \qquad \tan \Sigma = \frac{F}{D}$$

For N given:

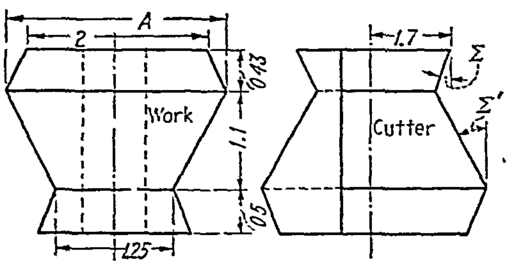
$$\sin \rho = \frac{S' \sin \alpha}{R}$$

$$\cos \phi = \frac{(R \cos \rho - S' \cos \alpha) \sin \alpha + B}{N}$$

$$\cot \omega' = \frac{N \sin \phi + (R \cos \rho - S' \cos \alpha) \cos \alpha}{B}$$

$$M' = B \csc \omega' \qquad F' = M' - N \qquad \tan \Sigma' = \frac{F'}{D'}$$

PROBLEMS



VARIABLE		
No.	Sym.	Value
1	A	2.125
2	A	2.25
3	A	2.375
4	A	2.5
5	A	2.625
6	A	2.75

$A = 2.875$

Form 1. *Ans.* $\begin{cases} \Sigma = 41° 56' 34'' \\ \Sigma' = 33° 42' 27'' \end{cases}$

Form 2. *Ans.* $\begin{cases} \Sigma = 38° 30' 19'' \\ \Sigma' = 30° 10' 16'' \end{cases}$

200. Determine (*a*) the angle Σ, and (*b*) the angle Σ'. Form 1. $B = 0.7$.

201. Determine (*a*) the angle Σ, and (*b*) the angle Σ'. Form 2. $B = 0.7$ and $\alpha = 10°$.

NATURAL TANGENTS AND COTANGENTS

′	0° tan	0° cotan	1° tan	1° cotan	2° tan	2° cotan	3° tan	3° cotan	′
0	00000	Infinite	01746	57 2900	03492	28 6363	05241	19 0811	60
1	00029	3437 750	01775	56 3506	03521	28 3994	05270	18 9755	59
2	00058	1718 870	01804	55 4415	03550	28 1664	05299	18 8711	58
3	00087	1145 920	01833	54 5613	03579	27 9372	05328	18 7678	57
4	00116	859 436	01862	53 7086	03609	27 7117	05357	18 6656	56
5	00145	687 549	01891	52 8821	03638	27 4899	05387	18 5645	55
6	00175	572 957	01920	52 0807	03667	27 2715	05416	18 4645	54
7	00204	491 106	01949	51 3032	03696	27 0566	05445	18 3655	53
8	00233	429 718	01978	50 5485	03725	26 8450	05474	18 2677	52
9	00262	381 971	02007	49 8157	03754	26 6367	05503	18 1708	51
10	00291	343 774	02036	49 1039	03783	26 4316	05533	18 0750	50
11	00320	312 521	02066	48 4121	03812	26 2296	05562	17 9802	49
12	00349	286 478	02095	47 7395	03842	26 0307	05591	17 8863	48
13	00378	264 441	02124	47 0853	03871	25 8348	05620	17 7934	47
14	00407	245 552	02153	46 4489	03900	25 6418	05649	17 7015	46
15	00436	229 182	02182	45 8294	03929	25 4517	05678	17 6106	45
16	00465	214 858	02211	45 2261	03958	25 2644	05708	17 5205	44
17	00495	202 219	02240	44 6386	03987	25 0798	05737	17 4314	43
18	00524	190 984	02269	44 0661	04016	24 8978	05766	17 3432	42
19	00553	180 932	02298	43 5081	04046	24 7185	05795	17 2558	41
20	00582	171 885	02328	42 9641	04075	24 5418	05824	17 1693	40
21	00611	163 700	02357	42 4335	04104	24 3675	05854	17 0837	39
22	00640	156 259	02386	41 9158	04133	24 1957	05883	16 9990	38
23	00669	149 465	02415	41 4106	04162	24 0263	05912	16 9150	37
24	00698	143 237	02444	40 9174	04191	23 8593	05941	16 8319	36
25	00727	137 507	02473	40 4358	04220	23 6945	05970	16 7496	35
26	00756	132 219	02502	39 9655	04250	23 5321	05999	16 6681	34
27	00785	127 321	02531	39 5059	04279	23 3718	06029	16 5874	33
28	00814	122 774	02560	39 0568	04308	23 2137	06058	16 5075	32
29	00844	118 540	02589	38 6177	04337	23 0577	06087	16 4283	31
30	00873	114 589	02619	38 1885	04366	22 9038	06116	16 3499	30
31	00902	110 892	02648	37 7686	04395	22 7519	06145	16 2722	29
32	00931	107 426	02677	37 3579	04424	22 6020	06175	16 1952	28
33	00960	104 171	02706	36 9560	04454	22 4541	06204	16 1190	27
34	00989	101 107	02735	36 5627	04483	22 3081	06233	16 0435	26
35	01018	98 2179	02764	36 1776	04512	22 1640	06262	15 9687	25
36	01047	95 4895	02793	35 8006	04541	22 0217	06291	15 8945	24
37	01076	92 9085	02822	35 4313	04570	21 8813	06321	15 8211	23
38	01105	90 4633	02851	35 0695	04599	21 7426	06350	15 7483	22
39	01135	88 1436	02881	34 7151	04628	21 6056	06379	15 6762	21
40	01164	85 9398	02910	34 3678	04658	21 4704	06408	15 6048	20
41	01193	83 8435	02939	34 0273	04687	21 3369	06437	15 5340	19
42	01222	81 8470	02968	33 6935	04716	21 2049	06467	15 4638	18
43	01251	79 9434	02997	33 3662	04745	21 0747	06496	15 3943	17
44	01280	78 1263	03026	33 0452	04774	20 9460	06525	15 3254	16
45	01309	76 3900	03055	32 7303	04803	20 8188	06554	15 2571	15
46	01338	74 7292	03084	32 4213	04832	20 6932	06584	15 1893	14
47	01367	73 1390	03114	32 1181	04862	20 5691	06613	15 1222	13
48	01396	71 6151	03143	31 8205	04891	20 4465	06642	15 0557	12
49	01425	70 1533	03172	31 5284	04920	20 3253	06671	14 9898	11
50	01455	68 7501	03201	31 2416	04949	20 2056	06700	14 9244	10
	01484	67 4019	03230	30 9599	04978	20 0872	06730	14 8596	9
	01513	66 1055	03259	30 6833	05007	19 9702	06759	14 7954	8
	542	64 8580	03288	30 4116	05037	19 8546	06788	14 7317	7
	71	63 6567	03317	30 1446	05066	19 7403	06817	14 6685	6
	[illegible]	62 4992	03346	29 8823	05095	19 6273	06847	14 6059	5
	[illegible]	61 3829	03376	29 6245	05124	19 5156	06876	14 5438	4
		60 3058	03405	29 3711	05153	19 4051	06905	14 4823	3
		59 2659	03434	29 1220	05182	19 2959	06934	14 4212	2
		2612	03463	28 8771	05212	19 1879	06963	14 3607	1
		[illegible]	03492	28 6363	05241	19 0811	06993	14 3007	0
				tan	cotan	tan	cotan	tan	′
					87°		86°		

′	4° tan	4° cotan	5° tan	5° cotan	6° tan	6° cotan	7° tan	7° cotan	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33154	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	19.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	16.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	85° cotan	85° tan	84° cotan	84° tan	83° cotan	83° tan	82° cotan	82° tan	′

′	8°		9°		10°		11°		′
	tan	cotan	tan	cotan	tan	cotan	tan	cotan	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18383	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91235	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5 68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	cotan	tan	cotan	tan	cotan	tan	cotan	tan	′
	81°		80°		79°		78°		

′	12° tan	12° cotan	13° tan	13° cotan	14° tan	14° cotan	15° tan	15° cotan	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27920	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03075	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	cotan 77°	tan 77°	cotan 76°	tan 76°	cotan 75°	tan 75°	cotan 74°	tan 74°	′

′	16° tan	16° cotan	17° tan	17° cotan	18° tan	18° cotan	19° tan	19° cotan	′
0	28675	3 48741	30573	3 27085	32492	3 07768	34433	2 90421	60
1	28706	3 48359	30605	3 26745	32524	3 07464	34465	2 90147	59
2	28738	3 47977	30637	3 26406	32556	3 07160	34498	2 89873	58
3	28769	3 47596	30669	3 26067	32588	3 06857	34530	2 89600	57
4	28800	3 47216	30700	3 25729	32621	3 06554	34563	2 89327	56
5	28832	3 46837	30732	3 25392	32653	3 06252	34596	2 89055	55
6	28864	3 46458	30764	3 25055	32685	3 05950	34628	2 88783	54
7	28895	3 46080	30796	3 24719	32717	3 05649	34661	2 88511	53
8	28927	3 45703	30828	3 24383	32749	3 05349	34693	2 88240	52
9	28958	3 45327	30860	3 24049	32782	3 05049	34726	2 87970	51
10	28990	3 44951	30891	3 23714	32814	3 04749	34758	2 87700	50
11	29021	3 44576	30923	3 23381	32846	3 04450	34791	2 87430	49
12	29053	3 44202	30955	3 23048	32878	3 04152	34824	2 87161	48
13	29084	3 43829	30987	3 22715	32911	3 03854	34856	2 86892	47
14	29116	3 43456	31019	3 22384	32943	3 03556	34889	2 86624	46
15	29147	3 43084	31051	3 22053	32975	3 03260	34922	2 86356	45
16	29179	3 42713	31083	3 21722	33007	3 02963	34954	2 86089	44
17	29210	3 42343	31115	3 21392	33040	3 02667	34987	2 85822	43
18	29242	3 41973	31147	3 21063	33072	3 02372	35019	2 85555	42
19	29274	3 41604	31178	3 20734	33104	3 02077	35052	2 85289	41
20	29305	3 41236	31210	3 20406	33136	3 01783	35085	2 85023	40
21	29337	3 40869	31242	3 20079	33169	3 01489	35117	2 84758	39
22	29368	3 40502	31274	3 19752	33201	3 01196	35150	2 84494	38
23	29400	3 40136	31306	3 19426	33233	3 00903	35183	2 84229	37
24	29432	3 39771	31338	3 19100	33266	3 00611	35216	2 83965	36
25	29463	3 39406	31370	3 18775	33298	3 00319	35248	2 83702	35
26	29495	3 39042	31402	3 18451	33330	3 00028	35281	2 83439	34
27	29526	3 38679	31434	3 18127	33363	2 99738	35314	2 83176	33
28	29558	3 38317	31466	3 17804	33395	2 99447	35346	2 82914	32
29	29590	3 37955	31498	3 17481	33427	2 99158	35379	2 82653	31
30	29621	3 37594	31530	3 17159	33460	2 98868	35412	2 82391	30
31	29653	3 37234	31562	3 16838	33492	2 98580	35445	2 82130	29
32	29685	3 36875	31594	3 16517	33524	2 98292	35477	2 81870	28
33	29716	3 36516	31626	3 16197	33557	2 98004	35510	2 81610	27
34	29748	3 36158	31658	3 15877	33589	2 97717	35543	2 81350	26
35	29780	3 35800	31690	3 15558	33621	2 97430	35576	2 81091	25
36	29811	3 35443	31722	3 15240	33654	2 97144	35608	2 80833	24
37	29843	3 35087	31754	3 14922	33686	2 96858	35641	2 80574	23
38	29875	3 34732	31786	3 14605	33718	2 96573	35674	2 80316	22
39	29906	3 34377	31818	3 14288	33751	2 96288	35707	2 80059	21
40	29938	3 34023	31850	3 13972	33783	2 96004	35740	2 79802	20
41	29970	3 33670	31882	3 13656	33816	2 95721	35772	2 79545	19
42	30001	3 33317	31914	3 13341	33848	2 95437	35805	2 79289	18
43	30033	3 32965	31946	3 13027	33881	2 95155	35838	2 79033	17
44	30065	3 32614	31978	3 12713	33913	2 94872	35871	2 78778	16
45	30097	3 32264	32010	3 12400	33945	2 94590	35904	2 78523	15
46	30128	3 31914	32042	3 12087	33978	2 94309	35937	2 78269	14
47	30160	3 31565	32074	3 11775	34010	2 94028	35969	2 78014	13
48	30192	3 31216	32106	3 11464	34043	2 93748	36002	2 77761	12
49	30224	3 30868	32139	3 11153	34075	2 93468	36035	2 77507	11
50	30255	3 30521	32171	3 10842	34108	2 93189	36068	2 77254	10
51	30287	3 30174	32203	3 10532	34140	2 92910	36101	2 77002	9
52	30319	3 29829	32235	3 10223	34173	2 92632	36134	2 76750	8
53	30351	3 29483	32267	3 09914	34205	2 92354	36167	2 76498	7
54	30382	3 29139	32299	3 09606	34238	2 92076	36199	2 76247	6
55	30414	3 28795	32331	3 09298	34270	2 91799	36232	2 75996	5
56	30446	3 28452	32363	3 08991	34303	2 91523	36265	2 75746	4
57	30478	3 28109	32396	3 08685	34335	2 91246	36298	2 75496	3
58	30509	3 27767	32428	3 08379	34368	2 90971	36331	2 75246	2
59	30541	3 27426	32460	3 08073	34400	2 90696	36364	2 74997	1
60	30573	3 27085	32492	3 07768	34433	2 90421	36397	2 74748	0
′	cotan 73°	tan 73°	cotan 72°	tan 72°	cotan 71°	tan 71°	cotan 70°	tan 70°	′

′	20°		21°		22°		23°		′
	tan	cotan	tan	cotan	tan	cotan	tan	cotan	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37124	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.29526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2 66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38862	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	cotan	tan	cotan	tan	cotan	tan	cotan	tan	′
	69°		68°		67°		66°		

′	32° tan	32° cotan	33° tan	33° cotan	34° tan	34° cotan	35° tan	35° cotan	′
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65023	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50799	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45138	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72166	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69966	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	cotan 57°	tan	cotan 56°	tan	cotan 55°	tan	cotan 54°	tan	′

′	36°		37°		38°		39°		′
	tan	cotan	tan	cotan	tan	cotan	tan	cotan	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36133	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34070	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	cotan	tan	cotan	tan	cotan	tan	cotan	tan	′
	53°		52°		51°		50°		

′	40°		41°		42°		43°		′
	tan	cotan	tan	cotan	tan	cotan	tan	cotan	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1 18894	87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1 14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1 18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	90834	1 10091	.94071	1.06303	45
16	.84706	1 18055	.87749	1.13961	90887	1 10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	87904	1.13761	.91046	1 09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1 13694	91099	1.09770	.94345	1.05994	40
21	.84956	1 17708	.88007	1 13627	91153	1 09706	.94400	1.05932	39
22	.85006	1 17638	.88059	1 13561	91206	1 09642	94455	1.05870	38
23	.85057	1 17569	88110	1 13494	91259	1 09578	.94510	1.05809	37
24	.85107	1.17500	88162	1 13428	.91313	1 09514	.94565	1.05747	36
25	.85157	1.17430	88214	1 13361	91366	1 09450	.94620	1.05685	35
26	.85207	1 17361	88265	1 13295	91419	1 09386	.94676	1.05624	34
27	.85257	1 17292	88317	1 13228	.91473	1 09322	94731	1.05562	33
28	.85307	1 17223	88369	1 13162	.91526	1 09258	.94786	1.05501	32
29	.85358	1 17154	88421	1 13096	91580	1.09195	.94841	1.05439	31
30	.85408	1 17085	88473	1.13029	91633	1 09131	.94896	1.05378	30
31	.85458	1 17016	88524	1.12963	91687	1 09067	.94952	1.05317	29
32	85509	1 16947	88576	1 12897	91740	1 09003	95007	1.05255	28
33	.85559	1 16878	88628	1 12831	91794	1 08940	.95062	1.05194	27
34	.85609	1 16809	88680	1 12765	91847	1 08876	.95118	1.05133	26
35	.85660	1 16741	88732	1 12699	91901	1 08813	95173	1.05072	25
36	.85710	1 16672	.88784	1 12633	91955	1 08749	.95229	1.05010	24
37	.85761	1 16603	.88836	1.12567	92008	1 08686	.95284	1.04949	23
38	85811	1 16535	.88888	1 12501	92062	1 08622	95340	1.04888	22
39	85862	1 16466	.88940	1 12435	92116	1 08559	95395	1.04827	21
40	.85912	1 16398	88992	1.12369	.92170	1 08496	95451	1.04766	20
41	.85963	1 16329	89045	1 12303	.92224	1 08432	.95506	1.04705	19
42	86014	1 16261	.89097	1 12238	92277	1 08369	95562	1.04644	18
43	86064	1 16192	89149	1 12172	92331	1 08306	95618	1.04583	17
44	86115	1 16124	89201	1 12106	.92385	1 08243	.95673	1.04522	16
45	86166	1 16056	.89253	1 12041	92439	1 08179	95729	1.04461	15
46	86216	1 15987	89306	1 11975	92493	1 08116	95785	1.04401	14
47	.86267	1 15919	89358	1 11909	92547	1 08053	95841	1.04340	13
48	.86318	1 15851	89410	1.11844	92601	1 07990	.95897	1.04279	12
49	.86368	1 15783	.89463	1 11778	92655	1 07927	.95952	1.04218	11
50	.86419	1 15715	89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	89567	1 11648	92763	1 07801	96064	1.04097	9
52	.86521	1 15579	89620	1 11582	92817	1 07738	96120	1.04036	8
53	86572	1 15511	89672	1 11517	92872	1 07676	96176	1 03976	7
54	.86623	1 15443	89725	1 11452	92926	1 07613	96232	1 03915	6
55	86674	1 15375	89777	1 11387	92980	1 07550	96288	1 03855	5
56	86725	1 15308	89830	1 11321	93034	1 07487	96344	1 03794	4
57	86776	1 15240	89883	1 11256	.93088	1.07425	96400	1.03734	3
58	86827	1 15172	.89935	1 11191	93143	1 07362	96457	1 03674	2
59	86878	1 15104	.89988	1 11126	93197	1 07299	96513	1.03613	1
60	86929	1.15037	.90040	1 11061	.93252	1.07237	.96569	1.03553	0
′	cotan	tan	cotan	tan	cotan	tan	cotan	tan	′
	49°		48°		47°		46°		

′	44° tan	44° cotan	′	′	44° tan	44° cotan	′	′	44° tan	44° cotan	′
0	.96569	1.03553	60	21	.97756	1.02295	39	41	.98901	1.01112	19
1	.96625	1.03493	59	22	.97813	1.02236	38	42	.98958	1.01053	18
2	.96681	1.03433	58	23	.97870	1.02176	37	43	.99016	1.00994	17
3	.96738	1.03372	57	24	.97927	1.02117	36	44	.99073	1.00935	16
4	.96794	1.03312	56	25	.97984	1.02057	35	45	.99131	1.00876	15
5	.96850	1.03252	55	26	.98041	1.01998	34	46	.99189	1.00818	14
6	.96907	1.03192	54	27	.98098	1.01939	33	47	.99247	1.00759	13
7	.96963	1.03132	53	28	.98155	1.01879	32	48	.99304	1.00701	12
8	.97020	1.03072	52	29	.98213	1.01820	31	49	.99362	1.00642	11
9	.97076	1.03012	51	30	.98270	1.01761	30	50	.99420	1.00583	10
10	.97133	1.02952	50	31	.98327	1.01702	29	51	.99478	1.00525	9
11	.97189	1.02892	49	32	.98384	1.01642	28	52	.99536	1.00467	8
12	.97246	1.02832	48	33	.98441	1.01583	27	53	.99594	1.00408	7
13	.97302	1.02772	47	34	.98499	1.01524	26	54	.99652	1.00350	6
14	.97359	1.02713	46	35	.98556	1.01465	25	55	.99710	1.00291	5
15	.97416	1.02653	45	36	.98613	1.01406	24	56	.99768	1.00233	4
16	.97472	1.02593	44	37	.98671	1.01347	23	57	.99826	1.00175	3
17	.97529	1.02533	43	38	.98728	1.01288	22	58	.99884	1.00116	2
18	.97586	1.02474	42	39	.98786	1.01229	21	59	.99942	1.00058	1
19	.97643	1.02414	41	40	.98843	1.01170	20	60	1	1	0
20	.97700	1.02355	40								
′	cotan 45°	tan 45°	′	′	cotan 45°	tan 45°	′	′	cotan 45°	tan 45°	′

NATURAL SINES AND COSINES

′	0° sine	0° cosine	′	′	0° sine	0° cosine	′	′	0° sine	0° cosine	′
0	.00000	1	60	21	.00611	.99998	39	41	.01193	.99993	19
1	.00029	1	59	22	.00640	.99998	38	42	.01222	.99993	18
2	.00058	1	58	23	.00669	.99998	37	43	.01251	.99992	17
3	.00087	1	57	24	.00698	.99998	36	44	.01280	.99992	16
4	.00116	1	56	25	.00727	.99997	35	45	.01309	.99991	15
5	.00145	1	55	26	.00756	.99997	34	46	.01338	.99991	14
6	.00175	1	54	27	.00785	.99997	33	47	.01367	.99991	13
7	.00204	1	53	28	.00814	.99997	32	48	.01396	.99990	12
8	.00233	1	52	29	.00844	.99996	31	49	.01425	.99990	11
9	.00262	1	51	30	.00873	.99996	30	50	.01454	.99989	10
10	.00291	1	50	31	.00902	.99996	29	51	.01483	.99989	9
11	.00320	.99999	49	32	.00931	.99996	28	52	.01513	.99989	8
12	.00349	.99999	48	33	.00960	.99995	27	53	.01542	.99988	7
13	.00378	.99999	47	34	.00989	.99995	26	54	.01571	.99988	6
14	.00407	.99999	46	35	.01018	.99995	25	55	.01600	.99987	5
15	.00436	.99999	45	36	.01047	.99995	24	56	.01629	.99987	4
16	.00465	.99999	44	37	.01076	.99994	23	57	.01658	.99986	3
17	.00495	.99999	43	38	.01105	.99994	22	58	.01687	.99986	2
18	.00524	.99999	42	39	.01134	.99994	21	59	.01716	.99985	1
19	.00553	.99998	41	40	.01164	.99993	20	60	.01745	.99985	0
20	.00582	.99998	40								
′	cosine 89°	sine 89°	′	′	cosine 89°	sine 89°	′	′	cosine 89°	sine 89°	′

′	1°		2°		3°		4°		′
	sine	cosine	sine	cosine	sine	cosine	sine	cosine	
0	01745	99985	03490	99939	05234	99863	06976	99756	60
1	01774	99984	03519	99938	05263	99861	07005	99754	59
2	01803	99984	03548	99937	05292	99860	07034	99752	58
3	01832	99983	03577	99936	05321	99858	07063	99750	57
4	01862	99983	03606	99935	05350	99857	07092	99748	56
5	01891	99982	03635	99934	05379	99855	07121	99746	55
6	01920	99982	03664	99933	05408	99854	07150	99744	54
7	01949	99981	03693	99932	05437	99852	07179	99742	53
8	01978	99980	03723	99931	05466	99851	07208	99740	52
9	02007	99980	03752	99930	05495	99849	07237	99738	51
10	02036	99979	03781	99929	05524	.99847	07266	99736	50
11	02065	99979	03810	99927	05553	99846	07295	99734	49
12	02094	99978	03839	99926	05582	99844	07324	99731	48
13	02123	99977	03868	99925	05611	99842	07353	99729	47
14	02152	99977	03897	99924	05640	99841	07382	99727	46
15	02181	99976	03926	99923	05669	99839	07411	99725	45
16	02211	99976	03955	99922	05698	99838	07440	99723	44
17	02240	99975	03984	99921	05727	99836	07469	99721	43
18	02269	99974	04013	99919	05756	99834	07498	99719	42
19	02298	99974	04042	99918	05785	99833	07527	99716	41
20	02327	99973	04071	99917	05814	99831	07556	99714	40
21	02356	99972	04100	99916	05844	99829	07585	99712	39
22	02385	99972	04129	99915	05873	99827	07614	99710	38
23	02414	99971	04159	99913	05902	99826	07643	99708	37
24	02443	99970	04188	99912	05931	99824	07672	99705	36
25	02472	99969	04217	99911	05960	99822	07701	99703	35
26	02501	99969	04246	99910	05989	99821	07730	99701	34
27	02530	99968	04275	99909	06018	99819	07759	99699	33
28	02560	99967	04304	99907	06047	99817	07788	99696	32
29	02589	99966	04333	99906	06076	99815	07817	99694	31
30	02618	99966	04362	99905	06105	99813	07846	99692	30
31	02647	99965	04391	99904	06134	99812	07875	99689	29
32	02676	99964	04420	99902	06163	99810	07904	99687	28
33	02705	99963	04449	99901	06192	99808	07933	99685	27
34	02734	99963	04478	99900	06221	99806	07962	99683	26
35	02763	99962	04507	99898	06250	99804	07991	99680	25
36	02792	99961	04536	99897	06279	99803	08020	99678	24
37	02821	99960	04565	99896	06308	99801	08049	99676	23
38	02850	99959	04594	99894	06337	99799	08078	99673	22
39	02879	99959	04623	99893	06366	99797	08107	99671	21
40	02908	99958	04653	99892	06395	99795	08136	99668	20
1	02938	99957	04682	99890	06424	99793	08165	99666	19
2	02967	99956	04711	99889	06453	99792	08194	99664	18
3	02996	99955	04740	99888	06482	99790	08223	99661	17
4	03025	99954	04769	99886	06511	99788	08252	99659	16
5	03054	99953	04798	99885	06540	99786	08281	99657	15
6	03083	99952	04827	99883	06569	99784	08310	99654	14
7	03112	99952	04856	99882	06598	99782	08339	99652	13
8	03141	99951	04885	99881	06627	99780	08368	99649	12
9	03170	99950	04914	99879	06656	99778	08397	99647	11
0	03199	99949	04943	99878	06685	99776	08426	99644	10
1	03228	99948	04972	99876	06714	99774	08455	99642	9
2	03257	99947	05001	99875	06743	99772	08484	99639	8
3	03286	99946	05030	99873	06773	99770	08513	99637	7
4	03316	99945	05059	99872	06802	99768	08542	99635	6
5	03345	99944	05088	99870	06831	99766	08571	99632	5
6	03374	99943	05117	99869	06860	99764	08600	99630	4
7	03403	99942	05146	99867	06889	99762	08629	99627	3
8	03432	99941	05175	99866	06918	99760	08658	99625	2
9	03461	99940	05205	99864	06947	99758	08687	99622	1
0	03490	99939	05234	99863	06976	99756	08716	99619	0
′	cosine	sine	cosine	sine	cosine	sine	cosine	sine	′
	88°		87°		86°		85°		

5°		6°		7°		8°		
e	cosine	sine	cosine	sine	cosine	sine	cosine	′
16	.99619	.10453	.99452	.12187	.99255	.13917	.99027	60
45	.99617	.10482	.99449	.12216	.99251	.13946	.99023	59
74	.99614	.10511	.99446	.12245	.99248	.13975	.99019	58
03	.99612	.10540	.99443	.12274	.99244	.14004	.99015	57
31	.99609	.10569	.99440	.12302	.99240	.14033	.99011	56
60	.99607	.10597	.99437	.12331	.99237	.14061	.99006	55
89	.99604	.10626	.99434	.12360	.99233	.14090	.99002	54
18	.99602	.10655	.99431	.12389	.99230	.14119	.98998	53
47	.99599	.10684	.99428	.12418	.99226	.14148	.98994	52
76	.99596	.10713	.99424	.12447	.99222	.14177	.98990	51
05	.99594	.10742	.99421	.12476	.99219	.14205	.98986	50
34	.99591	.10771	.99418	.12504	.99215	.14234	.98982	49
63	.99588	.10800	.99415	.12533	.99211	.14263	.98978	48
92	.99586	.10829	.99412	.12562	.99208	.14292	.98973	47
21	.99583	.10858	.99409	.12591	.99204	.14320	.98969	46
50	.99580	.10887	.99406	.12620	.99200	.14349	.98965	45
79	.99578	.10916	.99402	.12649	.99197	.14378	.98961	44
08	.99575	.10945	.99399	.12678	.99193	.14407	.98957	43
37	.99572	.10973	.99396	.12706	.99189	.14436	.98953	42
66	.99570	.11002	.99393	.12735	.99186	.14464	.98948	41
95	.99567	.11031	.99390	.12764	.99182	.14493	.98944	40
24	.99564	.11060	.99386	.12793	.99178	.14522	.98940	39
53	.99562	.11089	99383	.12822	.99175	.14551	.98936	38
82	.99559	.11118	.99380	.12851	.99171	.14580	.98931	37
11	.99556	.11147	.99377	.12880	.99167	.14608	.98927	36
40	.99553	.11176	.99374	.12908	.99163	.14637	.98923	35
69	.99551	.11205	.99370	.12937	.99160	.14666	.98919	34
98	.99548	.11234	.99367	.12966	.99156	.14695	.98914	33
27	.99545	.11263	.99364	.12995	.99152	.14723	.98910	32
56	.99542	.11291	.99360	.13024	.99148	.14752	.98906	31
85	.99540	.11320	.99357	.13053	.99144	.14781	.98902	30
14	.99537	.11349	.99354	.13081	.99141	.14810	.98897	29
42	.99534	.11378	.99351	.13110	.99137	.14838	.98893	28
71	.99531	.11407	99347	.13139	.99133	.14867	.98889	27
00	.99528	.11436	.99344	.13168	.99129	.14896	.98884	26
29	.99526	.11465	.99341	.13197	.99125	.14925	.98880	25
58	.99523	11494	99337	.13226	.99122	.14954	.98876	24
87	99520	.11523	99334	13254	.99118	.14982	.98871	23
16	.99517	11552	99331	.13283	.99114	.15011	.98867	22
45	.99514	.11580	.99327	.13312	.99110	.15040	.98863	21
74	.99511	.11609	.99324	.13341	.99106	.15069	.98858	20
03	.99508	.11638	.99320	.13370	.99102	.15097	.98854	19
32	.99506	.11667	.99317	.13399	.99098	.15126	.98849	18
61	.99503	.11696	.99314	.13427	.99094	.15155	.98845	17
90	.99500	.11725	.99310	.13456	.99091	.15184	.98841	16
19	.99497	.11754	.99307	.13485	.99087	.15212	.98836	15
48	.99494	.11783	.99303	.13514	.99083	.15241	.98832	14
77	99491	.11812	.99300	.13543	.99079	.15270	.98827	13
06	.99488	.11840	.99297	.13572	.99075	.15299	.98823	12
35	.99485	.11869	.99293	.13600	.99071	.15327	.98818	11
64	.99482	.11898	.99290	.13629	.99067	.15356	.98814	10
92	.99479	.11927	.99286	.13658	.99063	.15385	.98809	9
21	.99476	.11956	.99283	.13687	.99059	.15414	.98805	8
50	99473	.11985	.99279	.13716	.99055	.15442	.98800	7
79	.99470	.12014	.99276	.13744	.99051	.15471	.98796	6
08	.99467	.12043	.99272	.13773	.99047	.15500	.98791	5
37	.99464	.12071	.99269	.13802	.99043	.15529	.98787	4
66	.99461	.12100	.99265	.13831	.99039	.15557	.98782	3
95	.99458	.12129	.99262	.13860	.99035	.15586	.98778	2
24	.99455	.12158	.99258	.13889	.99031	.15615	.98773	1
53	.99452	.12187	.99255	.13917	.99027	.15643	.98769	0
ne	sine	cosine	sine	cosine	sine	cosine	sine	′
84°		83°		82°		81°		

′	9° sine	9° cosine	10° sine	10° cosine	11° sine	11° cosine	12° sine	12° cosine	′
0	15643	98769	17365	98481	19081	98163	20791	97815	60
1	15672	98764	17393	98476	19109	98157	20820	97809	59
2	15701	98760	17422	98471	19138	98152	20848	97803	58
3	15730	98755	17451	98466	19167	98146	20877	97797	57
4	15758	98751	17479	98461	19195	98140	20905	97791	56
5	15787	98746	17508	98455	19224	98135	20933	97784	55
6	15816	98741	17537	98450	19252	98129	20962	97778	54
7	15845	98737	17565	98445	19281	98124	20990	97772	53
8	15873	98732	17594	98440	19309	98118	21019	97766	52
9	15902	98728	17623	98435	19338	98112	21047	97760	51
10	15931	98723	17651	98430	19366	98107	21076	97754	50
11	15959	98718	17680	98425	19395	98101	21104	97748	49
12	15988	98714	17708	98420	19423	98096	21132	97742	48
13	16017	98709	17737	98414	19452	98090	21161	97735	47
14	16046	98704	17766	98409	19481	98084	21189	97729	46
15	16074	98700	17794	98404	19509	98079	21218	97723	45
16	16103	98695	17823	98399	19538	98073	21246	97717	44
17	16132	98690	17852	98394	19566	98067	21275	97711	43
18	16160	98686	17880	98389	19595	98061	21303	97705	42
19	16189	.98681	17909	98383	19623	98056	21331	97698	41
20	16218	98676	17937	98378	19652	98050	21360	97692	40
21	16246	98671	17966	98373	19680	98044	21388	97686	39
22	16275	98667	17995	98368	19709	98039	21417	97680	38
23	16304	98662	18023	98362	19737	98033	21445	97673	37
24	16333	98657	18052	98357	19766	98027	21474	97667	36
25	16361	98652	18081	98352	19794	98021	21502	97661	35
26	16390	98648	18109	98347	19823	98016	21530	97655	34
27	16419	98643	18138	98341	19851	98010	21559	97648	33
28	16447	98638	18166	98336	19880	98004	21587	97642	32
29	16476	98633	18195	98331	19908	97997	21616	97636	31
30	16505	98629	18224	98325	19937	97992	21644	97630	30
31	16533	98624	18252	98320	19965	97987	21672	97623	29
32	16562	98619	18281	98315	19994	97981	21701	97617	28
33	16591	98614	18309	98310	20022	97975	21729	97611	27
34	16620	98609	18338	98304	20051	97969	21758	97604	26
35	16648	98604	18367	98299	20079	97963	21786	97598	25
36	16677	98600	18395	98294	20108	97958	21814	97592	24
37	16706	98595	18424	98288	20136	97952	21843	97585	23
38	16734	98590	18452	98283	20165	97946	21871	97579	22
39	16763	98585	18481	98277	20193	97940	21899	97573	21
40	16792	98580	18509	98272	20222	97934	21928	97566	20
41	16820	98575	18538	98267	20250	97928	21956	97560	19
42	16849	98570	18567	98261	20279	97922	21985	97553	18
43	16878	98565	18595	98256	20307	97916	22013	97547	17
44	16906	98561	18624	98250	20336	97910	22041	97541	16
45	16935	98556	18652	98245	20364	97905	22070	97534	15
46	16964	98551	18681	98240	20393	97899	22098	97528	14
47	16992	98546	18710	98234	20421	97893	22126	97521	13
48	17021	98541	18738	98229	20450	97887	22155	97515	12
49	17050	98536	18767	98223	20478	97881	22183	97508	11
50	17078	98531	18795	98218	20507	97875	22212	97502	10
51	17107	98526	18824	98212	20535	97869	22240	97496	9
52	17136	98521	18852	98207	20563	97863	22268	97489	8
53	17164	98516	18881	98201	20592	97857	22297	97483	7
54	17193	98511	18910	98196	20620	97851	22325	97476	6
55	17222	98506	18938	98190	20649	97845	22353	97470	5
56	17250	98501	18967	98185	20677	97839	22382	97463	4
57	17279	98496	18995	98179	20706	97833	22410	97457	3
58	17308	98491	19024	98174	20734	97827	22438	97450	2
59	17336	98486	19052	98168	20763	97821	22467	97444	1
60	17365	98481	19081	98163	20791	97815	22495	97437	0
	cosine 80°	sine 80°	cosine 79°	sine 79°	cosine 78°	sine 78°	cosine 77°	sine 77°	′

′	13° sine	13° cosine	14° sine	14° cosine	15° sine	15° cosine	16° sine	16° cosine	′
0	22495	97437	24192	97030	25882	96593	27564	96126	60
1	22523	97430	24220	97023	25910	96585	27592	96118	59
2	22552	97424	24249	97015	25938	96578	27620	86110	58
3	22580	97417	24277	97008	25966	96570	27648	96102	57
4	22608	97411	24305	97001	25994	96562	27676	96094	56
5	22637	97404	24333	96994	26022	96555	27704	96086	55
6	22665	97398	24362	96987	26050	96547	27731	96078	54
7	22693	97391	24390	96980	26079	96540	27759	96070	53
8	22722	97384	24418	96973	26107	96532	27787	96062	52
9	22750	97378	24446	96966	26135	96524	27815	96054	51
10	22778	97371	24474	96959	26163	96517	27843	96046	50
11	22807	97365	24503	96952	26191	96509	27871	96037	49
12	22835	97358	24531	96945	26219	96502	27899	96029	48
13	22863	97351	24559	96937	26247	96494	27927	96021	47
14	22892	97345	24587	96930	26275	96486	27955	96013	46
15	22920	97338	24615	96923	26303	96479	27983	96005	45
16	22948	97331	24644	96916	26331	96471	28011	95997	44
17	22977	97325	24672	96909	26359	96463	28039	95989	43
18	23005	97318	24700	96902	26387	96456	28067	95981	42
19	23033	97311	24728	96894	26415	96448	28095	95972	41
20	23062	97304	24756	96887	26443	96440	28123	95964	40
21	23090	97298	24784	96880	26471	96433	28150	95956	39
22	23118	97291	24813	96873	26500	96425	28178	95948	38
23	23146	97284	24841	96866	26528	96417	28206	95940	37
24	23175	97278	24869	96858	26556	96410	28234	95931	36
25	23203	97271	24897	96851	26584	96402	28262	95923	35
26	23231	97264	24925	96844	26612	96394	28290	95915	34
27	23260	97257	24954	96837	26640	96386	28318	95907	33
28	23288	97251	24982	96829	26668	96379	28346	95898	32
29	23316	97244	25010	96822	26696	96371	28374	95890	31
30	23345	97237	25038	96815	26724	96363	28402	95882	30
31	23373	97230	25066	96807	26752	96355	28429	95874	29
32	23401	97223	25094	96800	26780	96347	28457	95865	28
33	23429	97217	25122	96793	26808	96340	28485	95857	27
34	23458	07210	25151	96786	26836	96332	28513	95849	26
35	23486	97203	25179	96778	26864	96324	28541	95841	25
36	23514	97196	25207	96771	26892	96316	28569	95832	24
37	23542	97189	25235	96764	26920	96308	28597	95824	23
38	23571	97182	25263	96756	26948	96301	28625	95816	22
39	23599	97176	25291	96749	26976	96293	28652	95807	21
40	23627	97169	25320	96742	27004	96285	28680	95799	20
41	23656	97162	25348	96734	27032	96277	28708	95791	19
42	23684	97155	25376	96727	27060	96269	28736	95782	18
43	23712	97148	25404	96719	27088	96261	28764	95774	17
44	23740	97141	25432	96712	27116	96253	28792	95766	16
45	23769	97134	25460	96705	27144	96246	28820	95757	15
46	23797	97127	25488	96697	27172	96238	28847	95749	14
47	23825	97120	25516	96690	27200	96230	28875	95740	13
48	23853	97113	25545	96682	27228	96222	28903	95732	12
49	23882	97106	25573	96675	27256	96214	28931	95724	11
50	23910	97100	25601	96667	27284	96206	28959	95715	10
51	23938	97093	25629	96660	27312	96198	28987	95707	9
52	23966	97086	25657	96653	27340	96190	29015	95698	8
53	23995	97079	25685	96645	27368	96182	29042	95690	7
54	24023	97072	25713	96638	27396	96174	29070	95681	6
55	24051	97065	25741	96630	27424	96166	29098	95673	5
56	24079	97058	25769	96623	27452	96158	29126	95664	4
57	24108	97051	25798	96615	27480	96150	29154	95656	3
58	24136	97044	25826	96608	27508	96142	29182	95647	2
59	24164	97037	25854	96600	27536	96134	29209	95639	1
60	24192	97030	25882	96593	27564	96126	29237	95630	0
′	cosine 76°	sine 76°	cosine 75°	sine 75°	cosine 74°	sine 74°	cosine 73°	sine 73°	′

′	17° sine	17° cosine	18° sine	18° cosine	19° sine	19° cosine	20° sine	20° cosine	′
0	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	60
1	.29265	.95622	.30929	.95097	.32584	.94542	.34229	.93959	59
2	.29293	.95613	.30957	.95088	.32612	.94533	.34257	.93949	58
3	.29321	.95605	.30985	.95079	.32639	.94523	.34284	.93939	57
4	.29348	.95596	.31012	.95070	.32667	.94514	.34311	.93929	56
5	.29376	.95588	.31040	.95061	.32694	.94504	.34339	.93919	55
6	.29404	.95579	.31068	.95052	.32722	.94495	.34366	.93909	54
7	.29432	.95571	.31095	.95043	.32749	.94485	.34393	.93899	53
8	.29460	.95562	.31123	.95033	.32777	.94476	.34421	.93889	52
9	.29487	.95554	.31151	.95024	.32804	.94466	.34448	.93879	51
10	.29515	.95545	.31178	.95015	.32832	.94457	.34475	.93869	50
11	.29543	.95536	.31206	.95006	.32859	.94447	.34503	.93859	49
12	.29571	.95528	.31233	.94997	.32887	.94438	.34530	.93849	48
13	.29599	.95519	.31261	.94988	.32914	.94428	.34557	.93839	47
14	.29626	.95511	.31289	.94979	.32942	.94418	.34584	.93829	46
15	.29654	.95502	.31316	.94970	.32969	.94409	.34612	.93819	45
16	.29682	.95493	.31344	.94961	.32997	.94399	.34639	.93809	44
17	.29710	.95485	.31372	.94952	.33024	.94390	.34666	.93799	43
18	.29737	.95476	.31399	.94943	.33051	.94380	.34694	.93789	42
19	.29765	.95467	.31427	.94933	.33079	.94370	.34721	.93779	41
20	.29793	.95459	.31454	.94924	.33106	.94361	.34748	.93769	40
21	.29821	.95450	.31482	.94915	.33134	.94351	.34775	.93759	39
22	.29849	.95441	.31510	.94906	.33161	.94342	.34803	.93748	38
23	.29876	.95433	.31537	.94897	.33189	.94332	.34830	.93738	37
24	.29904	.95424	.31565	.94888	.33216	.94322	.34857	.93728	36
25	.29932	.95415	.31593	.94878	.33244	.94313	.34884	.93718	35
26	.29960	.95407	.31620	.94869	.33271	.94303	.34912	.93708	34
27	.29987	.95398	.31648	.94860	.33298	.94293	.34939	.93698	33
28	.30015	.95389	.31675	.94851	.33326	.94284	.34966	.93688	32
29	.30043	.05380	.31703	.94842	.33353	.94274	.34993	.93677	31
30	.30071	.95372	.31730	.94832	.33381	.94264	.35021	.93667	30
31	.30098	.95363	.31758	.94823	.33408	.94254	.35048	.93657	29
32	.30126	.95354	.31786	.94814	.33436	.94245	.35075	.93647	28
33	.30154	.95345	.31813	.94805	.33463	.94235	.35102	.93637	27
34	.30182	.95337	.31841	.94795	.33490	.94225	.35130	.93626	26
35	.30209	.95328	.31868	.94786	.33518	.94215	.35157	.93616	25
36	.30237	.95319	.31896	.94777	.33545	.94206	.35184	.93606	24
37	.30265	.95310	.31923	.94768	.33573	.94196	.35211	.93596	23
38	.30292	.95301	.31951	.94758	.33600	.94186	.35239	.93585	22
39	.30320	.95293	.31979	.94749	.33627	.94176	.35266	.93575	21
40	.30348	.95284	.32006	.94740	.33655	.94167	.35293	.93565	20
41	.30376	.95275	.32034	.94730	.33682	.94157	.35320	.93555	19
42	.30403	.95266	.32061	.94721	.33710	.94147	.35347	.93544	18
43	.30431	.95257	.32089	.94712	.33737	.94137	.35375	.93534	17
44	.30459	.95248	.32116	.94702	.33764	.94127	.35402	.93524	16
45	.30486	.95240	.32144	.94693	.33792	.94118	.35429	.93514	15
46	.30514	.95231	.32171	.94684	.33819	.94108	.35456	.93503	14
47	.30542	.95222	.32199	.94674	.33846	.94098	.35484	.93493	13
48	.30570	.95213	.32227	.94665	.33874	.94088	.35511	.93483	12
49	.30597	.95204	.32254	.94656	.33901	.94078	.35538	.93472	11
50	.30625	.95195	.32282	.94646	.33929	.94068	.35565	.93462	10
51	.30653	.95186	.32309	.94637	.33956	.94058	.35592	.93452	9
52	.30680	.95177	.32337	.94627	.33983	.94049	.35619	.93441	8
53	.30708	.95168	.32364	.94618	.34011	.94039	.35647	.93431	7
54	.30736	.95159	.32392	.94609	.34038	.94029	.35674	.93420	6
55	.30763	.95150	.32419	.94599	.34065	.94019	.35701	.93410	5
56	.30791	.95142	.32447	.94590	.34093	.94009	.35728	.93400	4
57	.30819	.95133	.32474	.94580	.34120	.93999	.35755	.93389	3
58	.30846	.95124	.32502	.94571	.34147	.93989	.35782	.93379	2
59	.30874	.95115	.32529	.94561	.34175	.93979	.35810	.93368	1
60	.30902	.95106	.32557	.94552	.34202	.93969	.35837	.93358	0
′	72° cosine	72° sine	71° cosine	71° sine	70° cosine	70° sine	69° cosine	69° sine	′

′	21°		22°		23°		24°		′
	sine	cosine	sine	cosine	sine	cosine	sine	cosine	
0	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.36596	.93063	38215	.92410	.39822	.91729	.41416	.91020	32
29	36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	36758	.92999	38376	.92343	.39982	.91660	.41575	.90948	26
35	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	cosine	sine	cosine	sine	cosine	sine	cosine	sine	′
	68°		67°		66°		65°		

′	25° sine	25° cosine	26° sine	26° cosine	27° sine	27° cosine	28° sine	28° cosine	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	0
′	64° cosine	64° sine	63° cosine	63° sine	62° cosine	62° sine	61° cosine	61° sine	′

′	29° sine	29° cosine	30° sine	30° cosine	31° sine	31° cosine	32° sine	32° cosine	′
0	.48481	.87462	.50000	.86603	.51504	.85717	.52992	.84805	60
1	.48506	.87448	.50025	.86588	.51529	.85702	.53017	.84789	59
2	.48532	.87434	.50050	.86573	.51554	.85687	.53041	.84774	58
3	.48557	.87420	.50076	.86559	.51579	.85672	.53066	.84759	57
4	.48583	.87406	.50101	.86544	.51604	.85657	.53091	.84743	56
5	.48608	.87391	.50126	.86530	.51628	.85642	.53115	.84728	55
6	.48634	.87377	.50151	.86515	.51653	.85627	.53140	.84712	54
7	.48659	.87363	.50176	.86501	.51678	.85612	.53164	.84697	53
8	.48684	.87349	.50201	.86486	.51703	.85597	.53189	.84681	52
9	.48710	.87335	.50227	.86471	.51728	.85582	.53214	.84666	51
10	.48735	.87321	.50252	.86457	.51753	.85567	.53238	.84650	50
11	.48761	.87306	.50277	.86442	.51778	.85551	.53263	.84635	49
12	.48786	.87292	.50302	.86427	.51803	.85536	.53288	.84619	48
13	.48811	.87278	.50327	.86413	.51828	.85521	.53312	.84604	47
14	.48837	.87264	.50352	.86398	.51852	.85506	.53337	.84588	46
15	.48862	.87250	.50377	.86384	.51877	.85491	.53361	.84573	45
16	.48888	.87235	.50403	.86369	.51902	.85476	.53386	.84557	44
17	.48913	.87221	.50428	.86354	.51927	.85461	.53411	.84542	43
18	.48938	.87207	.50453	.86340	.51952	.85446	.53435	.84526	42
19	.48964	.87193	.50478	.86325	.51977	.85431	.53460	.84511	41
20	.48989	.87178	.50503	.86310	.52002	.85416	.53484	.84495	40
21	.49014	.87164	.50528	.86295	.52026	.85401	.53509	.84480	39
22	.49040	.87150	.50553	.86281	.52051	.85385	.53534	.84464	38
23	.49065	.87136	.50578	.86266	.52076	.85370	.53558	.84448	37
24	.49090	.87121	.50603	.86251	.52101	.85355	.53583	.84433	36
25	.49116	.87107	.50628	.86237	.52126	.85340	.53607	.84417	35
26	.49141	.87093	50654	.86222	.52151	.85325	.53632	.84402	34
27	.49166	.87079	.50679	.86207	.52175	.85310	.53656	.84386	33
28	49192	.87064	.50704	.86192	.52200	.85294	.53681	.84370	32
29	.49217	.87050	.50729	.86178	.52225	.85279	.53705	.84355	31
30	.49242	.87036	.50754	.86163	.52250	.85264	.53730	.84339	30
31	.49268	.87021	.50779	.86148	.52275	.85249	.53754	.84324	29
32	.49293	.87007	50804	.86133	.52299	.85234	.53779	.84308	28
33	.49318	.86993	50829	.86119	.52324	.85218	.53804	.84292	27
34	.49344	.86978	50854	.86104	.52349	.85203	.53828	.84277	26
35	49369	.86964	.50879	.86089	.52374	.85188	.53853	.84261	25
36	.49394	.86949	.50904	.86074	.52399	.85173	.53877	.84245	24
37	.49419	.86935	.50929	.86059	.52423	.85157	.53902	.84230	23
38	.49445	.86921	.50954	.86045	.52448	.85142	.53926	.84214	22
39	.49470	.86906	.50979	.86030	.52473	.85127	.53951	.84198	21
40	.49495	.86892	.51004	.86015	.52498	.85112	.53975	.84182	20
41	49521	.86878	.51029	.86000	.52522	.85096	.54000	.84167	19
42	.49546	.86863	.51054	.85985	.52547	.85081	.54024	.84151	18
43	.49571	.86849	.51079	.85970	.52572	.85066	.54049	.84135	17
44	.49596	.86834	.51104	.85956	.52597	.85051	.54073	.84120	16
45	.49622	.86820	.51129	.85941	.52621	.85035	.54097	.84104	15
46	.49647	.86805	.51154	.85926	.52646	.85020	.54122	.84088	14
47	.49672	.86791	.51179	.85911	.52671	.85005	.54146	.84072	13
48	.49697	.86777	.51204	.85896	.52696	.84989	.54171	.84057	12
49	.49723	.86762	.51229	.85881	.52720	.84974	.54195	.84041	11
50	.49748	.86748	.51254	.85866	.52745	.84959	.54220	.84025	10
51	.49773	.86733	.51279	.85851	.52770	.84943	.54244	.84009	9
52	.49798	.86719	.51304	.85836	.52794	.84928	.54269	.83994	8
53	.49824	.86704	.51329	.85821	.52819	.84913	.54293	.83978	7
54	.49849	.86690	.51354	.85806	.52844	.84897	.54317	.83962	6
55	.49874	.86675	.51379	.85792	.52869	.84882	.54342	.83946	5
56	.49899	.86661	.51404	.85777	.52893	.84866	.54366	.83930	4
57	.49924	.86646	.51429	.85762	.52918	.84851	.54391	.83915	3
58	.49950	.86632	.51454	.85747	.52943	.84836	.54415	.83899	2
59	.49975	.86617	.51479	.85732	.52967	.84820	.54440	.83883	1
60	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	0
′	cosine 60°	sine 60°	cosine 59°	sine 59°	cosine 58°	sine 58°	cosine 57°	sine 57°	′

′	33° sine	33° cosine	34° sine	34° cosine	35° sine	35° cosine	36° sine	36° cosine	′
0	54464	83867	55919	82904	57358	.81915	58779	80902	60
1	54488	83851	55943	82887	57381	81899	58802	80885	59
2	54513	83835	55968	82871	57405	81882	58826	80867	58
3	54537	83819	55992	82855	57429	81865	58849	80850	57
4	54561	83804	56016	82839	57453	81848	58873	80833	56
5	54586	83788	56040	82822	57477	81832	58896	80816	55
6	54610	83772	56064	82806	57501	81815	58920	80799	54
7	54635	83756	56088	82790	57524	81798	58943	80782	53
8	54659	83740	56112	82773	57548	81782	58967	80765	52
9	54683	83724	56136	82757	57572	81765	58990	80748	51
10	54708	83708	56160	82741	57596	81748	59014	80730	50
11	54732	83692	56184	82724	57619	81731	59037	80713	49
12	54756	83676	56208	82708	57643	81714	59061	80696	48
13	54781	83660	56232	82692	57667	81698	59084	80679	47
14	54805	83645	56256	82675	57691	81681	59108	80662	46
15	54829	83629	56280	82659	57715	81664	59131	80644	45
16	54854	83613	56305	82643	57738	81647	59154	80627	44
17	54878	83597	56329	82626	57762	81631	59178	80610	43
18	54902	83581	56353	82610	57786	81614	59201	80593	42
19	54927	83565	56377	82593	57810	81597	59225	80576	41
20	54951	83549	56401	82577	57833	81580	59248	80558	40
21	54975	83533	56425	82561	57857	81563	59272	80541	39
22	54999	83517	56449	82544	57881	81546	59295	80524	38
23	55024	83501	56473	82528	57904	81530	59318	80507	37
24	55048	83485	56497	82511	57928	81513	59342	80489	36
25	55072	83469	56521	82495	57952	81496	59365	80472	35
26	55097	83453	56545	82478	57976	81479	59389	80455	34
27	55121	83437	56569	82462	57999	81462	59412	80438	33
28	55145	83421	56593	82446	58023	81445	59436	80420	32
29	55169	83405	56617	82429	58047	81428	59459	80403	31
30	55194	83389	56641	82413	58070	81412	59482	80386	30
31	55218	83373	56665	82396	58094	81395	59506	80368	29
32	55242	83356	56689	82380	58118	81378	59529	80351	28
33	55266	83340	56713	82363	58141	81361	59552	80334	27
34	55291	83324	56736	82347	58165	81344	59576	80316	26
35	55315	83308	56760	82330	58189	81327	59599	80299	25
36	55339	83292	56784	82314	58212	81310	59622	80282	24
37	55363	83276	56808	82297	58236	81293	59646	80264	23
38	55388	83260	56832	82281	58260	81276	59669	80247	22
39	55412	83244	56856	82264	58283	81259	59693	80230	21
40	55436	83228	56880	82248	58307	81242	59716	80212	20
41	55460	83212	56904	82231	58330	81225	59739	80195	19
42	55484	83195	56928	82214	58354	81208	59763	80178	18
43	55509	83179	56952	82198	58378	81191	59786	80160	17
44	55533	83163	56976	82181	58401	81174	59809	80143	16
45	55557	83147	57000	82165	58425	81157	59832	80125	15
46	55581	83131	57024	82148	58449	81140	59856	80108	14
47	55605	83115	57047	82132	58472	81123	59879	80091	13
48	55630	83098	57071	82115	58496	81106	59902	80073	12
49	55654	83082	57095	82098	58519	81089	59926	80056	11
50	55678	83066	57119	82082	58543	81072	59949	80038	10
51	55702	83050	57143	82065	58567	81055	59972	80021	9
52	55726	83034	57167	82048	58590	81038	59995	80003	8
53	55750	83017	57191	82032	58614	81021	60019	79986	7
54	55775	83001	57215	82015	58637	81004	60042	79968	6
55	55799	82985	57238	81999	58661	80987	60065	79951	5
56	55823	82969	57262	81982	58684	80970	60089	79934	4
57	55847	82953	57286	81965	58708	80953	60112	79916	3
58	55871	82936	57310	81949	58731	80936	60135	79899	2
59	55895	82920	57334	81932	58755	80919	60158	79881	1
60	55919	82904	57358	81915	58779	80902	60182	79864	0
′	56° cosine	56° sine	55° cosine	55° sine	54° cosine	54° sine	53° cosine	53° sine	′

′	37° sine	37° cosine	38° sine	38° cosine	39° sine	39° cosine	40° sine	40° cosine	′
0	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	60
1	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76586	59
2	.60228	.79829	.61612	.78765	.62977	.77678	.64323	.76567	58
3	.60251	.79811	.61635	.78747	.63000	.77660	.64346	.76548	57
4	.60274	.79793	.61658	.78729	.63022	.77641	.64368	.76530	56
5	.60298	.79776	.61681	.78711	.63045	.77623	.64390	.76511	55
6	.60321	.79758	.61704	.78694	.63068	.77605	.64412	.76492	54
7	.60344	.79741	.61726	.78676	.63090	.77586	.64435	.76473	53
8	.60367	.79723	.61749	.78658	.63113	.77568	.64457	.76455	52
9	.60390	.79706	.61772	.78640	.63135	.77550	.64479	.76436	51
10	.60414	.79688	.61795	.78622	.63158	.77531	.64501	.76417	50
11	.60437	.79671	.61818	.78604	.63180	.77513	.64524	.76398	49
12	.60460	.79653	.61841	.78586	.63203	.77494	.64546	.76380	48
13	.60483	.79635	.61864	.78568	.63225	.77476	.64568	.76361	47
14	.60506	.79618	.61887	.78550	.63248	.77458	.64590	.76342	46
15	.60529	.79600	.61909	.78532	.63271	.77439	.64612	.76323	45
16	.60553	.79583	.61932	.78514	.63293	.77421	.64635	76304	44
17	.60576	.79565	.61955	.78496	.63316	.77402	.64657	.76286	43
18	.60599	.79547	.61978	.78478	.63338	.77384	.64679	.76267	42
19	.60622	.79530	.62001	.78460	.63361	.77366	.64701	.76248	41
20	.60645	.79512	.62024	.78442	.63383	.77347	.64723	.76229	40
21	.60668	.79494	.62046	.78424	.63406	.77329	.64746	.76210	39
22	.60691	.79477	.62069	.78405	.63428	.77310	.64768	.76192	38
23	.60714	.79459	.62092	.78387	.63451	.77292	.64790	.76173	37
24	60738	.79441	.62115	.78369	.63473	.77273	.64812	.76154	36
25	.60761	.79424	.62138	.78351	.63496	.77255	.64834	.76135	35
26	.60784	.79406	.62160	.78333	.63518	.77236	.64856	.76116	34
27	.60807	.79388	.62183	.78315	.63540	.77218	.64878	.76097	33
28	.60830	.79371	.62206	.78297	.63563	.77199	.64901	.76078	32
29	.60853	.79353	.62229	.78279	.63585	.77181	.64923	.76059	31
30	.60876	.79335	.62251	.78261	.63608	.77162	.64945	.76041	30
31	.60899	.79318	.62274	.78243	.63630	.77144	.64967	.76022	29
32	.60922	.79300	.62297	.78225	.63653	.77125	.64989	.76003	28
33	.60945	.79282	.62320	.78206	.63675	.77107	.65011	.75984	27
34	.60968	.79264	.62342	.78188	.63698	.77088	.65033	.75965	26
35	.60991	.79247	.62365	.78170	.63720	.77070	.65055	.75946	25
36	.61015	.79229	.62388	.78152	.63742	.77051	.65077	.75927	24
37	.61038	.79211	.62411	.78134	.63765	.77033	.65100	.75908	23
38	.61061	.79193	.62433	.78116	.63787	.77014	.65122	.75889	22
39	.61084	.79176	.62456	.78098	.63810	.76996	.65144	.75870	21
40	.61107	.79158	.62479	.78079	.63832	.76977	.65166	.75851	20
41	.61130	.79140	.62502	.78061	.63854	.76959	.65188	.75832	19
42	.61153	.79122	.62524	.78043	.63877	.76940	.65210	.75813	18
43	.61176	.79105	.62547	.78025	.63899	.76921	.65232	.75794	17
44	.61199	.79087	.62570	.78007	.63922	.76903	.65254	.75775	16
45	.61222	.79069	.62592	.77988	.63944	.76884	.65276	.75756	15
46	.61245	.79051	.62615	.77970	.63966	.76866	.65298	.75738	14
47	.61268	.79033	.62638	.77952	.63989	.76847	.65320	.75719	13
48	.61291	.79016	.62660	.77934	.64011	.76828	.65342	.75700	12
49	.61314	.78998	.62683	.77916	.64033	.76810	.65364	.75680	11
50	.61337	.78980	.62706	.77897	.64056	.76791	.65386	.75661	10
51	.61360	.78962	.62728	.77879	.64078	.76772	.65408	.75642	9
52	.61383	.78944	.62751	.77861	.64100	.76754	.65430	.75623	8
53	.61406	.78926	.62774	.77843	.64123	.76735	.65452	.75604	7
54	.61429	.78908	.62796	.77824	.64145	.76717	.65474	.75585	6
55	.61451	.78891	.62819	.77806	.64167	.76698	.65496	.75566	5
56	.61474	.78873	.62842	.77788	.64190	.76679	.65518	.75547	4
57	.61497	.78855	.62864	.77769	.64212	.76661	.65540	.75528	3
58	.61520	.78837	.62887	.77751	.64234	.76642	.65562	.75509	2
59	.61543	.78819	.62909	.77733	.64256	.76623	.65584	.75490	1
60	.61566	.78801	.62932	.77715	.64279	.76604	.65606	.75471	0
′	cosine 52°	sine 52°	cosine 51°	sine 51°	cosine 50°	sine 50°	cosine 49°	sine 49°	′

′	33° sine	33° cosine	34° sine	34° cosine	35° sine	35° cosine	36° sine	36° cosine	′
0	.54464	.83867	.55919	82904	57358	.81915	.58779	.80902	60
1	.54488	.83851	.55943	.82887	57381	.81899	58802	.80885	59
2	.54513	.83835	.55968	.82871	.57405	.81882	.58826	.80867	58
3	.54537	.83819	.55992	.82855	57429	.81865	.58849	.80850	57
4	.54561	.83804	.56016	82839	57453	.81848	58873	.80833	56
5	.54586	.83788	56040	82822	.57477	.81832	.58896	.80816	55
6	.54610	.83772	.56064	82806	.57501	.81815	.58920	.80799	54
7	.54635	.83756	.56088	82790	57524	.81798	.58943	.80782	53
8	.54659	83740	.56112	82773	57548	.81782	.58967	.80765	52
9	.54683	.83724	.56136	82757	57572	.81765	.58990	.80748	51
10	.54708	.83708	.56160	.82741	57596	.81748	.59014	.80730	50
11	54732	.83692	.56184	.82724	.57619	.81731	.59037	.80713	49
12	54756	83676	.56208	82708	.57643	.81714	.59061	.80696	48
13	54781	.83660	.56232	.82692	.57667	.81698	.59084	.80679	47
14	54805	83645	.56256	.82675	57691	.81681	59108	.80662	46
15	.54829	.83629	56280	.82659	.57715	.81664	59131	.80644	45
16	.54854	83613	.56305	.82643	.57738	.81647	59154	.80627	44
17	54878	83597	.56329	.82626	.57762	.81631	.59178	.80610	43
18	.54902	.83581	.56353	.82610	.57786	.81614	59201	.80593	42
19	.54927	.83565	.56377	.82593	.57810	.81597	.59225	.80576	41
20	.54951	.83549	.56401	.82577	.57833	.81580	.59248	.80558	40
21	.54975	.83533	.56425	.82561	.57857	.81563	.59272	.80541	39
22	.54999	.83517	.56449	.82544	57881	.81546	.59295	.80524	38
23	.55024	83501	.56473	.82528	57904	.81530	59318	.80507	37
24	.55048	.83485	56497	82511	57928	.81513	59342	.80489	36
25	55072	.83469	56521	.82495	.57952	.81496	59365	.80472	35
26	.55097	.83453	56545	82478	.57976	.81479	59389	.80455	34
27	.55121	.83437	56569	82462	.57999	.81462	.59412	.80438	33
28	55145	83421	56593	82446	58023	.81445	59436	.80420	32
29	55169	.83405	.56617	82429	.58047	.81428	.59459	.80403	31
30	.55194	83389	.56641	.82413	58070	.81412	.59482	.80386	30
31	55218	83373	56665	.82396	58094	.81395	59506	.80368	29
32	55242	83356	56689	82380	58118	.81378	59529	.80351	28
33	55266	83340	56713	82363	58141	.81361	59552	80334	27
34	55291	83324	56736	82347	58165	.81344	59576	80316	26
35	55315	83308	56760	82330	58189	.81327	59599	80299	25
36	55339	83292	56784	82314	58212	.81310	59622	.80282	24
37	55363	83276	56808	82297	58236	.81293	59646	.80264	23
38	55388	83260	56832	82281	58260	.81276	59669	.80247	22
39	55412	83244	.56856	82264	58283	.81259	.59693	.80230	21
40	55436	83228	56880	82248	58307	.81242	59716	80212	20
41	55460	83212	56904	82231	58330	.81225	59739	.80195	19
42	55484	83195	56928	82214	58354	.81208	.59763	80178	18
43	55509	83179	56952	.82198	58378	.81191	59786	.80160	17
44	55533	83163	56976	.82181	58401	81174	59809	.80143	16
45	55557	.83147	57000	.82165	58425	.81157	59832	80125	15
46	55581	83131	57024	.82148	58449	.81140	59856	80108	14
47	55605	83115	57047	82132	58472	.81123	59879	80091	13
48	55630	83098	57071	.82115	58496	.81106	59902	.80073	12
49	55654	83082	57095	.82098	58519	.81089	59926	.80056	11
50	55678	83066	57119	.82082	58543	.81072	59949	.80038	10
51	55702	.83050	57143	.82065	58567	.81055	59972	80021	9
52	55726	.83034	57167	.82048	58590	81038	59995	80003	8
53	55750	.83017	57191	.82032	58614	81021	60019	.79986	7
54	55775	83001	57215	.82015	58637	81004	60042	79968	6
55	55799	82985	57238	81999	58661	80987	60065	79951	5
56	55823	82969	57262	.81982	58684	.80970	60089	79934	4
57	55847	82953	57286	81965	58708	80953	60112	79916	3
58	55871	82936	57310	.81949	58731	80936	60135	79899	2
59	55895	.82920	57334	.81932	58755	80919	60158	.79881	1
60	55919	.82904	.57358	.81915	.58779	.80902	.60182	.79864	0
′	cosine 56°	sine 56°	cosine 55°	sine 55°	cosine 54°	sine 54°	cosine 53°	sine 53°	′

′	37° sine	37° cosine	38° sine	38° cosine	39° sine	39° cosine	40° sine	40° cosine	′
0	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	60
1	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76586	59
2	.60228	.79829	.61612	.78765	.62977	.77678	.64323	.76567	58
3	.60251	.79811	.61635	.78747	.63000	.77660	.64346	.76548	57
4	.60274	.79793	.61658	.78729	.63022	.77641	.64368	.76530	56
5	.60298	.79776	.61681	.78711	.63045	.77623	.64390	.76511	55
6	.60321	.79758	.61704	.78694	.63068	.77605	.64412	.76492	54
7	.60344	.79741	.61726	.78676	.63090	.77586	.64435	.76473	53
8	.60367	.79723	.61749	.78658	.63113	.77568	.64457	.76455	52
9	.60390	.79706	.61772	.78640	.63135	.77550	.64479	.76436	51
10	.60414	.79688	.61795	.78622	.63158	.77531	.64501	.76417	50
11	.60437	.79671	.61818	.78604	.63180	.77513	.64524	.76398	49
12	.60460	.79653	.61841	.78586	.63203	.77494	.64546	.76380	48
13	.60483	.79635	.61864	.78568	.63225	.77476	.64568	.76361	47
14	.60506	.79618	.61887	.78550	.63248	.77458	.64590	.76342	46
15	.60529	.79600	.61909	.78532	.63271	.77439	.64612	.76323	45
16	.60553	.79583	.61932	.78514	.63293	.77421	.64635	76304	44
17	.60576	.79565	.61955	.78496	.63316	.77402	.64657	.76286	43
18	.60599	.79547	.61978	.78478	.63338	.77384	.64679	.76267	42
19	.60622	.79530	.62001	.78460	.63361	.77366	.64701	.76248	41
20	.60645	.79512	.62024	.78442	.63383	.77347	.64723	.76229	40
21	.60668	.79494	.62046	.78424	.63406	.77329	.64746	.76210	39
22	.60691	.79477	.62069	.78405	.63428	.77310	.64768	.76192	38
23	.60714	.79459	.62092	.78387	.63451	.77292	.64790	.76173	37
24	.60738	.79441	.62115	.78369	.63473	.77273	.64812	.76154	36
25	.60761	.79424	.62138	.78351	.63496	.77255	.64834	.76135	35
26	60784	.79406	.62160	.78333	.63518	.77236	.64856	.76116	34
27	.60807	.79388	.62183	.78315	.63540	.77218	.64878	.76097	33
28	.60830	.79371	.62206	.78297	.63563	.77199	.64901	.76078	32
29	.60853	.79353	.62229	.78279	.63585	.77181	.64923	.76059	31
30	.60876	.79335	.62251	.78261	.63608	.77162	.64945	.76041	30
31	.60899	.79318	.62274	.78243	.63630	.77144	.64967	.76022	29
32	.60922	.79300	.62297	.78225	.63653	.77125	.64989	.76003	28
33	.60945	.79282	.62320	.78206	.63675	.77107	.65011	.75984	27
34	.60968	.79264	.62342	.78188	.63698	.77088	.65033	.75965	26
35	.60991	.79247	.62365	.78170	.63720	.77070	.65055	.75946	25
36	.61015	.79229	62388	.78152	.63742	.77051	.65077	.75927	24
37	.61038	.79211	.62411	.78134	.63765	.77033	.65100	.75908	23
38	.61061	.79193	.62433	.78116	.63787	.77014	.65122	.75889	22
39	.61084	.79176	.62456	.78098	.63810	.76996	.65144	.75870	21
40	.61107	.79158	.62479	.78079	.63832	.76977	.65166	.75851	20
41	.61130	.79140	.62502	.78061	.63854	.76959	.65188	.75832	19
42	.61153	.79122	.62524	.78043	.63877	.76940	.65210	.75813	18
43	.61176	.79105	.62547	.78025	.63899	.76921	.65232	.75794	17
44	.61199	.79087	.62570	.78007	.63922	.76903	.65254	.75775	16
45	.61222	.79069	.62592	.77988	.63944	.76884	.65276	.75756	15
46	.61245	.79051	.62615	.77970	.63966	.76866	.65298	.75738	14
47	.61268	.79033	.62638	.77952	.63989	.76847	.65320	.75719	13
48	.61291	.79016	.62660	.77934	.64011	.76828	.65342	.75700	12
49	.61314	.78998	.62683	.77916	.64033	.76810	.65364	.75680	11
50	.61337	.78980	.62706	.77897	.64056	.76791	.65386	.75661	10
51	.61360	.78962	.62728	.77879	.64078	.76772	.65408	.75642	9
52	.61383	.78944	.62751	.77861	.64100	.76754	.65430	.75623	8
53	.61406	.78926	.62774	.77843	.64123	.76735	.65452	.75604	7
54	.61429	.78908	.62796	.77824	.64145	.76717	.65474	.75585	6
55	.61451	.78891	.62819	.77806	.64167	.76698	.65496	.75566	5
56	.61474	.78873	.62842	.77788	.64190	.76679	.65518	.75547	4
57	.61497	.78855	.62864	.77769	.64212	.76661	.65540	.75528	3
58	.61520	.78837	.62887	.77751	.64234	.76642	.65562	.75509	2
59	.61543	.78819	.62909	.77733	.64256	.76623	.65584	.75490	1
60	.61566	.78801	.62932	.77715	.64279	.76604	.65606	.75471	0
′	52° cosine	52° sine	51° cosine	51° sine	50° cosine	50° sine	49° cosine	49° sine	′

′	41° sine	41° cosine	42° sine	42° cosine	43° sine	43° cosine	44° sine	44° cosine	′
0	.65606	.75471	.66913	.74314	.68200	.73135	69466	.71934	60
1	.65628	.75452	.66935	.74295	68221	.73116	.69487	.71914	59
2	.65650	.75433	.66956	.74276	68242	.73096	.69508	.71894	58
3	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.65694	.75395	66999	.74237	.68285	.73056	.69549	.71853	56
5	65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.65738	.75356	67043	.74198	.68327	.73016	.69591	.71813	54
7	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.65781	.75318	.67086	.74159	68370	.72976	.69633	.71772	52
9	65803	.75299	67107	.74139	.68391	.72957	.69654	.71752	51
10	65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	65891	.75222	67194	.74061	.68476	.72877	.69737	.71671	47
14	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.65935	75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.65956	.75165	67258	74002	68539	.72817	.69800	.71610	44
17	.65978	.75146	67280	.73983	.68561	.72797	.69821	.71590	43
18	.66000	.75126	67301	73963	68582	.72777	.69842	.71569	42
19	66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	66088	75050	67387	.73885	.68666	72697	69925	.71488	38
23	66109	75030	67409	73865	68688	.72677	.69946	.71468	37
24	66131	75011	67430	73846	68709	.72657	.69966	.71447	36
25	66153	74992	67452	73826	68730	.72637	69987	.71427	35
26	66175	74973	67473	.73806	68751	.72617	.70008	.71407	34
27	66197	74953	67495	73787	68772	.72597	70029	.71386	33
28	66218	74934	67516	73767	.68793	.72577	70049	.71366	32
29	66240	74915	67538	73747	68814	.72557	70070	.71345	31
30	.66262	.74896	.67559	73728	.68835	.72537	.70091	.71325	30
31	66284	.74876	67580	73708	68857	.72517	.70112	.71305	29
32	66306	74857	67602	73688	68878	.72497	.70132	.71284	28
33	66327	74838	67623	73669	68899	72477	.70153	71264	27
34	66349	74818	67645	73649	68920	72457	.70174	.71243	26
35	66371	74799	67666	73629	68941	72437	.70195	.71223	25
36	66393	74780	67688	73610	68962	72417	.70215	.71203	24
37	66414	74760	67709	73590	68983	72397	70236	.71182	23
38	66436	74741	67730	73570	69004	72377	70257	.71162	22
39	66458	74722	67752	73551	69025	72357	.70277	.71141	21
40	66480	74703	.67773	73531	69046	72337	.70298	.71121	20
41	66501	74683	67795	73511	69067	72317	.70319	71100	19
42	66523	74664	67816	73491	69088	72297	70339	.71080	18
43	66545	74644	67837	73472	69109	72277	.70360	.71059	17
44	66566	74625	67859	73452	69130	72257	70381	.71039	16
45	66588	74606	67880	73432	69151	72236	70401	71019	15
46	66610	74586	67901	73413	69172	.72216	70422	.70998	14
47	66632	74567	67923	73393	69193	72196	70443	.70978	13
48	66653	74548	67944	73373	69214	72176	70463	.70957	12
49	66675	74528	67965	73353	69235	72156	.70484	.70937	11
50	66697	74509	67987	73333	69256	72136	.70505	.70916	10
51	66718	.74489	68008	73314	69277	72116	70525	.70896	9
52	66740	74470	68029	73294	69298	72095	70546	.70875	8
53	66762	74451	68051	73274	69319	72075	70567	70855	7
54	66783	74431	68072	73254	69340	72055	70587	.70834	6
55	66805	74412	68093	73234	69361	72035	70608	.70813	5
56	66827	74392	68115	73215	69382	72015	70628	70793	4
57	66848	74373	68136	73195	69403	71995	70649	.70772	3
58	66870	74353	68157	73175	69424	71974	70670	70752	2
59	66891	74334	.68179	.73155	69445	.71954	.70690	.70731	1
60	66913	.74314	68200	.73135	.69466	71934	.70711	.70711	0
′	48° cosine	48° sine	47° cosine	47° sine	46° cosine	46° sine	45° cosine	45° sine	′

SECANTS AND COSECANTS

′	0° sec	0° cosec	1° sec	1° cosec	2° sec	2° cosec	3° sec	3° cosec	′
0	1	Infinite.	1.0001	57.299	1.0006	28.654	1.0014	19.107	60
1	1	3437.70	1.0001	56.359	1.0006	28.417	1.0014	19.002	59
2	1	1718.90	1.0002	55.450	1.0006	28.184	1.0014	18.897	58
3	1	1145.90	1.0002	54.570	1.0006	27.955	1.0014	18.794	57
4	1	859.44	1.0002	53.718	1.0006	27.730	1.0014	18.692	56
5	1	687.55	1.0002	52.891	1.0007	27.508	1.0014	18.591	55
6	1	572.96	1.0002	52.090	1.0007	27.290	1.0015	18.491	54
7	1	491.11	1.0002	51.313	1.0007	27.075	1.0015	18.393	53
8	1	429.72	1.0002	50.558	1.0007	26.864	1.0015	18.295	52
9	1	381.97	1.0002	49.826	1.0007	26.655	1.0015	18.198	51
10	1	343.77	1.0002	49.114	1.0007	26.450	1.0015	18.103	50
11	1	312.52	1.0002	48.422	1.0007	26.249	1.0015	18.008	49
12	1	286.48	1.0002	47.750	1.0007	26.050	1.0016	17.914	48
13	1	264.44	1.0002	47.096	1.0007	25.854	1.0016	17.821	47
14	1	245.55	1.0002	46.460	1.0008	25.661	1.0016	17.730	46
15	1	229.18	1.0002	45.840	1.0008	25.471	1.0016	17.639	45
16	1	214.86	1.0002	45.237	1.0008	25.284	1.0016	17.549	44
17	1	202.22	1.0002	44.650	1.0008	25.100	1.0016	17.460	43
18	1	190.99	1.0002	44.077	1.0008	24.918	1.0017	17.372	42
19	1	180.73	1.0003	43.520	1.0008	24.739	1.0017	17.285	41
20	1	171.89	1.0003	42.976	1.0008	24.562	1.0017	17.198	40
21	1	163.70	1.0003	42.445	1.0008	24.388	1.0017	17.113	39
22	1	156.26	1.0003	41.928	1.0008	24.216	1.0017	17.028	38
23	1	149.47	1.0003	41.423	1.0009	24.047	1.0017	16.944	37
24	1	143.24	1.0003	40.930	1.0009	23.880	1.0018	16.861	36
25	1	137.51	1.0003	40.448	1.0009	23.716	1.0018	16.779	35
26	1	132.22	1.0003	39.978	1.0009	23.553	1.0018	16.698	34
27	1	127.32	1.0003	39.518	1.0009	23.393	1.0018	16.617	33
28	1	122.78	1.0003	39.069	1.0009	23.235	1.0018	16.538	32
29	1	118.54	1.0003	38.631	1.0009	23.079	1.0018	16.459	31
30	1	114.59	1.0003	38.201	1.0009	22.925	1.0019	16.380	30
31	1	110.90	1.0003	37.782	1.0010	22.774	1.0019	16.303	29
32	1	107.43	1.0003	37.371	1.0010	22.624	1.0019	16.226	28
33	1	104.17	1.0004	36.969	1.0010	22.476	1.0019	16.150	27
34	1	101.11	1.0004	36.576	1.0010	22.330	1.0019	16.075	26
35	1	98.223	1.0004	36.191	1.0010	22.186	1.0019	16.000	25
36	1	95.495	1.0004	35.814	1.0010	22.044	1.0020	15.926	24
37	1	92.914	1.0004	35.445	1.0010	21.904	1.0020	15.853	23
38	1.0001	90.469	1.0004	35.084	1.0010	21.765	1.0020	15.780	22
39	1.0001	88.149	1.0004	34.729	1.0011	21.629	1.0020	15.708	21
40	1.0001	85.946	1.0004	34.382	1.0011	21.494	1.0020	15.637	20
41	1.0001	83.849	1.0004	34.042	1.0011	21.360	1.0021	15.566	19
42	1.0001	81.853	1.0004	33.708	1.0011	21.228	1.0021	15.496	18
43	1.0001	79.950	1.0004	33.381	1.0011	21.098	1.0021	15.427	17
44	1.0001	78.133	1.0004	33.060	1.0011	20.970	1.0021	15.358	16
45	1.0001	76.396	1.0005	32.745	1.0011	20.843	1.0021	15.290	15
46	1.0001	74.736	1.0005	32.437	1.0012	20.717	1.0022	15.222	14
47	1.0001	73.146	1.0005	32.134	1.0012	20.593	1.0022	15.155	13
48	1.0001	71.622	1.0005	31.836	1.0012	20.471	1.0022	15.089	12
49	1.0001	70.160	1.0005	31.544	1.0012	20.350	1.0022	15.023	11
50	1.0001	68.757	1.0005	31.257	1.0012	20.230	1.0022	14.958	10
51	1.0001	67.409	1.0005	30.976	1.0012	20.112	1.0023	14.893	9
52	1.0001	66.113	1.0005	30.699	1.0012	19.995	1.0023	14.829	8
53	1.0001	64.866	1.0005	30.428	1.0013	19.880	1.0023	14.765	7
54	1.0001	63.664	1.0005	30.161	1.0013	19.766	1.0023	14.702	6
55	1.0001	62.507	1.0005	29.899	1.0013	19.653	1.0023	14.640	5
56	1.0001	61.391	1.0006	29.641	1.0013	19.541	1.0024	14.578	4
57	1.0001	60.314	1.0006	29.388	1.0013	19.431	1.0024	14.517	3
58	1.0001	59.274	1.0006	29.139	1.0013	19.322	1.0024	14.456	2
59	1.0001	58.270	1.0006	28.894	1.0013	19.214	1.0024	14.395	1
60	1.0001	57.299	1.0006	28.654	1.0014	19.107	1.0024	14.335	0
′	89° cosec	89° sec	88° cosec	88° sec	87° cosec	87° sec	86° cosec	86° sec	′

′	4° sec	4° cosec	5° sec	5° cosec	6° sec	6° cosec	7° sec	7° cosec	′
0	1 0024	14 335	1 0038	11 474	1 0055	9 5668	1 0075	8 2055	60
1	1 0025	14 276	1 0038	11 436	1 0055	9 5404	1 0075	8 1861	59
2	1 0025	14 217	1 0039	11 398	1 0056	9 5141	1 0076	8 1668	58
3	1 0025	14 159	1 0039	11 360	1 0056	9 4880	1 0076	8 1476	57
4	1 0025	14 101	1 0039	11 323	1 0056	9 4620	1 0076	8 1285	56
5	1 0025	14 043	1 0039	11 286	1 0057	9 4362	1 0077	8 1094	55
6	1 0026	13 986	1 0040	11 249	1 0057	9 4105	1 0077	8 0905	54
7	1 0026	13 930	1 0040	11 213	1 0057	9 3850	1 0078	8 0717	53
8	1 0026	13 874	1 0040	11 176	1 0057	9 3596	1 0078	8 0529	52
9	1 0026	13 818	1 0040	11 140	1 0058	9 3343	1 0078	8 0342	51
10	1 0026	13 763	1 0041	11 104	1 0058	9 3092	1 0079	8 0156	50
11	1 0027	13 708	1 0041	11 069	1 0058	9 2842	1 0079	7 9971	49
12	1 0027	13 654	1 0041	11 033	1 0059	9 2593	1 0079	7 9787	48
13	1 0027	13 600	1 0041	10 988	1 0059	9 2346	1 0080	7 9604	47
14	1 0027	13 547	1 0042	10 963	1 0059	9 2100	1 0080	7 9421	46
15	1 0027	13 494	1 0042	10 929	1 0060	9 1855	1 0080	7 9240	45
16	1 0028	13 441	1 0042	10 894	1 0060	9 1612	1 0081	7 9059	44
17	1 0028	13 389	1 0043	10 860	1 0060	9 1370	1 0081	7 8879	43
18	1 0028	13 337	1 0043	10 826	1 0061	9 1129	1 0082	7 8700	42
19	1 0028	13 286	1 0043	10 792	1 0061	9 0890	1 0082	7 8522	41
20	1 0029	13 235	1 0043	10 758	1 0061	9 0651	1 0082	7 8344	40
21	1 0029	13 184	1 0044	10 725	1 0062	9 0414	1 0083	7 8168	39
22	1 0029	13 134	1 0044	10 692	1 0062	9 0179	1 0083	7 7992	38
23	1 0029	13 084	1 0044	10 659	1 0062	8 9944	1 0084	7 7817	37
24	1 0029	13 034	1 0044	10 626	1 0063	8 9711	1 0084	7 7642	36
25	1 0030	12 985	1 0045	10 593	1 0063	8 9479	1 0084	7 7469	35
26	1 0030	12 937	1 0045	10 561	1 0063	8 9248	1 0085	7 7296	34
27	1 0030	12 888	1 0045	10 529	1 0064	8 9018	1 0085	7 7124	33
28	1 0030	12 840	1 0046	10 497	1 0064	8 8790	1 0085	7 6953	32
29	1 0031	12 793	1 0046	10 465	1 0064	8 8563	1 0086	7 6783	31
30	1 0031	12 745	1 0046	10 433	1 0065	8 8337	1 0086	7 6613	30
31	1 0031	12 698	1 0046	10 402	1 0065	8 8112	1 0087	7 6444	29
32	1 0031	12 652	1 0047	10 371	1 0065	8 7888	1 0087	7 6276	28
33	1 0032	12 606	1 0047	10 340	1 0066	8 7665	1 0087	7 6108	27
34	1 0032	12 560	1 0047	10 309	1 0066	8 7444	1 0088	7 5942	26
35	1 0032	12 514	1 0048	10 278	1 0066	8 7223	1 0088	7 5776	25
36	1 0032	12 469	1 0048	10 248	1 0067	8 7004	1 0089	7 5611	24
37	1 0032	12 424	1 0048	10 217	1 0067	8 6786	1 0089	7 5446	23
38	1 0033	12 379	1 0048	10 187	1 0067	8 6569	1 0089	7 5282	22
39	1 0033	12 335	1 0049	10 157	1 0068	8 6353	1 0090	7 5119	21
40	1 0033	12 291	1 0049	10 127	1 0068	8 6138	1 0090	7 4957	20
41	1 0033	12 248	1 0049	10 098	1 0068	8 5924	1 0090	7 4795	19
42	1 0034	12 204	1 0050	10 068	1 0069	8 5711	1 0091	7 4634	18
43	1 0034	12 161	1 0050	10 039	1 0069	8 5499	1 0091	7 4474	17
44	1 0034	12 118	1 0050	10 010	1 0069	8 5289	1 0092	7 4315	16
45	1 0034	12 076	1 0050	9 9812	1 0070	8 5079	1 0092	7 4156	15
46	1 0035	12 034	1 0051	9 9525	1 0070	8 4871	1 0092	7 3998	14
47	1 0035	11 992	1 0051	9 9239	1 0070	8 4663	1 0093	7 3840	13
48	1 0035	11 950	1 0051	9 8955	1 0071	8 4457	1 0093	7 3683	12
49	1 0035	11 909	1 0052	9 8672	1 0071	8 4251	1 0094	7 3527	11
50	1 0036	11 868	1 0052	9 8391	1 0071	8 4046	1 0094	7 3372	10
51	1 0036	11 828	1 0052	9 8112	1 0072	8 3843	1 0094	7 3217	9
52	1 0036	11 787	1 0053	9 7834	1 0072	8 3640	1 0095	7 3063	8
53	1 0036	11 747	1 0053	9 7558	1 0073	8 3439	1 0095	7 2909	7
54	1 0037	11 707	1 0053	9 7283	1 0073	8 3238	1 0096	7 2757	6
55	1 0037	11 668	1 0053	9 7010	1 0073	8 3039	1 0096	7 2604	5
56	1 0037	11 628	1 0054	9 6739	1 0074	8 2840	1 0097	7 2453	4
57	1 0037	11 589	1 0054	9 6469	1 0074	8 2642	1 0097	7 2302	3
58	1 0038	11 550	1 0054	9 6200	1 0074	8 2446	1 0097	7 2152	2
59	1 0038	11 512	1 0055	9 5933	1 0075	8 2250	1 0098	7 2002	1
60	1 0038	11 474	1 0055	9 5668	1 0075	8 2055	1 0098	7 1853	0
	cosec 85°	sec 85°	cosec 84°	sec 84°	cosec 83°	sec 83°	cosec 82°	sec 82°	

′	8° sec	8° cosec	9° sec	9° cosec	10° sec	10° cosec	11° sec	11° cosec	′
0	1.0098	7.1853	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	60
1	1.0099	7.1704	1.0125	6.3807	1.0155	5.7493	1.0188	5.2330	59
2	1.0099	7.1557	1.0125	6.3690	1.0155	5.7398	1.0188	5.2252	58
3	1.0099	7.1409	1.0126	6.3574	1.0156	5.7304	1.0189	5.2174	57
4	1.0100	7.1263	1.0126	6.3458	1.0156	5.7210	1.0189	5.2097	56
5	1.0100	7.1117	1.0127	6.3343	1.0157	5.7117	1.0190	5.2019	55
6	1.0101	7.0972	1.0127	6.3228	1.0157	5.7023	1.0191	5.1942	54
7	1.0101	7.0827	1.0128	6.3113	1.0158	5.6930	1.0191	5.1865	53
8	1.0102	7.0683	1.0128	6.2999	1.0158	5.6838	1.0192	5.1788	52
9	1.0102	7.0539	1.0129	6.2885	1.0159	5.6745	1.0192	5.1712	51
10	1.0102	7.0396	1.0129	6.2772	1.0159	5.6653	1.0193	5.1636	50
11	1.0103	7.0254	1.0130	6.2659	1.0160	5.6561	1.0193	5.1560	49
12	1.0103	7.0112	1.0130	6.2546	1.0160	5.6470	1.0194	5.1484	48
13	1.0104	6.9971	1.0131	6.2434	1.0161	5.6379	1.0195	5.1409	47
14	1.0104	6.9830	1.0131	6.2322	1.0162	5.6288	1.0195	5.1333	46
15	1.0104	6.9690	1.0132	6.2211	1.0162	5.6197	1.0196	5.1258	45
16	1.0105	6.9550	1.0132	6.2100	1.0163	5.6107	1.0196	5.1183	44
17	1.0105	6.9411	1.0133	6.1990	1.0163	5.6017	1.0197	5.1109	43
18	1.0106	6.9273	1.0133	6.1880	1.0164	5.5928	1.0198	5.1034	42
19	1.0106	6.9135	1.0134	6.1770	1.0164	5.5838	1.0198	5.0960	41
20	1.0107	6.8998	1.0134	6.1661	1.0165	5.5749	1.0199	5.0886	40
21	1.0107	6.8861	1.0135	6.1552	1.0165	5.5660	1.0199	5.0812	39
22	1.0107	6.8725	1.0135	6.1443	1.0166	5.5572	1.0200	5.0739	38
23	1.0108	6.8589	1.0136	6.1335	1.0166	5.5484	1.0201	5.0666	37
24	1.0108	6.8454	1.0136	6.1227	1.0167	5.5396	1.0201	5.0593	36
25	1.0109	6.8320	1.0136	6.1120	1.0167	5.5308	1.0202	5.0520	35
26	1.0109	6.8185	1.0137	6.1013	1.0168	5.5221	1.0202	5.0447	34
27	1.0110	6.8052	1.0137	6.0906	1.0169	5.5134	1.0203	5.0375	33
28	1.0110	6.7919	1.0138	6.0800	1.0169	5.5047	1.0204	5.0302	32
29	1.0111	6.7787	1.0138	6.0694	1.0170	5.4960	1.0204	5.0230	31
30	1.0111	6.7655	1.0139	6.0588	1.0170	5.4874	1.0205	5.0158	30
31	1.0111	6.7523	1.0139	6.0483	1.0171	5.4788	1.0205	5.0087	29
32	1.0112	6.7392	1.0140	6.0379	1.0171	5.4702	1.0206	5.0015	28
33	1.0112	6.7262	1.0140	6.0274	1.0172	5.4617	1.0207	4.9944	27
34	1.0113	6.7132	1.0141	6.0170	1.0172	5.4532	1.0207	4.9873	26
35	1.0113	6.7003	1.0141	6.0066	1.0173	5.4447	1.0208	4.9802	25
36	1.0114	6.6874	1.0142	5.9963	1.0174	5.4362	1.0208	4.9732	24
37	1.0114	6.6745	1.0142	5.9860	1.0174	5.4278	1.0209	4.9661	23
38	1.0115	6.6617	1.0143	5.9758	1.0175	5.4194	1.0210	4.9591	22
39	1.0115	6.6490	1.0143	5.9655	1.0175	5.4110	1.0210	4.9521	21
40	1.0015	6.6363	1.0144	5.9554	1.0176	5.4026	1.0211	4.9452	20
41	1.0116	6.6237	1.0144	5.9452	1.0176	5.3943	1.0211	4.9382	19
42	1.0116	6.6111	1.0145	5.9351	1.0177	5.3860	1.0212	4.9313	18
43	1.0117	6.5985	1.0145	5.9250	1.0177	5.3777	1.0213	4.9243	17
44	1.0117	6.5860	1.0146	5.9150	1.0178	5.3695	1.0213	4.9175	16
45	1.0118	6.5736	1.0146	5.9049	1.0179	5.3612	1.0214	4.9106	15
46	1.0118	6.5612	1.0147	5.8950	1.0179	5.3530	1.0215	4.9037	14
47	1.0119	6.5488	1.0147	5.8850	1.0180	5.3449	1.0215	4.8969	13
48	1.0119	6.5365	1.0148	5.8751	1.0180	5.3367	1.0216	4.8901	12
49	1.0119	6.5243	1.0148	5.8652	1.0181	5.3286	1.0216	4.8833	11
50	1.0120	6.5121	1.0149	5.8554	1.0181	5.3205	1.0217	4.8765	10
51	1.0120	6.4999	1.0150	5.8456	1.0182	5.3124	1.0218	4.8697	9
52	1.0121	6.4878	1.0150	5.8358	1.0182	5.3044	1.0218	4.8630	8
53	1.0121	6.4757	1.0151	5.8261	1.0183	5.2963	1.0219	4.8563	7
54	1.0122	6.4637	1.0151	5.8163	1.0184	5.2883	1.0220	4.8496	6
55	1.0122	6.4517	1.0152	5.8067	1.0184	5.2803	1.0220	4.8429	5
56	1.0123	6.4398	1.0152	5.7970	1.0185	5.2724	1.0221	4.8362	4
57	1.0123	6.4279	1.0153	5.7874	1.0185	5.2645	1.0221	4.8296	3
58	1.0124	6.4160	1.0153	5.7778	1.0186	5.2566	1.0222	4.8229	2
59	1.0124	6.4042	1.0154	5.7683	1.0186	5.2487	1.0223	4.8163	1
60	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	1.0223	4.8097	0
′	cosec 81°	sec 81°	cosec 80°	sec 80°	cosec 79°	sec 79°	cosec 78°	sec 78°	′

′	12° sec	12° cosec	13° sec	13° cosec	14° sec	14° cosec	15° sec	15° cosec	′
0	1 0223	4 8097	1 0263	4 4454	1 0306	4 1336	1 0353	3 8637	60
1	1 0224	4 8032	1 0264	4 4398	1 0307	4 1287	1 0353	3 8595	59
2	1 0225	4 7966	1 0264	4 4342	1 0308	4 1239	1 0354	3 8553	58
3	1 0225	4 7901	1 0265	4 4287	1 0308	4 1191	1 0355	3 8512	57
4	1 0226	4 7835	1 0266	4 4231	1 0309	4 1144	1 0356	3 8470	56
5	1 0226	4 7770	1 0266	4 4176	1 0310	4 1096	1 0357	3 8428	55
6	1 0227	4 7706	1 0267	4 4121	1 0311	4 1048	1 0358	3 8387	54
7	1 0228	4 7641	1 0268	4 4065	1 0311	4 1001	1 0358	3 8346	53
8	1 0228	4 7576	1 0268	4 4011	1 0312	4 0953	1 0359	3 8304	52
9	1 0229	4 7512	1 0269	4 3956	1 0313	4 0906	1 0360	3 8263	51
10	1 0230	4 7448	1 0270	4 3901	1 0314	4 0859	1 0361	3 8222	50
11	1 0230	4 7384	1 0271	4 3847	1 0314	4 0812	1 0362	3 8181	49
12	1 0231	4 7320	1 0271	4 3792	1 0315	4 0765	1 0362	3 8140	48
13	1 0232	4 7257	1 0272	4 3738	1 0316	4 0718	1 0363	3 8100	47
14	1 0232	4 7193	1 0273	4 3684	1 0317	4 0672	1 0364	3 8059	46
15	1 0233	4 7130	1 0273	4 3630	1 0317	4 0625	1 0365	3 8018	45
16	1 0234	4 7067	1 0274	4 3576	1 0318	4 0579	1 0366	3 7978	44
17	1 0234	4 7004	1 0275	4 3522	1 0319	4 0532	1 0367	3 7937	43
18	1 0235	4 6942	1 0276	4 3469	1 0320	4 0486	1 0367	3 7897	42
19	1 0235	4 6879	1 0276	4 3415	1 0320	4 0440	1 0368	3 7857	41
20	1 0236	4 6817	1 0277	4 3362	1 0321	4 0394	1 0369	3 7816	40
21	1 0237	4 6754	1 0278	4 3309	1 0322	4 0348	1 0370	3 7776	39
22	1 0237	4 6692	1 0278	4 3256	1 0323	4 0302	1 0371	3 7736	38
23	1 0238	4 6631	1 0279	4 3203	1 0323	4 0256	1 0371	3 7697	37
24	1 0239	4 6569	1 0280	4 3150	1 0324	4 0211	1 0372	3 7657	36
25	1 0239	4 6507	1 0280	4 3098	1 0325	4 0165	1 0373	3 7617	35
26	1 0240	4 6446	1 0281	4 3045	1 0326	4 0120	1 0374	3 7577	34
27	1 0241	4 6385	1 0282	4 2993	1 0327	4 0074	1 0375	3 7538	33
28	1 0241	4 6324	1 0283	4 2941	1 0327	4 0029	1 0376	3 7498	32
29	1 0242	4 6263	1 0283	4 2888	1 0328	3 9984	1 0376	3 7459	31
30	1 0243	4 6202	1 0284	4 2836	1 0329	3 9939	1 0377	3 7420	30
31	1 0243	4 6142	1 0285	4 2785	1 0330	3 9894	1 0378	3 7380	29
32	1 0244	4 6081	1 0285	4 2733	1 0330	3 9850	1 0379	3 7341	28
33	1 0245	4 6021	1 0286	4 2681	1 0331	3 9805	1 0380	3 7302	27
34	1 0245	4 5961	1 0287	4 2630	1 0332	3 9760	1 0381	3 7263	26
35	1 0246	4 5901	1 0288	4 2579	1 0333	3 9716	1 0382	3 7224	25
36	1 0247	4 5841	1 0288	4 2527	1 0334	3 9672	1 0382	3 7186	24
37	1 0247	4 5782	1 0289	4 2476	1 0334	3 9627	1 0383	3 7147	23
38	1 0248	4 5722	1 0290	4 2425	1 0335	3 9583	1 0384	3 7108	22
39	1 0249	4 5663	1 0291	4 2375	1 0336	3 9539	1 0385	3 7070	21
40	1 0249	4 5604	1 0291	4 2324	1 0337	3 9495	1 0386	3 7031	20
41	1 0250	4 5545	1 0292	4 2273	1 0338	3 9451	1 0387	3 6993	19
42	1 0251	4 5486	1 0293	4 2223	1 0338	3 9408	1 0387	3 6955	18
43	1 0251	4 5428	1 0293	4 2173	1 0339	3 9364	1 0388	3 6917	17
44	1 0252	4 5369	1 0294	4 2122	1 0340	3 9320	1 0389	3 6878	16
45	1 0253	4 5311	1 0295	4 2072	1 0341	3 9277	1 0390	3 6840	15
46	1 0253	4 5253	1 0296	4 2022	1 0341	3 9234	1 0391	3 6802	14
47	1 0254	4 5195	1 0296	4 1972	1 0342	3 9190	1 0392	3 6765	13
48	1 0255	4 5137	1 0297	4 1923	1 0343	3 9147	1 0393	3 6727	12
49	1 0255	4 5079	1 0298	4 1873	1 0344	3 9104	1 0393	3 6689	11
50	1 0256	4 5021	1 0299	4 1824	1 0345	3 9061	1 0394	3 6651	10
51	1 0257	4 4964	1 0299	4 1774	1 0345	3 9018	1 0395	3 6614	9
52	1 0257	4 4907	1 0300	4 1725	1 0346	3 8976	1 0396	3 6576	8
53	1 0258	4 4850	1 0301	4 1676	1 0347	3 8933	1 0397	3 6539	7
54	1 0259	4 4793	1 0302	4 1627	1 0348	3 8890	1 0398	3 6502	6
55	1 0260	4 4736	1 0302	4 1578	1 0349	3 8848	1 0399	3 6464	5
56	1 0260	4 4679	1 0303	4 1529	1 0349	3 8805	1 0399	3 6427	4
57	1 0261	4 4623	1 0304	4 1481	1 0350	3 8763	1 0400	3 6390	3
58	1 0262	4 4566	1 0305	4 1432	1 0351	3 8721	1 0401	3 6353	2
59	1 0262	4 4510	1 0305	4 1384	1 0352	3 8679	1 0402	3 6316	1
60	1 0263	4 4454	1 0306	4 1336	1 0353	3 8637	1 0403	3 6279	0
′	77° cosec	77° sec	76° cosec	76° sec	75° cosec	75° sec	74° cosec	74° sec	′

′	16° sec	16° cosec	17° sec	17° cosec	18° sec	18° cosec	19° sec	19° cosec	′
0	1.0403	3.6279	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	60
1	1.0404	3.6243	1.0458	3.4170	1.0516	3.2332	1.0577	3.0690	59
2	1.0405	3.6206	1.0459	3.4138	1.0517	3.2303	1.0578	3.0664	58
3	1.0406	3.6169	1.0460	3.4106	1.0518	3.2274	1.0579	3.0638	57
4	1.0406	3.6133	1.0461	3.4073	1.0519	3.2245	1.0580	3.0612	56
5	1.0407	3.6096	1.0461	3.4041	1.0520	3.2216	1.0581	3.0586	55
6	1.0408	3.6060	1.0462	3.4009	1.0521	3.2188	1.0582	3.0561	54
7	1.0409	3.6024	1.0463	3.3977	1.0522	3.2159	1.0584	3.0535	53
8	1.0410	3.5987	1.0464	3.3945	1.0523	3.2131	1.0585	3.0509	52
9	1.0411	3.5951	1.0465	3.3913	1.0524	3.2102	1.0586	3.0484	51
10	1.0412	3.5915	1.0466	3.3881	1.0525	3.2074	1.0587	3.0458	50
11	1.0413	3.5879	1.0467	3.3849	1.0526	3.2045	1.0588	3.0433	49
12	1.0413	3.5843	1.0468	3.3817	1.0527	3.2017	1.0589	3.0407	48
13	1.0414	3.5807	1.0469	3.3785	1.0528	3.1989	1.0590	3.0382	47
14	1.0415	3.5772	1.0470	3.3754	1.0529	3.1960	1.0591	3.0357	46
15	1.0416	3.5736	1.0471	3.3722	1.0530	3.1932	1.0592	3.0331	45
16	1.0417	3.5700	1.0472	3.3690	1.0531	3.1904	1.0593	3.0306	44
17	1.0418	3.5665	1.0473	3.3659	1.0532	3.1876	1.0594	3.0281	43
18	1.0419	3.5629	1.0474	3.3627	1.0533	3.1848	1.0595	3.0256	42
19	1.0420	3.5594	1.0475	3.3596	1.0534	3.1820	1.0596	3.0231	41
20	1.0420	3.5559	1.0476	3.3565	1.0535	3.1792	1.0598	3.0206	40
21	1.0421	3.5523	1.0477	3.3534	1.0536	3.1764	1.0599	3.0181	39
22	1.0422	3.5488	1.0478	3.3502	1.0537	3.1736	1.0600	3.0156	38
23	1.0423	3.5453	1.0478	3.3471	1.0538	3.1708	1.0601	3.0131	37
24	1.0424	3.5418	1.0479	3.3440	1.0539	3.1681	1.0602	3.0106	36
25	1.0425	3.5383	1.0480	3.3409	1.0540	3.1653	1.0603	3.0081	35
26	1.0426	3.5348	1.0481	3.3378	1.0541	3.1625	1.0604	3.0056	34
27	1.0427	3.5313	1.0482	3.3347	1.0542	3.1598	1.0605	3.0031	33
28	1.0428	3.5279	1.0483	3.3316	1.0543	3.1570	1.0606	3.0007	32
29	1.0428	3.5244	1.0484	3.3286	1.0544	3.1543	1.0607	2.9982	31
30	1.0429	3.5209	1.0485	3.3255	1.0545	3.1515	1.0608	2.9957	30
31	1.0430	3.5175	1.0486	3.3224	1.0546	3.1488	1.0609	2.9933	29
32	1.0431	3.5140	1.0487	3.3194	1.0547	3.1461	1.0611	2.9908	28
33	1.0432	3.5106	1.0488	3.3163	1.0548	3.1433	1.0612	2.9884	27
34	1.0433	3.5072	1.0489	3.3133	1.0549	3.1406	1.0613	2.9859	26
35	1.0434	3.5037	1.0490	3.3102	1.0550	3.1379	1.0614	2.9835	25
36	1.0435	3.5003	1.0491	3.3072	1.0551	3.1352	1.0615	2.9810	24
37	1.0436	3.4969	1.0492	3.3042	1.0552	3.1325	1.0616	2.9786	23
38	1.0437	3.4935	1.0493	3.3011	1.0553	3.1298	1.0617	2.9762	22
39	1.0438	3.4901	1.0494	3.2981	1.0554	3.1271	1.0618	2.9738	21
40	1.0438	3.4867	1.0495	3.2951	1.0555	3.1244	1.0619	2.9713	20
41	1.0439	3.4833	1.0496	3.2921	1.0556	3.1217	1.0620	2.9689	19
42	1.0440	3.4799	1.0497	3.2891	1.0557	3.1190	1.0622	2.9665	18
43	1.0441	3.4766	1.0498	3.2861	1.0558	3.1163	1.0623	2.9641	17
44	1.0442	3.4732	1.0499	3.2831	1.0559	3.1137	1.0624	2.9617	16
45	1.0443	3.4698	1.0500	3.2801	1.0560	3.1110	1.0625	2.9593	15
46	1.0444	3.4665	1.0501	3.2772	1.0561	3.1083	1.0626	2.9569	14
47	1.0445	3.4632	1.0502	3.2742	1.0562	3.1057	1.0627	2.9545	13
48	1.0446	3.4598	1.0503	3.2712	1.0563	3.1030	1.0628	2.9521	12
49	1.0447	3.4565	1.0504	3.2683	1.0565	3.1004	1.0629	2.9497	11
50	1.0448	3.4532	1.0505	3.2653	1.0566	3.0977	1.0630	2.9474	10
51	1.0448	3.4498	1.0506	3.2624	1.0567	3.0951	1.0632	2.9450	9
52	1.0449	3.4465	1.0507	3.2594	1.0568	3.0925	1.0633	2.9426	8
53	1.0450	3.4432	1.0508	3.2565	1.0569	3.0898	1.0634	2.9402	7
54	1.0451	3.4399	1.0509	3.2535	1.0570	3.0872	1.0635	2.9379	6
55	1.0452	3.4366	1.0510	3.2506	1.0571	3.0846	1.0636	2.9355	5
56	1.0453	3.4334	1.0511	3.2477	1.0572	3.0820	1.0637	2.9332	4
57	1.0454	3.4301	1.0512	3.2448	1.0573	3.0793	1.0638	2.9308	3
58	1.0455	3.4268	1.0513	3.2419	1.0574	3.0767	1.0639	2.9285	2
59	1.0456	3.4236	1.0514	3.2390	1.0575	3.0741	1.0641	2.9261	1
60	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	1.0642	2.9238	0
′	cosec 73°	sec	cosec 72°	sec	cosec 71°	sec	cosec 70°	sec	′

′	20°		21°		22°		23°		′
	sec	cosec	sec	cosec	sec	cosec	sec	cosec	
0	1 0642	2 9238	1 0711	2 7904	1 0785	2 6695	1 0864	2 5593	60
1	1 0643	2 9215	1 0713	2 7883	1 0787	2 6675	1 0865	2 5575	59
2	1 0644	2 9191	1 0714	2 7862	1 0788	2 6656	1 0866	2 5558	58
3	1 0645	2 9168	1 0715	2 7841	1 0789	2 6637	1 0868	2 5540	57
4	1 0646	2 9145	1 0716	2 7820	1 0790	2 6618	1 0869	2 5523	56
5	1 0647	2 9122	1 0717	2 7799	1 0792	2 6599	1 0870	2 5506	55
6	1 0648	2 9098	1 0719	2 7778	1 0793	2 6580	1 0872	2 5488	54
7	1 0650	2 9075	1 0720	2 7757	1 0794	2 6561	1 0873	2 5471	53
8	1 0651	2 9052	1 0721	2 7736	1 0795	2 6542	1 0874	2 5453	52
9	1 0652	2 9029	1 0722	2 7715	1 0797	2 6523	1 0876	2 5436	51
10	1 0653	2 9006	1 0723	2 7694	1 0798	2 6504	1 0877	2 5419	50
11	1 0654	2 8983	1 0725	2 7674	1 0799	2 6485	1 0878	2 5402	49
12	1 0655	2 8960	1 0726	2 7653	1 0801	2 6466	1 0880	2 5384	48
13	1 0656	2 8937	1 0727	2 7632	1 0802	2 6447	1 0881	2 5367	47
14	1 0658	2 8915	1 0728	2 7611	1 0803	2 6428	1 0882	2 5350	46
15	1 0659	2 8892	1 0729	2 7591	1 0804	2 6410	1 0884	2 5333	45
16	1 0660	2 8869	1 0731	2 7570	1 0806	2 6391	1 0885	2 5316	44
17	1 0661	2 8846	1 0732	2 7550	1 0807	2 6372	1 0886	2 5299	43
18	1 0662	2 8824	1 0733	2 7529	1 0808	2 6353	1 0888	2 5281	42
19	1 0663	2 8801	1 0734	2 7509	1 0810	2 6335	1 0889	2 5264	41
20	1 0664	2 8778	1 0736	2 7488	1 0811	2 6316	1 0891	2 5247	40
21	1 0666	2 8756	1 0737	2 7468	1 0812	2 6297	1 0892	2 5230	39
22	1 0667	2 8733	1 0738	2 7447	1 0813	2 6279	1 0893	2 5213	38
23	1 0668	2 8711	1 0739	2 7427	1 0815	2 6260	1 0895	2 5196	37
24	1 0669	2 8688	1 0740	2 7406	1 0816	2 6242	1 0896	2 5179	36
25	1 0670	2 8666	1 0742	2 7386	1 0817	2 6223	1 0897	2 5163	35
26	1 0671	2 8644	1 0743	2 7366	1 0819	2 6205	1 0899	2 5146	34
27	1 0673	2 8621	1 0744	2 7346	1 0820	2 6186	1 0900	2 5129	33
28	1 0674	2 8599	1 0745	2 7325	1 0821	2 6168	1 0902	2 5112	32
29	1 0675	2 8577	1 0747	2 7305	1 0823	2 6150	1 0903	2 5095	31
30	1 0676	2 8554	1 0748	2 7285	1 0824	2 6131	1 0904	2 5078	30
31	1 0677	2 8532	1 0749	2 7265	1 0825	2 6113	1 0906	2 5062	29
32	1 0678	2 8510	1 0750	2 7245	1 0826	2 6095	1 0907	2 5045	28
33	1 0679	2 8488	1 0751	2 7225	1 0828	2 6076	1 0908	2 5028	27
34	1 0681	2 8466	1 0753	2 7205	1 0829	2 6058	1 0910	2 5011	26
35	1 0682	2 8444	1 0754	2 7185	1 0830	2 6040	1 0911	2 4995	25
36	1 0683	2 8422	1 0755	2 7165	1 0832	2 6022	1 0913	2 4978	24
37	1 0684	2 8400	1 0756	2 7145	1 0833	2 6003	1 0914	2 4961	23
38	1 0685	2 8378	1 0758	2 7125	1 0834	2 5985	1 0915	2 4945	22
39	1 0686	2 8356	1 0759	2 7105	1 0836	2 5967	1 0917	2 4928	21
40	1 0688	2 8334	1 0760	2 7085	1 0837	2 5949	1 0918	2 4912	20
41	1 0689	2 8312	1 0761	2 7065	1 0838	2 5931	1 0920	2 4895	19
42	1 0690	2 8290	1 0763	2 7045	1 0840	2 5913	1 0921	2 4879	18
43	1 0691	2 8269	1 0764	2 7026	1 0841	2 5895	1 0922	2 4862	17
44	1 0692	2 8247	1 0765	2 7006	1 0842	2 5877	1 0924	2 4846	16
45	1 0694	2 8225	1 0766	2 6986	1 0844	2 5859	1 0925	2 4829	15
46	1 0695	2 8204	1 0768	2 6967	1 0845	2 5841	1 0927	2 4813	14
47	1 0696	2 8182	1 0769	2 6947	1 0846	2 5823	1 0928	2 4797	13
48	1 0697	2 8160	1 0770	2 6927	1 0847	2 5805	1 0929	2 4780	12
49	1 0698	2 8139	1 0771	2 6908	1 0849	2 5787	1 0931	2 4764	11
50	1 0699	2 8117	1 0773	2 6888	1 0850	2 5770	1 0932	2 4748	10
51	1 0701	2 8096	1 0774	2 6869	1 0851	2 5752	1 0934	2 4731	9
52	1 0702	2 8074	1 0775	2 6849	1 0853	2 5734	1 0935	2 4715	8
53	1 0703	2 8053	1 0776	2 6830	1 0854	2 5716	1 0936	2 4699	7
54	1 0704	2 8032	1 0778	2 6810	1 0855	2 5699	1 0938	2 4683	6
55	1 0705	2 8010	1 0779	2 6791	1 0857	2 5681	1 0939	2 4666	5
56	1 0707	2 7989	1 0780	2 6772	1 0858	2 5663	1 0941	2 4650	4
57	1 0708	2 7968	1 0781	2 6752	1 0859	2 5646	1 0942	2 4634	3
58	1 0709	2 7947	1 0783	2 6733	1 0861	2 5628	1 0943	2 4618	2
59	1 0710	2 7925	1 0784	2 6714	1 0862	2 5610	1 0945	2 4602	1
60	1 0711	2 7904	1 0785	2 6695	1 0864	2 5593	1 0946	2 4586	0
′	cosec	sec	cosec	sec	cosec	sec	cosec	sec	′
	69°		68°		67°		66°		

′	24° sec	24° cosec	25° sec	25° cosec	26° sec	26° cosec	27° sec	27° cosec	′
0	1 0946	2 4586	1 1034	2 3662	1 1126	2 2812	1 1223	2 2027	60
1	1 0948	2 4570	1 1035	2 3647	1 1127	2 2798	1 1225	2 2014	59
2	1 0949	2 4554	1 1037	2 3632	1 1129	2 2784	1 1226	2 2002	58
3	1 0951	2 4538	1 1038	2 3618	1 1131	2 2771	1 1228	2 1989	57
4	1 0952	2 4522	1 1040	2 3603	1 1132	2 2757	1 1230	2 1977	56
5	1 0953	2 4506	1 1041	2 3588	1 1134	2 2744	1 1231	2 1964	55
6	1 0955	2 4490	1 1043	2 3574	1 1135	2 2730	1 1233	2 1952	54
7	1 0956	2 4474	1 1044	2 3559	1 1137	2 2717	1 1235	2 1939	53
8	1 0958	2 4458	1 1046	2 3544	1 1139	2 2703	1 1237	2 1927	52
9	1 0959	2 4442	1 1047	2 3530	1 1140	2 2690	1 1238	2 1914	51
10	1 0961	2 4426	1 1049	2 3515	1 1142	2 2676	1 1240	2 1902	50
11	1 0962	2 4411	1 1050	2 3501	1 1143	2 2663	1 1242	2 1889	49
12	1 0963	2 4395	1 1052	2 3486	1 1145	2 2650	1 1243	2 1877	48
13	1 0965	2 4379	1 1053	2 3472	1 1147	2 2636	1 1245	2 1865	47
14	1 0966	2 4363	1 1055	2 3457	1 1148	2 2623	1 1247	2 1852	46
15	1 0968	2 4347	1 1056	2 3443	1 1150	2 2610	1 1248	2 1840	45
16	1 0969	2 4332	1 1058	2 3428	1 1151	2 2596	1 1250	2 1828	44
17	1 0971	2 4316	1 1059	2 3414	1 1153	2 2583	1 1252	2 1815	43
18	1 0972	2 4300	1 1061	2 3399	1 1155	2 2570	1 1253	2 1803	42
19	1 0973	2 4285	1 1062	2 3385	1 1156	2 2556	1 1255	2 1791	41
20	1 0975	2 4269	1 1064	2 3371	1 1158	2 2543	1 1257	2 1778	40
21	1 0976	2 4254	1 1065	2 3356	1 1159	2 2530	1 1258	2 1766	39
22	1 0978	2 4238	1 1067	2 3342	1 1161	2 2517	1 1260	2 1754	38
23	1 0979	2 4222	1 1068	2 3328	1 1163	2 2503	1 1262	2 1742	37
24	1 0981	2 4207	1 1070	2 3313	1 1164	2 2490	1 1264	2 1730	36
25	1 0982	2 4191	1 1072	2 3299	1 1166	2 2477	1 1265	2 1717	35
26	1 0984	2 4176	1 1073	2 3285	1 1167	2 2464	1 1267	2 1705	34
27	1 0985	2 4160	1 1075	2 3271	1 1169	2 2451	1 1269	2 1693	33
28	1 0986	2 4145	1 1076	2 3256	1 1171	2 2438	1 1270	2 1681	32
29	1 0988	2 4130	1 1078	2 3242	1 1172	2 2425	1 1272	2 1669	31
30	1 0989	2 4114	1 1079	2 3228	1 1174	2 2411	1 1274	2 1657	30
31	1 0991	2 4099	1 1081	2 3214	1 1176	2 2398	1 1275	2 1645	29
32	1 0992	2 4083	1 1082	2 3200	1 1177	2 2385	1 1277	2 1633	28
33	1 0994	2 4068	1 1084	2 3186	1 1179	2 2372	1 1279	2 1620	27
34	1 0995	2 4053	1 1085	2 3172	1 1180	2 2359	1 1281	2 1608	26
35	1 0997	2 4037	1 1087	2 3158	1 1182	2 2346	1 1282	2 1596	25
36	1 0998	2 4022	1 1088	2 3143	1 1184	2 2333	1 1284	2 1584	24
37	1 1000	2 4007	1 1090	2 3129	1 1185	2 2320	1 1286	2 1572	23
38	1 1001	2 3992	1 1092	2 3115	1 1187	2 2307	1 1287	2 1560	22
39	1 1003	2 3976	1 1093	2 3101	1 1189	2 2294	1 1289	2 1548	21
40	1 1004	2 3961	1 1095	2 3087	1 1190	2 2282	1 1291	2 1536	20
41	1 1005	2 3946	1 1096	2 3073	1 1192	2 2269	1 1293	2 1525	19
42	1 1007	2 3931	1 1098	2 3059	1 1193	2 2256	1 1294	2 1513	18
43	1 1008	2 3916	1 1099	2 3046	1 1195	2 2243	1 1296	2 1501	17
44	1 1010	2 3901	1 1101	2 3032	1 1197	2 2230	1 1298	2 1489	16
45	1 1011	2 3886	1 1102	2 3018	1 1198	2 2217	1 1299	2 1477	15
46	1 1013	2 3871	1 1104	2 3004	1 1200	2 2204	1 1301	2 1465	14
47	1 1014	2 3856	1 1106	2 2990	1 1202	2 2192	1 1303	2 1453	13
48	1 1016	2 3841	1 1107	2 2976	1 1203	2 2179	1 1305	2 1441	12
49	1 1017	2 3826	1 1109	2 2962	1 1205	2 2166	1 1306	2 1430	11
50	1 1019	2 3811	1 1110	2 2949	1 1207	2 2153	1 1308	2 1418	10
51	1 1020	2 3796	1 1112	2 2935	1 1208	2 2141	1 1310	2 1406	9
52	1 1022	2 3781	1 1113	2 2921	1 1210	2 2128	1 1312	2 1394	8
53	1 1023	2 3766	1 1115	2 2907	1 1212	2 2115	1 1313	2 1382	7
54	1 1025	2 3751	1 1116	2 2894	1 1213	2 2103	1 1315	2 1371	6
55	1 1026	2 3736	1 1118	2 2880	1 1215	2 2090	1 1317	2 1359	5
56	1 1028	2 3721	1 1120	2 2866	1 1217	2 2077	1 1319	2 1347	4
57	1 1029	2 3706	1 1121	2 2853	1 1218	2 2065	1 1320	2 1335	3
58	1 1031	2 3691	1 1123	2 2839	1 1220	2 2052	1 1322	2 1324	2
59	1 1032	2 3677	1 1124	2 2825	1 1222	2 2039	1 1324	2 1312	1
60	1 1034	2 3662	1 1126	2 2812	1 1223	2 2027	1 1326	2 1300	0
′	65° cosec	65° sec	64° cosec	64° sec	63° cosec	63° sec	62° cosec	62° sec	′

′	36° sec	36° cosec	37° sec	37° cosec	38° sec	38° cosec	39° sec	39° cosec	′
0	1 2361	1 7013	1 2521	1 6616	1 2690	1 6243	1 2867	1 5890	60
1	1 2363	1 7006	1 2524	1 6610	1 2693	1 6237	1 2871	1 5884	59
2	1 2366	1 6999	1 2527	1 6603	1 2696	1 6231	1 2874	1 5879	58
3	1 2368	1 6993	1 2530	1 6597	1 2699	1 6224	1 2877	1 5873	57
4	1 2371	1 6986	1 2532	1 6591	1 2702	1 6218	1 2880	1 5867	56
5	1 2374	1 6979	1 2535	1 6584	1 2705	1 6212	1 2883	1 5862	55
6	1 2376	1 6972	1 2538	1 6578	1 2707	1 6206	1 2886	1 5856	54
7	1 2379	1 6965	1 2541	1 6572	1 2710	1 6200	1 2889	1 5850	53
8	1 2382	1 6959	1 2543	1 6565	1 2713	1 6194	1 2892	1 5845	52
9	1 2384	1 6952	1 2546	1 6559	1 2716	1 6188	1 2895	1 5839	51
10	1 2387	1 6945	1 2549	1 6552	1 2719	1 6182	1 2898	1 5833	50
11	1 2389	1 6938	1 2552	1 6546	1 2722	1 6176	1 2901	1 5828	49
12	1 2392	1 6932	1 2554	1 6540	1 2725	1 6170	1 2904	1 5822	48
13	1 2395	1 6925	1 2557	1 6533	1 2728	1 6164	1 2907	1 5816	47
14	1 2397	1 6918	1 2560	1 6527	1 2731	1 6159	1 2910	1 5811	46
15	1 2400	1 6912	1 2563	1 6521	1 2734	1 6153	1 2913	1 5805	45
16	1 2403	1 6905	1 2565	1 6514	1 2737	1 6147	1 2916	1 5799	44
17	1 2405	1 6898	1 2568	1 6508	1 2739	1 6141	1 2919	1 5794	43
18	1 2408	1 6891	1 2571	1 6502	1 2742	1 6135	1 2922	1 5788	42
19	1 2411	1 6885	1 2574	1 6496	1 2745	1 6129	1 2926	1 5783	41
20	1 2413	1 6878	1 2577	1 6489	1 2748	1 6123	1 2929	1 5777	40
21	1 2416	1 6871	1 2579	1 6483	1 2751	1 6117	1 2932	1 5771	39
22	1 2419	1 6865	1 2582	1 6477	1 2754	1 6111	1 2935	1 5766	38
23	1 2421	1 6858	1 2585	1 6470	1 2757	1 6105	1 2938	1 5760	37
24	1 2424	1 6851	1 2588	1 6464	1 2760	1 6099	1 2941	1 5755	36
25	1 2427	1 6845	1 2591	1 6458	1 2763	1 6093	1 2944	1 5749	35
26	1 2429	1 6838	1 2593	1 6452	1 2766	1 6087	1 2947	1 5743	34
27	1 2432	1 6831	1 2596	1 6445	1 2769	1 6081	1 2950	1 5738	33
28	1 2435	1 6825	1 2599	1 6439	1 2772	1 6077	1 2953	1 5732	32
29	1 2437	1 6818	1 2602	1 6433	1 2775	1 6070	1 2956	1 5727	31
30	1 2440	1 6812	1 2605	1 6427	1 2778	1 6064	1 2960	1 5721	30
31	1 2443	1 6805	1 2607	1 6420	1 2781	1 6058	1 2963	1 5716	29
32	1 2445	1 6798	1 2610	1 6414	1 2784	1 6052	1 2966	1 5710	28
33	1 2448	1 6792	1 2613	1 6408	1 2787	1 6046	1 2969	1 5705	27
34	1 2451	1 6785	1 2616	1 6402	1 2790	1 6040	1 2972	1 5699	26
35	1 2453	1 6779	1 2619	1 6396	1 2793	1 6034	1 2975	1 5694	25
36	1 2456	1 6772	1 2622	1 6389	1 2795	1 6029	1 2978	1 5688	24
37	1 2459	1 6766	1 2624	1 6383	1 2798	1 6023	1 2981	1 5683	23
38	1 2461	1 6759	1 2627	1 6377	1 2801	1 6017	1 2985	1 5677	22
39	1 2464	1 6752	1 2630	1 6371	1 2804	1 6011	1 2988	1 5672	21
40	1 2467	1 6746	1 2633	1 6365	1 2807	1 6005	1 2991	1 5666	20
41	1 2470	1 6739	1 2636	1 6359	1 2810	1 6000	1 2994	1 5661	19
42	1 2472	1 6733	1 2639	1 6352	1 2813	1 5994	1 2997	1 5655	18
43	1 2475	1 6726	1 2641	1 6346	1 2816	1 5988	1 3000	1 5650	17
44	1 2478	1 6720	1 2644	1 6340	1 2819	1 5982	1 3003	1 5644	16
45	1 2480	1 6713	1 2647	1 6334	1 2822	1 5976	1 3006	1 5639	15
46	1 2483	1 6707	1 2650	1 6328	1 2825	1 5971	1 3010	1 5633	14
47	1 2486	1 6700	1 2653	1 6322	1 2828	1 5965	1 3013	1 5628	13
48	1 2488	1 6694	1 2656	1 6316	1 2831	1 5959	1 3016	1 5622	12
49	1 2490	1 6687	1 2659	1 6309	1 2834	1 5953	1 3019	1 5617	11
50	1 2494	1 6681	1 2661	1 6303	1 2837	1 5947	1 3022	1 5611	10
51	1 2497	1 6674	1 2664	1 6297	1 2840	1 5942	1 3025	1 5606	9
52	1 2499	1 6668	1 2667	1 6291	1 2843	1 5936	1 3029	1 5600	8
53	1 2502	1 6661	1 2670	1 6285	1 2846	1 5930	1 3032	1 5595	7
54	1 2505	1 6655	1 2673	1 6279	1 2849	1 5924	1 3035	1 5590	6
55	1 2508	1 6648	1 2676	1 6273	1 2852	1 5919	1 3038	1 5584	5
56	1 2510	1 6642	1 2679	1 6267	1 2855	1 5913	1 3041	1 5579	4
57	1 2513	1 6636	1 2681	1 6261	1 2858	1 5907	1 3044	1 5573	3
58	1 2516	1 6629	1 2684	1 6255	1 2861	1 5901	1 3048	1 5568	2
59	1 2519	1 6623	1 2687	1 6249	1 2864	1 5896	1 3051	1 5563	1
60	1 2521	1 6616	1 2690	1 6243	1 2867	1 5890	1 3054	1 5557	0
′	cosec 53°	sec 53°	cosec 52°	sec 52°	cosec 51°	sec 51°	cosec 50°	sec 50°	′

′	40° sec	40° cosec	41° sec	41° cosec	42° sec	42° cosec	43° sec	43° cosec	′
0	1.3054	1.5557	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	60
1	1.3057	1.5552	1.3253	1.5237	1.3460	1.4940	1.3677	1.4658	59
2	1.3060	1.5546	1.3257	1.5232	1.3463	1.4935	1.3681	1.4654	58
3	1.3064	1.5541	1.3260	1.5227	1.3467	1.4930	1.3684	1.4649	57
4	1.3067	1.5536	1.3263	1.5222	1.3470	1.4925	1.3688	1.4644	56
5	1.3070	1.5530	1.3267	1.5217	1.3474	1.4921	1.3692	1.4640	55
6	1.3073	1.5525	1.3270	1.5212	1.3477	1.4916	1.3695	1.4635	54
7	1.3076	1.5520	1.3274	1.5207	1.3481	1.4911	1.3699	1.4631	53
8	1.3080	1.5514	1.3277	1.5202	1.3485	1.4906	1.3703	1.4626	52
9	1.3083	1.5509	1.3280	1.5197	1.3488	1.4901	1.3707	1.4622	51
10	1.3086	1.5503	1.3284	1.5192	1.3492	1.4897	1.3710	1.4617	50
11	1.3089	1.5498	1.3287	1.5187	1.3495	1.4892	1.3714	1.4613	49
12	1.3092	1.5493	1.3290	1.5182	1.3499	1.4887	1.3718	1.4608	48
13	1.3096	1.5487	1.3294	1.5177	1.3502	1.4882	1.3722	1.4604	47
14	1.3099	1.5482	1.3297	1.5171	1.3506	1.4877	1.3725	1.4599	46
15	1.3102	1.5477	1.3301	1.5166	1.3509	1.4873	1.3729	1.4595	45
16	1.3105	1.5471	1.3304	1.5161	1.3513	1.4868	1.3733	1.4590	44
17	1.3109	1.5466	1.3307	1.5156	1.3517	1.4863	1.3737	1.4586	43
18	1.3112	1.5461	1.3311	1.5151	1.3520	1.4858	1.3740	1.4581	42
19	1.3115	1.5456	1.3314	1.5146	1.3524	1.4854	1.3744	1.4577	41
20	1.3118	1.5450	1.3318	1.5141	1.3527	1.4849	1.3748	1.4572	40
21	1.3121	1.5445	1.3321	1.5136	1.3531	1.4844	1.3752	1.4568	39
22	1.3125	1.5440	1.3324	1.5131	1.3534	1.4839	1.3756	1.4563	38
23	1.3128	1.5434	1.3328	1.5126	1.3538	1.4835	1.3759	1.4559	37
24	1.3131	1.5429	1.3331	1.5121	1.3542	1.4830	1.3763	1.4554	36
25	1.3134	1.5424	1.3335	1.5116	1.3545	1.4825	1.3767	1.4550	35
26	1.3138	1.5419	1.3338	1.5111	1.3549	1.4821	1.3771	1.4545	34
27	1.3141	1.5413	1.3342	1.5106	1.3552	1.4816	1.3774	1.4541	33
28	1.3144	1.5408	1.3345	1.5101	1.3556	1.4811	1.3778	1.4536	32
29	1.3148	1.5403	1.3348	1.5096	1.3560	1.4806	1.3782	1.4532	31
30	1.3151	1.5398	1.3352	1.5092	1.3563	1.4802	1.3786	1.4527	30
31	1.3154	1.5392	1.3355	1.5087	1.3567	1.4797	1.3790	1.4523	29
32	1.3157	1.5387	1.3359	1.5082	1.3571	1.4792	1.3794	1.4518	28
33	1.3161	1.5382	1.3362	1.5077	1.3574	1.4788	1.3797	1.4514	27
34	1.3164	1.5377	1.3366	1.5072	1.3578	1.4783	1.3801	1.4510	26
35	1.3167	1.5371	1.3369	1.5067	1.3581	1.4778	1.3805	1.4505	25
36	1.3170	1.5366	1.3372	1.5062	1.3585	1.4774	1.3809	1.4501	24
37	1.3174	1.5361	1.3376	1.5057	1.3589	1.4769	1.3813	1.4496	23
38	1.3177	1.5356	1.3379	1.5052	1.3592	1.4764	1.3816	1.4492	22
39	1.3180	1.5351	1.3383	1.5047	1.3596	1.4760	1.3820	1.4487	21
40	1.3184	1.5345	1.3386	1.5042	1.3600	1.4755	1.3824	1.4483	20
41	1.3187	1.5340	1.3390	1.5037	1.3603	1.4750	1.3828	1.4479	19
42	1.3190	1.5335	1.3393	1.5032	1.3607	1.4746	1.3832	1.4474	18
43	1.3193	1.5330	1.3397	1.5027	1.3611	1.4741	1.3836	1.4470	17
44	1.3197	1.5325	1.3400	1.5022	1.3614	1.4736	1.3839	1.4465	16
45	1.3200	1.5319	1.3404	1.5018	1.3618	1.4732	1.3843	1.4461	15
46	1.3203	1.5314	1.3407	1.5013	1.3622	1.4727	1.3847	1.4457	14
47	1.3207	1.5309	1.3411	1.5008	1.3625	1.4723	1.3851	1.4452	13
48	1.3210	1.5304	1.3414	1.5003	1.3629	1.4718	1.3855	1.4448	12
49	1.3213	1.5299	1.3418	1.4998	1.3633	1.4713	1.3859	1.4443	11
50	1.3217	1.5294	1.3421	1.4993	1.3636	1.4709	1.3863	1.4439	10
51	1.3220	1.5289	1.3425	1.4988	1.3640	1.4704	1.3867	1.4435	9
52	1.3223	1.5283	1.3428	1.4983	1.3644	1.4699	1.3870	1.4430	8
53	1.3227	1.5278	1.3432	1.4979	1.3647	1.4695	1.3874	1.4426	7
54	1.3230	1.5273	1.3435	1.4974	1.3651	1.4690	1.3878	1.4422	6
55	1.3233	1.5268	1.3439	1.4969	1.3655	1.4686	1.3882	1.4417	5
56	1.3237	1.5263	1.3442	1.4964	1.3658	1.4681	1.3886	1.4413	4
57	1.3240	1.5258	1.3446	1.4959	1.3662	1.4676	1.3890	1.4408	3
58	1.3243	1.5253	1.3449	1.4954	1.3666	1.4672	1.3894	1.4404	2
59	1.3247	1.5248	1.3453	1.4949	1.3669	1.4667	1.3898	1.4400	1
60	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	1.3902	1.4395	0
′	49° cosec	49° sec	48° cosec	48° sec	47° cosec	47° sec	46° cosec	46° sec	′

	44°				44°				44°		
′	sec	cosec	′	′	sec	cosec	′	′	sec	cosec	′
0	1 3902	1 4395	60	21	1 3984	1 4305	39	41	1 4065	1 4221	19
1	1 3905	1 4391	59	22	1 3988	1 4301	38	42	1 4069	1 4217	18
2	1 3909	1 4387	58	23	1 3992	1 4297	37	43	1 4073	1 4212	17
3	1 3913	1 4382	57	24	1 3996	1 4292	36	44	1 4077	1 4208	16
4	1 3917	1 4378	56	25	1 4000	1 4288	35	45	1 4081	1 4204	15
5	1 3921	1 4374	55	26	1 4004	1 4284	34	46	1 4085	1 4200	14
6	1 3925	1 4370	54	27	1 4008	1 4280	33	47	1 4089	1 4196	13
7	1 3929	1 4365	53	28	1 4012	1 4276	32	48	1 4093	1 4192	12
8	1 3933	1 4361	52	29	1 4016	1 4271	31	49	1 4097	1 4188	11
9	1 3937	1 4357	51	30	1 4020	1 4267	30	50	1 4101	1 4183	10
10	1 3941	1 4352	50								
				31	1 4024	1 4263	29	51	1 4105	1 4179	9
11	1 3945	1 4348	49	32	1 4028	1 4259	28	52	1 4109	1 4175	8
12	1 3949	1 4344	48	33	1 4032	1 4254	27	53	1 4113	1 4171	7
13	1 3953	1 4339	47	34	1 4036	1 4250	26	54	1 4117	1 4167	6
14	1 3957	1 4335	46	35	1 4040	1 4246	25	55	1 4122	1 4163	5
15	1 3960	1 4331	45	36	1 4044	1 4242	24	56	1 4126	1 4159	4
16	1 3964	1 4327	44	37	1 4048	1 4238	23	57	1 4130	1 4154	3
17	1 3968	1 4322	43	38	1 4052	1 4233	22	58	1 4134	1 4150	2
18	1 3972	1 4318	42	39	1 4056	1 4229	21	59	1 4138	1 4146	1
19	1 3976	1 4314	41	40	1 4060	1 4225	20	60	1 4142	1 4142	0
20	1 3980	1 4310	40								
′	cosec	sec	′	′	cosec	sec	′	′	cosec	sec	′
	45°				45°				45°		

Greek Alphabet

Alpha (ăl′fa)	Α α	Nu (nū)	Ν ν
Beta (bā′ta)	Β β	Xi (ksē)	Ξ ξ
Gamma (găm′a)	Γ γ	Omicron (ŏm′ĭkrŏn)	Ο ο
Delta (dĕl′ta)	Δ δ or ∂	Pi (pī)	Π π
Epsilon (ĕp′sĭlŏn)	Ε ϵ	Rho (rō)	Ρ ρ
Zeta (zā′ta)	Ζ ζ	Sigma (sĭg′ma)	Σ σ or ς
Eta (ā′ta)	Η η	Tau (tô)	Τ τ
Theta (thā′ta)	Θ θ	Upsilon (ūp′sĭlŏn)	Υ υ
Iota (īō′ta)	Ι ι	Phi (fē)	Φ φ or ϕ
Kappa (kăp′a)	Κ κ	Chi (kē)	Χ χ
Lambda (lăm′da)	Λ λ	Psi (psē)	Ψ ψ
Mu (mū)	Μ μ	Omega (ō′mĕga)	Ω ω

ANSWERS

Page	Problem	Symbol	Variable	Answer
17	1	*D*	748653	6
	2	*E*	439267	4
	3	*F*	254273	5
	4	*G*	532581	6
	5	*H*	896247	0
	6	*J*	573862	0
	7	*K*	7823	4
	8	*L*	43875	8
	9	*M*	8236	4
19	1	*A*	.0982	$\frac{491}{5000}$
	2	*B*	9.542	$9\frac{271}{500}$
	3	*C*	.0053	$\frac{53}{10000}$
	4	*D*	$\frac{932}{1000}$	.932
	5	*E*	$\frac{31}{10000}$	.0031
	6	*F*	$\frac{95}{100}$	.95
	1	*A*	3.4	1.67
20	2	*A*	7.5323	5.8531
	3	*A*	11.746	.708
	4	*A*	.4631	5.9811
	5	*A*	4.6273	2.4077
21	6	*A*	2.4285	6.3368
	7	*A*	2.4285	3.2908
22	1	*A*	4.3927	38.015
	2	*B*	8.3576	90.730
	3	*C*	6.2594	154.46
	4	*D*	.73826	6.2727
	5	*E*	.87543	4.2875
	6	*F*	.46937	2.7999
	7	*G*	3.4278	23.075
	8	*H*	7.3492	59.231
	9	*J*	.93748	10.112
	10	*K*	1.9	58.872
24	1	*B*	6.3268	8.2591
	2	*C*	.85924	7.9892
	3	*D*	.09387	84.962
	4	*E*	8.1245	2.3875
	5	*F*	.83945	1.8458
	6	*G*	4.3768	1.5994

Page	Problem	Symbol	Variable	Answer
24	7	*H*	.89537	6.6872
	8	*J*	9.2843	121.29
	9	*K*	.07659	.87978
25	10	*L*	9.3854	.03958
	11	*M*	4.5876	1.6654
	12	*N*	.74382	.94319
	13	*P*	8.2953	.59016
	14	*R*	.09437	.09287
	15	*S*	2.4895	2.3557
	16	*T*	.06382	.43442
	17	*U*	19	.89473
	18	*V*	2186	11.566
	19	*W*	691	.37916
	20	*A*	.9738	2.3864
	21	*A*	12.438	1.8545
26	22	*A*	1.4	1.535
	23	*A*	.59286	1.3224
	24	*A*	.4183	1.2517
27	25	*A*	3.2	.5241
	26	*A*	2.1	.33
	27	*A*	2.2589	1.4759
	28	*A*	6.843	6.652
28	29	*A*	9.498	1.609
	30	*A*	1.73	3.78
	31	*B*	3.705	1.0775
	32	*B*	.085	11
32	1	*N*	16	.241
	2	*N*	11	1.261
	3	*N*	12	2.487
	4	*N*	15	3.665
33	5	*N*	10	.170
	6	*N*	15	.495
	7	*N*	12	.632
	8	*N*	.19	.539
	9	*N*	19	.759
	10	*N*	17	.837
36	1	*L*	8	6° 40′
	2	*L*	4	23° 20′
	3	*L*	11	37° 55′
	4	*L*	6	44° 30′
	5	*L*	10	53° 50′
	6	*L*	7	63° 35′
	7	*L*	11	76° 55′
	8	*L*	7	87° 35′

Page	Problem	Symbol	Variable	Answer
36	9	θ	26°	4° 20′
	10	θ	22°	2° 12′
	11	θ	25°	2° 22′
	12	θ	28°	1° 30′
37	13	U	12	2.137
	14	L	7	57° 35′
40	2	A	52	32
	3	B	28	88°
	4	C	22	5
41	7	D	10	4
	8	E	17	41
	9	F	92	−46
	10	G	19	14
	11	H	26	16
	12	J	24	42
	13	K	14	34
	14	L	45	−15
	15	M	17	−12
	16	N	30	17
	17	P	53	−14
	18	Q	11	−12
	19	R	16	−6
	20	S	17	22
	21	T	8	−9
	22	U	62	−11
	23	V	37	−108
	24	W	42	−21
	25	A	23	−16
43	1	M	78	80
	2	N	12.8	104
	3	L	98	93
	4	K	88	85.5
	5	P	25	121
	6	Q	104	113
	7	R	18	17
	8	A	8	−165
	9	B	2	−56
	10	C	15	388
44	11	D	8	−471
	12	E	20	−5870
	13	F	27	21
	14	G	11	71
	15	H	19	−27
	16	J	20	61

Page	Problem	Symbol	Variable	Answer	
44	17	*S*	17	−390	
	18	*T*	6	85	
54	1	*N*	32	1.3125	
	2	*M*	20	12	
	3	*R*	65	31.787	
	4	*T*	16.75	$10.659	
	5	*S*	11.325	.661	
55	6	*F*	20.2	38.037	
	7	*G*	2.458	.4115	
	8	*K*	67	$\frac{35}{67}$	
	9	*L*	38	21.966	
	10	*H*	38	$\frac{3}{268}$	
	11	*D*	$\frac{8}{11}$	$\frac{10}{11}$	
	12	*C*	53	1.1886	
	13	*P*	57	20.357	
	14	*P*	98	333.2	
57	1	*B*	1280	2596.1	
	2	*C*	63	854.49	
	3	*D*	5.875	1.7279	
	4	*E*	14	24.444	
	5	*G*	105.5	71.990	
	6	*R*	375.5	44.08	
	7	*S*	27.5	2799.2	
	8	*Q*	28600	2.2309	
58	9	*T*	350.5	1602.2	
	10	*H*	.645	.129	
59	1	*N*	21	28.767 %	
	2	*L*	225	9.3333 %	
	3	*G*	$\frac{2}{9}$	77.777 %	
	4	*M*	10	130	
	5	*H*	29	966.66	
	6	No Variable		1.042 %	
	7	*F*	13.1	93.893 %	
	8	*C*	8.25	$6.64	
	9	*S*	59.75	$36.12	
	10	*T*	168	.01176	
	11	*N*	48.25	$53.459	
60	12	*D*	.965	.96443	
				Tin	8.19
	13	*A*	9.75	Copper	.4875
				Antimony	.975
				Lead	.0975
	14	*B*	519	Bismuth	259.5
				Lead	129.75

Page	Problem	Symbol	Variable	Answer	
60	14	*B*	519	Tin	64.875
				Cadmium	64.875
61	15	*E*	1995	798	
	16	*J*	255	Yellow	63.75
				Green	19.125
				Red	0
				Black	12.75
				Blue	31.875
63	1	*A*	6.7	3.4388	
	2	*B*	3.12	.76923	
	3	*C*	12.1	1.3884	
	4	*D*	6.7	17.42	
64	5	*E*	16.3	.9196	
	6	*F*	11.2	.84625	
	7	*G*	.468	1.8756	
	8	*H*	.406	.50246	
65	9	*J*	1.75	.00102	
	1	*F*	.615	.61536	
	2	*G*	4.625	2.9143	
	3	*H*	31	34.065 %	
	4	*J*	77.2	989.74	
	5	*K*	579.89	26.674	
	6	*L*	165.9	11.115	
	7	*S*	33.4	.57485	
	8	*T*	6900	23.474	
	9	*M*	.545	.71322	
66	10	*N*	3.500	.53485	
	11	*F*	.328	3.5367	
	12	*G*	.663	1.4956	
67	13	*H*	4.25	18.288	
	14	*J*	34	18.307	
	15	*K*	11.25	46.875 %	
	16	*L*	20	$168.00	
	17	*M*	97	3055.5	
	18	*N*	16	$49.12	
	19	*P*	74.75	$51.839	
	20	*Q*	909	3370.0	
71	1	*A*	38296	195.69	
	2	*B*	642934	801.83	
	3	*C*	29 ÷ 43	.82122	
	4	*D*	62895	250.78	
	5	*E*	46.658	6.8306	
	6	*F*	.00547	.07395	
	7	*G*	9.5386	3.0884	

Page	Problem	Symbol	Variable	Answer
71	8	H	537.69	23.188
	9	J	.00367	.06058
	10	K	.36528	.60438
	11	L	.05986	.24466
	12	M	19.473	4.4128
	13	N	.000084	.00916
	14	P	.85423	.92424
	15	P	91.876	9.5851
	16	S	6329.2	79.556
	17	N	59.875	7.7378
	18	P	6.7982	2.6073
	19	R	.26574	.51549
	20	S	$\frac{573}{726}$	.88839
	21	A	302.68	17.397
	22	B	8.762	2.9500
	23	C	12.381	3.5186
	24	D	21.296	4.6149
72	25	T	5.8767	1.9793
	26	U	8.9326	1.9924
73	1	c	17.8	161.2
	2	r	9.8	19.045
74	3	s	9	14.345
	4	B	8.9	13.6
	5	n	21	5.1754
	6	P	14	.04764
	7	N	62	2.5625
	8	R	10	13
	9	P	9	.29508
	10	T	35	13.221
	11	B	9.7	33.184
	12	B	8.9	313.22
	13	N	27	3.7556
	14	L	9.3	1.6468
	15	D	21.7	20.434
	16	N	43	7.5079
75	17	H	8.5	39.1
	18	D	5.3	30.151
	19	H	8.5	6.0762
	20	B	19.9	6.2956
	21	D	1.988	1.706
	22	D	1.988	.032
	23	F	2.487	2.8717
	24	F	3.7	4.2723

Page	Problem	Symbol	Variable	Answer
75	25	*E*	2.225	3.1465
	26	*E*	2.458	3.4761
76	27	*H*	1.27	1.8407
	28	*H*	.687	2.5723
	29	*d*	.987	1.6055
	30	*d*	.758	1.262
	31	*W*	.571	.31525
	32	*W*	.783	.63325
77	33	*D*	3.125	3.3976
	34	*D*	2.375	2.6476
	35	*T*	9	1.5336
	36	*L*	11.7	3.3832
99	1	*A*	6.82	56.7
	2	*B*	146.	47.5
	3	*C*	7.42	7.99
	4	*D*	14.44	.493
	5	*E*	34.7	93.7
	6	*F*	.317	383.
	7	*G*	4.21	17.7
	8	*H*	42.1	1770.
	9	*I*	66.4	8.15
	10	*J*	664.	25.8
	11	*K*	12.6	146.
	12	*L*	28.9	5.00
	13	*M*	49.4	.267
	14	*N*	7.42	1.75
	15	*P*	8.47	17.2
	16	*Q*	2.18	86.1
	17	*R*	.132	18.0
	18	*S*	6.41	999.
	19	*T*	9.68	2.30
	20	*U*	34.7	.319
	21	*V*	6.31	2.06
	22	*W*	5.72	4.12
	23	*A*	28.6	.0537
	24	*B*	31.7	2.40
	25	*C*	19.4	14700.
	26	*D*	94.6	.286
	27	*E*	462.	5.27
	28	*F*	28.7	36.1
	29	*G*	30.8	1620.
	30	*H*	43.2	1.03
	31	*J*	3° 12′	.0558
	32	*K*	37° 20′	.606

Page	Problem	Symbol	Variable	Answer
207	11	*P*	21.26	17.968
	12	*Q*	9.147	68° 44′ 3″
	13	*R*	10.39	49° 12′ 2″
	14	*S*	3.395	11.377
228	1	*J*	11.7	35° 19′ 40″
	2	*J*	11.7	6.6737
	3	*U*	6.68	7.4529
	4	*U*	6.68	7.9652
	5	*T*	9.53	11.682
	6	*P*	9.97	12.115
	7	*S*	15.8	22.168
	8	*F*	4.23	5.5374
233	1	*G*	10.9	99° 28′ 18″
	2	*G*	10.9	31° 3′ 7″
	3	*H*	18.9	57° 55′ 1″
	4	*H*	18.9	73° 30′ 58″

INDEX

Q

R

S

T

V